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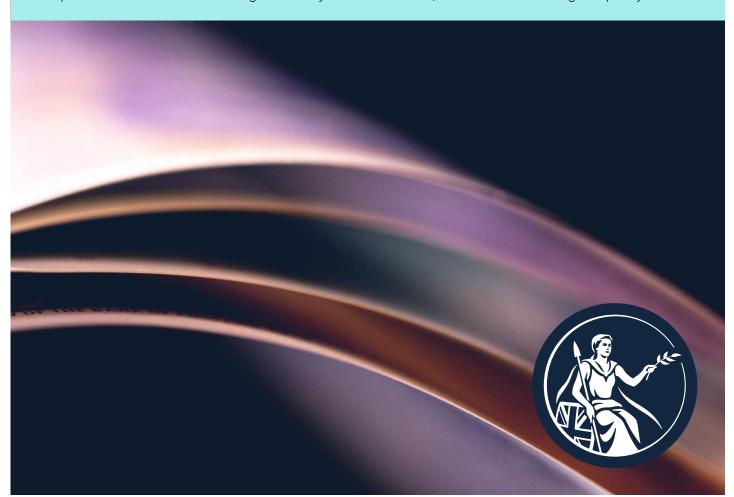
# Trade fragmentation, inflationary pressures and monetary policy

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# Trade fragmentation, inflationary pressures and monetary policy

Ludovica Ambrosino,<sup>(1)</sup> Jenny Chan<sup>(2)</sup> and Silvana Tenreyro<sup>(3)</sup>

#### **Abstract**

How does trade fragmentation affect inflationary pressures? What is the response of monetary policy needed to sustain inflation at target? To answer these questions, we develop a two-sector, small open-economy model featuring imperfect international risk-sharing and household heterogeneity that captures both the supply-side and demand-side effects of fragmentation. The impact of fragmentation on inflationary pressures, and the appropriate policy response, depend not only on the direct effect of higher import prices on supply but, crucially, on how aggregate demand adjusts in response to lower real incomes and productivity. In turn, this depends on the pace of fragmentation (whether it is gradual or front-loaded), as well as several other structural factors elucidated by the model analysis. We compare the outcomes resulting from a central bank following Taylor-type monetary policy rules to a constrained-efficient allocation.

**Key words:** Monetary policy, trade fragmentation, open economies, inflation, heterogeneity, globalisation.

JEL classification: F12, F15, F41, F62.

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#### 1 Introduction

Global trends have shifted noticeably in recent decades. The protracted postwar increase in trade openness has stalled, amidst a resurgence in trade wars and protectionism. This shift is visible in Figure 1, which plots the long-term trajectory of global trade flows relative to world GDP, as well as in Figure 2, which tracks a broad index of economic fragmentation since the 1970s (Fernández-Villaverde, Mineyama, and Song (2024)). Both figures show a turning point around the global financial crisis, with trade openness plateauing and fragmentation steadily increasing, before rising sharply during the pandemic and Russia's invasion of Ukraine. The geopolitical factors driving these changes are likely to persist. New trade paradigms, such as friendshoring or fragmentation into trading blocs of geopolitically aligned countries are becoming normalised (Yellen (2022)). This reconfiguration of trade patterns raises concerns about potential losses in efficiency and aggregate output (Javorcik, Kitzmüller, Schweiger, and Yıldırım (2024); Georgieva (2023)).

A key question for policy makers is how trade fragmentation will affect inflationary pressures, and, in turn, what should be the monetary policy response. The conventional view suggests that as nations retreat from global integration and supply chains duplicate, production costs will rise, exerting upward pressure on inflation (e.g., Lagarde (2023), Goodhart and Pradhan (2020)). Indeed, disinflationary trends of the 1990s and 2000s have often been linked to the rapid increase in trade integration during that period, leading to expectations that its reversal will be inflationary. However, this relationship remains contentious. Other forces besides globalisation may have contributed to the era of disinflation, including the shift to inflation-targeting regimes (Roberts (2006)) and the lower bound constraint on interest rates in many countries (Attinasi and Balatti (2021)). Taking the United States as an example, estimates of the disinflationary effects of increased trade integration appear modest at best (Yellen (2006)).<sup>1</sup>

The conventional view is built around the direct or partial-equilibrium impact of trade integration on supply, abstracting from its indirect impact on aggregate demand. This paper addresses this gap by studying the broader, general equilibrium effects of trade fragmentation in a setting in which aggregate demand is also affected through changes in real incomes. We model fragmentation in two ways: the first consists of an increase in the price of imported goods (which could result, for example, when firms switch from the cheapest or more efficient suppliers to a geopolitically-aligned, but more expensive supplier; or when tariffs or non-tariff barriers are imposed). The second form of fragmentation entails a fall in tradable sector productivity in the domestic economy (relative to its original trend). Through these alternative forms of fragmentation, we illustrate how the inflationary impact

<sup>&</sup>lt;sup>1</sup>Kamin, Marazzi, and Schindler (2004), for example, show that the impact of Chinese exports on global prices has been fairly modest. Moreover, these studies do not explicitly take into account real exchange-rate adjustments. As Kohn (2005) argues, during the second half of the 1990s, the dollar experienced a substantial appreciation, driven by increased investment flows drawn to the prospect of higher productivity growth. This may have amplified the downward trend in dollar prices of U.S. imports.

Figure 1: Sum of exports and imports,% of GDP

*Source*: Peterson Institute for International Economics; Jordà-Schularick-Taylor Macrohistory Database; Penn World Table (10.0); World Bank; OWID.

depends crucially on the adjustment of aggregate demand. Higher import prices or lower tradable sector productivity not only constrain supply through higher marginal costs, but also demand through lower real incomes and consumption - the general equilibrium effects. Consequently, the net impact on inflationary pressures is a priori ambiguous.

We capture these competing channels in a two-sector (tradable and non-tradable), small open economy New Keynesian model featuring household heterogeneity and home bias in consumption. Specifically, following Debortoli and Galí (2017), the economy features two types of consumers. The first type consists of *unconstrained* agents, with access to international and domestic security markets. To account for frictions in international financial markets, we introduce imperfect international risk sharing: unconstrained agents trade risk-free foreign bonds, with a convex cost of holding assets in quantities that deviate from some long-run level (Schmitt-Grohé and Uribe (2003)). The second type consists of *constrained* hand-to-mouth households, who consume only out of their labour income and have no access to financial markets. The domestic economy trades with the rest of the world, importing goods for direct consumption, for use as intermediate inputs, or both.

We consider three scenarios to show how different forms of fragmentation can have distinct macroeconomic implications. First, we consider a *gradual* (and permanent) increase in the price of imported goods. This yields a persistent increase in imported inflation, which lasts until the import price stabilises at a higher level in the medium-to-longer term. Aggregate consumption falls in response to fragmentation, as both financially constrained and unconstrained households suffer losses in real income: the real disposable income of hand-to-

1.5 - Fragmentation Index

1.0 - 0.5 - 0.5 - 1975 1980 1985 1990 1995 2000 2005 2010 2015 2020

Figure 2: Fragmentation has increased since 2008.

Source: Fernández-Villaverde, Mineyama, and Song (2024)

mouth consumers falls as a direct consequence of higher prices, restricting their spending; in turn, financially unconstrained households, who take into account their permanent-income losses, also reduce their consumption in anticipation of lower future incomes. This further accentuates the fall in aggregate demand, spilling over to hand-to-mouth consumers. Real wages fall both because of the negative terms-of-trade effect and because of the fall in domestic demand. Financially constrained households make up for some of the fall in income by increasing their labour supply. The fall in aggregate demand pushes down on domestic inflation. Aggregate CPI (consumer price index) inflation, a composite of domestic and imported goods inflation, falls, given the larger weight of domestic components on the basket. The reduction in demand is reflected in the real natural rate of interest, which decreases with the fragmentation shock. This suggests that when demand adjusts, the overall effect is not inflationary. This scenario leads to a long period of stagnation, with weak demand and subdued inflation. In this setting, monetary policy needs to loosen in order to bring inflation back to target.

Next, we consider a fully *front-loaded*, permanent increase in the price of imported goods.<sup>2</sup> The shock creates a sharp temporary trade-off, with inflation increasing and aggregate demand falling on impact. Both financially unconstrained and constrained households lower

<sup>&</sup>lt;sup>2</sup>This is akin to the recent U.S. and E.U. tariffs on Chinese electric vehicles, reaching up to 100 percent and 38 percent, respectively, as well as the increase in energy prices after Russia's invasion of Ukraine.

their consumption. The fall in real wages (relative to the price of imported inputs) triggers an increase in labour supply from both types of consumers. On impact, the short-term real interest rate increases, as a tightening in monetary policy is required to bring inflation back to target. The result is a temporary overshoot in inflation, with longer-term losses in income and consumption.

Finally, we study a *gradual and permanent* fall in the total factor productivity (TFP) of tradable goods as a potential consequence of increased fragmentation. As in the standard New Keynesian model, this shock results in the following changes in the domestic tradable sector: more employment per unit of output and an increase in inflation as marginal costs rise. Compared to the gradual import price shock, a gradual deterioration in TFP redistributes resources differently. Real wages do not fall by as much, which mitigates the fall in consumption. Aggregate employment also increases by less, suggesting that labour income is less adversely affected by a TFP shock, relative to an import price shock with a similar trajectory. In this scenario, the fall in non-tradable inflation is not enough to outweigh the increase in home tradable inflation and aggregate CPI inflation increases moderately. In principle, the impact of this shock on the natural real rate is ambiguous. In our calibration, there is a small decrease in the natural rate, a fall in non-tradable components of inflation and a moderate, temporary increase on CPI inflation.

In summary, all three fragmentation scenarios lead to a contraction in aggregate supply, but their effects on demand differ. Conventional assessments of the impact of fragmentation on inflation often abstract from the demand-side or general-equilibrium impact that fragmentation can have through lower real incomes, focusing primarily on its adverse supply-side effects. While the direct (or partial equilibrium) effect of fragmentation might be inflationary, the general equilibrium effect could dampen inflation, as lower real incomes weigh on aggregate demand. The effects of fragmentation on inflation dynamics and the direction of monetary policy cannot be decoupled from its impact on the natural real interest rate ( $r^*$ ). As trade fragmentation affects the desired levels of savings and spending, the balance between these supply and demand forces ultimately determines the sign and size of changes in the natural rate of interest.

To build intuition, we start the analysis with a representative-agent New Keynesian (RANK) version of our model as a special case of our TANK baseline model, where there are no constrained households. Next, we compare the RANK model to our TANK baseline: the presence of hand-to-mouth households leads to a larger fall in consumption and a smaller fall in output. This is because constrained households increase their labour supply more than unconstrained ones in response to the import price shocks. Our conclusions across the three scenarios remain the same and in particular, the demand adjustment in the gradual scenario is still sufficient to lower domestic inflationary pressures and offset the increase in imported goods inflation.

To sharpen our understanding of the underlying mechanisms, we vary two key parameters in our simulations: the degree of home bias in consumption and the Cobb-Douglas

weight of foreign inputs in domestic production. The extent of home bias in consumption appears to play an important role. More open economies are more exposed to shocks in foreign prices, which is reflected in the responses of consumption and production. In the scenarios with a persistent increase in foreign prices, we see a deeper fall in the natural real rate in the more open economy. However, in the case of a negative and permanent TFP shock, openness mitigates supply pressures by allowing for diversification away from home tradables and facilitating reallocation between the two sectors domestically. The permanent nature of the shock also affects the consumption of forward-looking households, which weighs on demand.

We show that our main results are robust to extensions that incorporate additional domestic supply-side constraints: a higher share of imported inputs in domestic production and wage rigidities. Increased reliance on imported inputs leads to higher employment in our various scenarios, driven by factor substitution as import prices increase or as productivity deteriorates. However, the demand-side impact remains similar to our baseline calibration, where labour demand in the non-tradable sector already starts from a higher level.

An extension of the RANK model with nominal wage rigidities moderates the degree of disinflationary pressures in the gradual fragmentation scenario and worsens the policy trade-off in the front-loaded scenario. However, the demand-side impacts are largely unchanged, as a more modest decline in real wages is offset by a sharper fall in output and employment.

Finally, following Drechsel, McLeay, Tenreyro, and Turri (2025), we benchmark the small open economy's response to various fragmentation shocks against a constrained-efficient allocation to isolate the sources of inefficiencies in the decentralised equilibrium. Nominal frictions that delay the adjustment of non-tradables prices to shocks and frictions in the adjustment of foreign bond holdings limit the efficient reallocation of resources within and across sectors, as well as intertemporally and externally in response to shocks. As a result, there is too much reallocation towards home tradables following the import price shocks and too little towards non-tradables after an adverse productivity shock in the home tradable sector.

Related Literature We build on a rich literature studying monetary policy in small open economies (SOEs), including the seminal work of Benigno and Benigno (2003) and Gali and Monacelli (2005). Other important contributions to this line of research include but are not limited to, Santacreu (2005) and De Paoli (2009), who study tradable and non-tradable sectors in SOEs, and Schmitt-Grohé and Uribe (2003), who introduce imperfect international risk sharing and price stability.

We also draw on an extensive literature that studies the impact of external shocks on macroeconomic outcomes using structural models, such as Romero (2008), Catao (2013), Hevia and Nicolini (2013), Wills (2014), Bergholt (2014), Ferrero and Seneca (2019), Drech-

sel, McLeay, and Tenreyro (2019), Siena (2021), Comin, Johnson, and Jones (2023), Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro (2023), Guerrieri, Marcussen, Reichlin, and Tenreyro (2023) and Nispi Landi and Moro (2024).<sup>3</sup> Recent contributions to this literature have explored the transmission of external shocks in models with household heterogeneity (Ferra, Mitman, and Romei (2020), Auclert, Rognlie, Souchier, and Straub (2021), Auclert, Monnery, Rognlie, and Straub (2023), and Chan, Diz, and Kanngiesser (2024)). As in Comin and Johnson (2020), we highlight general-equilibrium effects. Our focus is on fragmentation rather than offshoring, and our scenarios capture a broader range of possibilities: instead of modelling a transition to a steady state with a different degrees of openness, we consider fragmentation shocks that differ in the speed of adjustment, their origin (domestic versus foreign), and the variable directly affected (foreign input prices versus productivity).

Finally, we build on the vast literature that has examined the macroeconomic effects of globalisation. While increased competition in import prices has placed downward pressure on prices of manufactured goods, studies have shown that globalisation has had a negative, but economically small (if not negligible) effect on core inflation (Carluccio, Gautier, and Guilloux-Nefussi (2023)). Moreover, there is evidence that global disinflationary forces, such as the shift to inflation targeting regimes (ECB (2021), Roberts (2006)) or the lower bound constraint on interest rates (Attinasi and Balatti (2021)) may offer a better explanation for the observed disinflationary trends. Theoretical results provide support for these findings. Sbordone (2008) shows that in a model in which firms' desired markup is a function of its relative market share, an increase in the number of traded goods can generate real rigidities that affect the slope of the Phillips curve. As the economy reaches a steady state with higher trade, the elasticity of demand that firms face increases, but the elasticity of the desired markup declines. These opposing forces determine how the inflation-marginal cost component of the Phillips curve slope varies. Estimates on trade data from 1960 to 2006 suggest that it remains uncertain whether trade growth observed during the globalisation era is sufficient to have driven a decline in this component of the slope. On the empirical front, Chen, Imbs, and Scott (2009) provide evidence of short-run pro-competitive effects from increased openness. They also show that trade liberalization can have ambiguous effects in the long run, as firms can respond to increased competition by locating to protected markets.4

<sup>&</sup>lt;sup>3</sup>The balance between global demand and supply pressures shapes aggregate outcomes and inflationary dynamics. Guerrieri, Lorenzoni, and Werning (2025) show that while individual central banks typically take global supply conditions as given, their collective actions influence global demand and the transmission of supply shocks to global inflation.

<sup>&</sup>lt;sup>4</sup>Recent developments in global trade policy have motivated a growing body of research on the macroeconomic effects of tariffs (Bergin and Corsetti (2023), Bianchi and Coulibaly (2025), Meng, Russ, and Singh (2023), Campos, Estefania-Flores, Furceri, and Timini (2023), Auclert, Rognlie, and Straub (2025), Kalemli-Özcan, Soylu, and Yildirim (2025), Werning, Lorenzoni, and Guerrieri (2025), Cuba-Borda, Queralto, Reyes-Heroles, and Scaramucci (2025), Mehrotra and Waugh (2025), Mix and Hoang (2025), and Gnocato, Montes-Galdón, and Stamato (2025)). This paper studies the macroeconomic consequences of a realignment in global trading patterns that may be less efficient overall. While tariffs can contribute to this inefficiency, realignment may also

Our paper also relates to the strand of literature pioneered by Rogoff (2003) and Rogoff (2007), which looks at how economic integration affects global inflationary trends. We abstract, however, from the political economy factors studied by Afrouzi, Halac, Rogoff, and Yared (2024), who suggest that globalisation would worsen the trade-offs faced by central banks, leading them to succumb to political pressures and deviate from or abandon their inflation targets. The question we ask in this paper is a different one: what would it take for central banks to bring inflation back to target under different fragmentation scenarios? As we show, in some scenarios, activity and inflation both fall, leading to stagnation (that is, without a trade-off); in others, activity and inflation move in opposite directions, creating short-term trade-offs or temporary stagflation. What is required of monetary policy to return inflation to target depends on how aggregate demand responds to lower incomes in general equilibrium. This is contingent on a number of structural parameters that we consider, as well as on the trajectory of fragmentation, particularly on the extent to which the impact on import prices is gradual or front-loaded.

**Outline** Section 2 develops our theoretical framework. Section 3 calibrates the model and analyses shocks that are linked to trade fragmentation in a RANK and TANK framework. Section 4 varies key parameters: the relative importance of home bias and the utilisation of imported inputs in domestic production. Section 5 studies the efficient allocation of a benevelent social planner. Section 6 studies an extension with nominal wage rigidity. Finally, Section 7 presents concluding remarks and potential directions for future research.

#### 2 Baseline Model

The goal of this section is to deliver qualitative insights into shocks associated with deglobalisation. We present a small open economy model that builds on Drechsel, McLeay, and Tenreyro (2019) and Ferrero and Seneca (2019). To capture a more realistic response of aggregate demand to international shocks, we introduce constrained and unconstrained households as in Debortoli and Galí (2017). Finally, to study the impact of fragmentation, we introduce an imported input used in the production of domestic goods. Figure 3 presents an illustration of the model described in this section.

#### 2.1 Households

There is a continuum of households with identical preferences at any given point in time t. A constant measure  $(1-\lambda)$  of households are *unconstrained* (U): they are able to smooth

reflect a reorientation toward less efficient suppliers, making our framework applicable to a range of factors that may increase import costs.

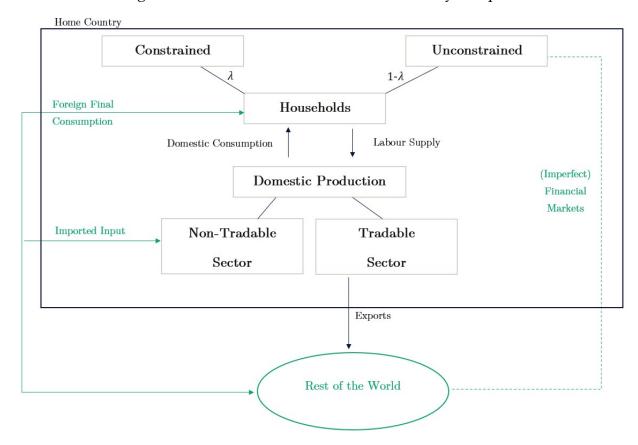


Figure 3: Model Structure from Home Country Perspective.

consumption through their access to international and domestic financial markets. The remaining fraction ( $\lambda$ ) of households are fully *constrained* (C), meaning that they have no access to financial markets.

Each type of household  $j \in \{U, C\}$  consumes  $C_t^j$  and supplies labour  $N_t^j$ , at wage  $W_t$ , leading to an expected utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t^j)^{1-\sigma}}{1-\sigma} - \kappa_\ell \frac{(N_t^j)^{1+\phi}}{1+\phi} \right\}.$$

The parameters  $\beta$ ,  $\sigma$ , and  $\phi$  capture the discount factor, the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity, respectively, while  $\kappa_{\ell}$  is the disutility weight placed on labour.

**Unconstrained households:** The period budget constraint of these households is given by:

$$P_{t}C_{t}^{U} + B_{t} + \mathcal{E}_{t}B_{t}^{*} = B_{t-1}(1 + i_{t-1}) + \mathcal{E}_{t}B_{t-1}^{*}(1 + i_{t-1}^{*}) + W_{t}N_{t}^{U} + P_{t}\Psi_{t} - \frac{\chi}{2}\mathcal{E}_{t}P_{t}^{*}\left(\frac{B_{t}^{*}}{P_{t}^{*}} - b^{*}\right)^{2}$$

$$\tag{1}$$

where  $P_t$  is the aggregate price level,  $B_t$  denotes the holdings of a risk-free one-period nominal bond in domestic currency, which pays the nominal interest rate  $i_t$ . The risk-free one-

period nominal bond in foreign currency is denoted by  $B_t^*$ , where  $i_t^*$  is the foreign interest rate and  $\mathcal{E}_t$  is the nominal exchange rate (expressed in terms of domestic currency relative to foreign currency). The real profits from firms in both tradable and non-tradable sectors  $(\Psi_t)$  are rebated to the unconstrained agents. Following Schmitt-Grohé and Uribe (2003), we assume that there is a quadratic cost in changing the real bond position relative to a real steady-state value  $(b^*)$  when trading in the foreign bond market. These costs are a common feature of small open economy models to ensure that the model returns to a unique steady-state net foreign asset position following a transitory shock. This cost (in units of the consumption index) is denoted by a non-negative parameter,  $\chi$ , while  $P_t^*$  is the aggregate price level in the foreign country. For simplicity, we can rewrite the budget constraint in real terms as

$$C_t^{U} + b_t + \mathcal{S}_t b_t^* = b_{t-1} \frac{(1+i_{t-1})}{(1+\pi_t)} + \mathcal{S}_t b_{t-1}^* \frac{(1+i_{t-1}^*)}{(1+\pi_t^*)} + w_t N_t^{U} + \Psi_t - \frac{\chi}{2} \mathcal{S}_t (b_t^* - b^*)^2$$
 (2)

where  $b_t = \frac{B_t}{P_t}$ ,  $b_t^* = \frac{B_t^*}{P_t^*}$ ,  $w_t = \frac{W_t}{P_t}$ ,  $S_t = \frac{\mathcal{E}_t P_t^*}{P_t}$  is the real exchange rate, and  $\pi_t = \frac{P_t}{P_{t-1}} - 1$  is the net inflation rate.

Unconstrained households maximise their expected lifetime utility by choosing a sequence  $\{C_t^U, N_t^U, b_t, b_t^*\}_{t=0}^{\infty}$  subject to the series of budget constraints (2). Consequently, the optimality conditions are as follows,

$$\kappa_{\ell}(N_t^U)^{\phi} = (C_t^U)^{-\sigma} \frac{W_t}{P_t} \tag{3}$$

$$\frac{1}{(1+i_t)} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1}{(1+\pi_{t+1})} \right]$$
(4)

$$[1 + \chi(b_t^* - b^*)] = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1 + i_t^*}{(1 + \pi_{t+1}^*)} \frac{S_{t+1}}{S_t} \right]$$
 (5)

where  $\Pi_{t+1} = (1+\pi_{t+1}) = \frac{P_{t+1}}{P_t}$  is the gross inflation rate. We define  $\Lambda^U_{t,t+1} = \beta \left(\frac{C^U_{t+1}}{C^U_t}\right)^{-\sigma}$  as the relevant stochastic discount factor, since only the unconstrained households have access to financial markets. The household's optimality condition for labour yields the labour supply relation (3). The Euler equation (4) follows from the first order condition for  $b_t$ . Finally, households' choices of foreign and domestic bonds give rise to an uncovered interest rate parity condition, which links the expected change in the exchange rate to the differential between the domestic and foreign interest rates. The conditions on  $b_t$  and  $b_t^*$  imply equation (5), the deviation from the uncovered interest-rate parity (UIP). According to this equation, international risk sharing will generally be imperfect, and aggregate demand across countries will fluctuate inefficiently.

$$\chi(b_t^* - b^*) = \mathbb{E}_t \left[ \Lambda_{t,t+1}^U \left( \frac{(1+i_t^*)}{(1+\pi_{t+1}^*)} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} - \frac{(1+i_t)}{((1+\pi_{t+1}))} \right) \right].$$

**Constrained households:** These households do not have access to financial markets and therefore they cannot smooth their consumption over time without adjusting their leisure. Instead, they only consume their labour income in each period,

$$P_t C_t^C = W_t N_t^C$$

$$C_t^C = \frac{W_t}{P_t} N_t^C \tag{6}$$

and they supply their labour optimally,

$$\kappa_{\ell}(N_t^C)^{\phi} = (C_t^C)^{-\sigma} \frac{W_t}{P_t}.$$
 (7)

Aggregate consumption is a weighted average of the consumption of the two types of households,  $C_t = (1 - \lambda)C_t^U + \lambda C_t^C$ . Similarly, aggregate labour is given by  $N_t = (1 - \lambda)N_t^U + \lambda N_t^C$ .

As in Santacreu (2005), aggregate consumption is a CES aggregate of both tradable (T) and non-tradable (N) goods:

$$C_{t} \equiv \left[ (1 - \varsigma)^{\frac{1}{\iota}} C_{T,t}^{\frac{\iota - 1}{\iota}} + \varsigma^{\frac{1}{\iota}} C_{N,t}^{\frac{\iota - 1}{\iota}} \right]^{\frac{\iota}{\iota - 1}},$$

where  $\varsigma$  is the share of non-tradable goods in domestic consumption.  $C_{T,t}$  is a CES aggregate of tradable goods produced in the domestic and foreign economy,

$$C_{T,t} = \left[ (1 - \theta)^{\frac{1}{\mu}} C_{H,t}^{\frac{\mu - 1}{\mu}} + \theta^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu - 1}{\mu}} \right]^{\frac{\mu}{\mu - 1}}, \tag{8}$$

where  $1 - \theta$  captures the home bias: smaller values of  $\theta$  imply that the economy consumes less foreign goods.

The aggregate CPI price index ( $P_t$ ) and the tradable good price index ( $P_{T,t}$ ) are respectively given by

$$P_{t} \equiv \left[ (1 - \varsigma) P_{T,t}^{1-\iota} + \varsigma P_{N,t}^{1-\iota} \right]^{\frac{1}{1-\iota}}$$

$$P_{T,t} \equiv \left[ (1 - \theta) P_{H,t}^{1-\mu} + \theta P_{F,t}^{1-\mu} \right]^{\frac{1}{1-\mu}}$$

where  $P_{F,t} = \mathcal{E}_t P_{F,t}^*$  and  $P_{F,t}^*$  follows an AR(1) process.<sup>5</sup> In our simulations, we will study different processes for this variable to capture the increase in fragmentation.

Aggregate prices are a function of the prices for both tradable and nontradable goods. In turn, tradable prices are a composite of prices for domestically-produced tradable goods

<sup>&</sup>lt;sup>5</sup>See Appendix A for the specifications of all exogenous variables.

and foreign-produced tradable goods, with weights reflecting the degree of home bias. The non-tradable goods and price index are given, respectively, by

$$C_{N,t} \equiv \left(\int_0^1 C_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}, \quad P_{N,t} \equiv \left(\int_0^1 P_{N,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

where  $\epsilon$  captures the substitutability across different nontradable consumption varieties i. Total consumption expenditure by households is given by the sum of expenditures on domestic and foreign goods,

$$P_tC_t = P_{T,t}C_{T,t} + P_{N,t}C_{N,t} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t}$$

The system of demand functions is given by

$$C_{H,t} = (1 - \theta) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\mu} C_{T,t}$$

$$C_{F,t} = \theta \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\mu} C_{T,t}$$

$$C_{N,t} = \varsigma \left(\frac{P_{N,t}}{P_t}\right)^{-\iota} C_t$$

$$C_{T,t} = (1 - \varsigma) \left(\frac{P_{T,t}}{P_t}\right)^{-\iota} C_t$$

$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\epsilon} C_{N,t}.$$

The terms of trade are defined as the price of imports in terms of the price of domestic goods,

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}.\tag{9}$$

#### 2.2 Firms

Households supply labour to both the tradable and non-tradable sectors, such that

$$N_t = N_{H,t} + N_{N,t} = \lambda N_t^C + (1 - \lambda) N_t^U.$$
 (10)

Labour is completely mobile across sectors, therefore there is only one wage rate in equilibrium.

#### 2.2.1 Non-tradable goods sector

**Final Goods Producers** Competitive final goods producers assemble intermediate goods  $Y_{N,t}(i)$ , where  $P_{N,t}(i)$  is the price charged by the individual firm producing variety i. Their optimisation problem is to choose  $\{Y_{N,t}(i)\}, i \in [0,1]$  to maximise profits

$$\max_{Y_{N,t}(i)} \left\{ P_{N,t} Y_{N,t} - \int_0^1 P_{N,t}(i) Y_{N,t}(i) di \right\}$$

subject to an aggregation technology with constant elasticity of substitution,

$$Y_{N,t} = \left(\int_0^1 Y_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

where  $P_{N,t}(i)$  is the price charged by the individual firm i. Profit maximization, taking as given the final goods price  $P_{N,t}$  and the prices for the intermediate goods  $P_{N,t}(i)$ , yields the set of demand schedules

$$Y_{N,t}(i) = \left[\frac{P_{N,t}(i)}{P_{N,t}}\right]^{-\epsilon} Y_{N,t}.$$
(11)

**Intermediate Goods Producers** Intermediate goods firms use labour and an *intermediate imported input*  $M_{F,t}$  in production,

$$Y_{N,t}(i) = A_{N,t} M_{F,t}^{\kappa}(i) N_{N,t}^{1-\kappa}(i)$$
(12)

where  $A_{N,t}$  is the exogenous sector-specific productivity.

Firms are monopolistically competitive and adjust prices according to Rotemberg (1982), incurring an adjustment cost each time,

$$AC_t(i) = \frac{\xi}{2} \left( \frac{P_{N,t}(i)}{P_{N,t-1}(i)} - \Pi_N \right)^2 Y_{N,t} P_{N,t},$$

where  $\xi$  summarises the degree of nominal rigidity in the economy. Let  $\Pi_N$  denote steady state non-tradable inflation. The total factor cost function is equal to

$$TC_{t}(i) (W_{t}, Y_{N,t}(i), P_{F,t}, A_{N,t}) = (1 - \tau) (W_{t}N_{N,t} + P_{F,t}M_{F,t})$$

$$= (1 - \tau) \left(\frac{Y_{N,t}(i)}{A_{t}}W_{t}^{1-\kappa}P_{F,t}^{\kappa}\left[\left(\frac{\kappa}{(1 - \kappa)}\right)^{1-\kappa} + \left(\frac{(1 - \kappa)}{\kappa}\right)^{\kappa}\right]\right),$$
(13)

where  $\tau = \frac{1}{\epsilon}$  is a subsidy to correct for inefficiencies introduced by the presence of monopolistic competition. Finally, we obtain the Phillips Curve for non-tradable goods,

$$\Pi_{N,t}\left(\Pi_{N,t} - \Pi_{N}\right) = \mathbb{E}_{t} \left[\Lambda_{t,t+1}^{U} \Pi_{N,t+1} \left(\Pi_{N,t+1} - \Pi_{N}\right) \frac{Y_{N,t+1} P_{N,t+1}}{Y_{N,t} P_{N,t}}\right] + \frac{\epsilon}{\xi} \left(\frac{MC_{t}}{P_{N,t}} - \frac{\epsilon - 1}{\epsilon}\right), \tag{14}$$

where  $MC_t$  is the marginal cost obtained from (13). Using the demand relation and the labour market clearing condition  $N_{N,t} = \int_0^1 N_{N,t}(i)di$ , we can write the aggregate production function as

$$Y_{N,t}\Delta_t = A_{N,t}M_{F,t}^{\kappa}N_{N,t}^{1-\kappa}$$

where  $\Delta_t = \left(1 - \frac{\xi}{2}(\Pi_{N,t} - \Pi_N)^2\right)$  captures the output loss caused by the costly adjustment of prices.

#### 2.2.2 Tradable goods sector

The tradable sector is internationally competitive. Domestic firms in this sector take prices as given at  $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ . We assume that the foreign price dynamics  $(P_{H,t}^*)$  is driven by developments in world markets and therefore exogenous from the perspective of our small open economy. Tradable sector production is given by

$$Y_{H,t} = A_{H,t} N_{H,t}^{1-\zeta}, (15)$$

where  $A_{H,t}$  is the exogenous sector-specific productivity. The problem of a firm in the tradable sector is to maximise profits

$$\max_{N_{H,t}} P_{H,t} Y_{H,t} - W_t N_{H,t},$$

subject to the production technology (15). This yields

$$\frac{W_t}{P_{H,t}} = (1 - \zeta) A_{H,t} N_{H,t}^{-\zeta}.$$
 (16)

#### 2.3 Monetary Policy

The monetary policy authority sets the interest rate according to the following Taylor rule,

$$\frac{I_t}{I} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_{\pi}} \left(\frac{Y_t}{Y}\right)^{\phi_y},\tag{17}$$

where I,  $\Pi$ , Y are the steady state level of nominal interest rate, inflation and output, respectively.

#### 2.4 Equilibrium

Given the tradable and foreign imported prices  $P_{T,t}^*$ ,  $P_{F,t}^*$ , the monetary policy rule determining  $i_t$ , foreign output, inflation and interest rates  $Y_t^*$ ,  $\Pi_t^*$ ,  $i_t^*$ , and an initial condition on price dispersion, the equilibrium is given by a sequence of quantities  $\{C_{H,t}, C_{T,t}, C_{N,t}, C_{F,t}, C_t, C_t^U, C_t^C, N_t^U, N_t^C, N_{H,t}, N_{N,t}, B_{t+1}^H, B_{t+1}^*, Y_{N,t}, Y_{H,t}, M_{F,t}, \Psi_t\}_{t=0}^{\infty}$  and prices  $\{\Lambda_{t,t+1}^U, \Pi_{H,t}, \Pi_{N,t}, \Pi_{T,t}, \Pi_{T,t}, \Pi_t, W_t, \mathcal{T}_t, \mathcal{S}_t, \mathcal{E}_t, \Delta_t\}_{t=0}^{\infty}$  such that firms and households maximise their objectives, and the goods, labour and financial markets clear,

$$Y_{t} = C_{H,t} + C_{H,t}^{*} + C_{N,t}$$

$$Y_{H,t} = C_{H,t} + C_{H,t}^{*}$$

$$C_{N,t} = Y_{N,t}$$

$$B_{t} = 0$$

$$B_{t}^{*} = B^{*}$$

$$N_{t} = N_{H,t} + N_{N,t} = N_{t}^{C} \lambda + N_{t}^{U} (1 - \lambda)$$

$$Y_{t}^{*} = C_{t}^{*}.$$

#### 2.5 Natural Real Interest Rate and Natural Level of Output

To understand the impact of our shocks and to determine the appropriate real policy interest rate, we calculate the natural level of output  $(Y_t^n)$  and the natural real interest rate  $(r_t^n)$ .

 $Y_t^n$  is the level of output that would arise under flexible prices. To derive it, we need to determine the profit-maximizing flexible price for the domestic non-tradable good firms, which is the only sector facing nominal rigidities. Profit-maximising firms set the flexible optimal price in order to equalise marginal cost and marginal revenue. This is equivalent to setting the real marginal cost to the inverse of the desired markup,

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}.$$

This yields

$$\frac{\epsilon - 1}{\epsilon} = \frac{(1 - \tau)}{A_{N,t}} \left( (C_t^n)^{\sigma} (N_t^n)^{\phi} \right)^{1 - \kappa} \left( \frac{P_{F,t}}{P_t} \right)^{\kappa} \left[ \left( \frac{\kappa}{(1 - \kappa)} \right)^{1 - \kappa} + \left( \frac{(1 - \kappa)}{\kappa} \right)^{\kappa} \right]. \tag{18}$$

Using the UIP condition as well as the equilibrium condition for output, we can express the natural real interest rate as the risk-free real interest rate consistent with the Euler equation when output is at its natural level at all times,

$$(C_{t+1}^n)^{\sigma} = \beta (1 + r_t^n) (C_t^n)^{\sigma}$$
$$(1 + r_t^n) = \frac{1}{\beta} \left[ \left( \frac{Y_{t+1}^n}{Y_t^n} \right) \frac{\Sigma_{\zeta\theta,t}}{\Sigma_{\zeta\theta,t+1}} \right]^{\sigma}$$

where 
$$C_t = \frac{1}{\Sigma_{\varsigma\theta,t}} Y_t$$
 and  $\Sigma_{\varsigma\theta,t} \equiv \left[\varsigma\left(\frac{P_t}{P_{N,t}}\right)^{\iota} + (1-\theta)(1-\varsigma)\left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu}\left(\frac{P_t}{P_{T,t}}\right)^{\iota} + \theta\left(\frac{P_t}{P_{H,t}}\right)^{\mu} \frac{\mathcal{S}_t^{\mu-1}}{\mathcal{D}_t^{\frac{1}{\sigma}}}\right].6$ 

#### 3 Calibration

We calibrate the model at a quarterly frequency. Our aggregate baseline calibration is standard, as several preference and technology parameters are shared with the standard New Keynesian literature. We assume a discount factor  $\beta=0.9877$  which implies an annual nominal rate of 5% in steady state. In our baseline scenario, we set the share of hand-to-mouth consumers in the population equal to 30%, following Kaplan, Violante, and Weidner (2014), Kaplan, Moll, and Violante (2018), Kaplan and Violante (2022). We set the elasticity of intertemporal substitution  $\sigma=2$ . This aligns with the recent literature (Jones (2023), Kimball, Reck, Zhang, Ohtake, and Tsutsui (2024), etc.) that discusses the limitations of logutility assumptions in economics models. For simplicity, we consider unitary elasticities of substitution between foreign and domestic tradable goods ( $\mu$ ), and between tradable and

<sup>&</sup>lt;sup>6</sup>See Appendix A for a full derivation.

non-tradable goods ( $\iota$ ), and the Frisch elasticity.<sup>7</sup> In the baseline model,  $\theta$  equals 0.6, which implies a weight on foreign goods in the economy of approximately 25 percent, following Harrison and Oomen (2010). We set  $\kappa$ , the income share of foreign primitive input in the production of non-tradable goods, close to 0 in the baseline. A positive  $\kappa$  (around 0.3 to match the non-labour share of income) amplifies the quantitative impact of the fragmentation scenarios we study, while leaving the qualitative interpretation unchanged.<sup>8</sup> Finally, we calibrate  $\varsigma$ , the share of non-tradable goods in the consumption basket to be 0.6.

Parameter	Definition	Value	Source / Target
β	Household discount factor	0.9877	Annual net nominal rate $r_{ss} \approx 5\%$
$\sigma$	Household risk aversion	2	Corsetti, Dedola, and Leduc (2009)
$\kappa_\ell$	Labour disutility	$\approx 5$	Literature
$\phi$	Inverse Frisch elasticity	1	Gali and Monacelli (2005)
$\lambda$	Share of constrained households	0.3	Kaplan, Moll, and Violante (2018)
$\theta$	Share of foreign tradables	0.6	Harrison and Oomen (2010)
ς	Share of non-tradables	0.6	Literature
μ	Elasticity of substitution between home and foreign goods	1	Literature
ι	Elasticity of substitution between tradables and non-tradables	1	Literature
$\epsilon$	Elasticity of substitution between non-tradable varieties	11	10% gross final markup
$\phi_\pi$	Interest sensitivity to inflation	1.5	Literature
$\phi_{y}$	Interest sensitivity to output	0	Literature
$\xi$	Rotemberg adjustment cost	57	Avg lifetime of prices 3Q
ζ	Labour share in tradables	0.8	Literature
κ	Foreign input share in non-tradables	$\approx 0;0.3$	Literature
$ ho_s$	Persistence coefficient	0.9	$s \in \{N, H^*, C^*, r^*\}$
$ ho_H$	Tradable TFP persistence coefficient	0.85	
$ ho_F$	Persistence coefficient of foreign price	0.75;1	
χ	Portfolio adjustment cost	0.001	Literature

Table 1: This table presents the baseline quarterly calibration.

#### 3.1 Special Case: RANK

To establish some basic intuition for our results, we study our three fragmentation scenarios in a representative agent model. This corresponds to a special case of our baseline model, where the share of constrained households is  $\lambda = 0$ .

**Gradual Fragmentation** To simulate a gradual shift towards a more restrictive trade environment, Figure 4 plots the impulse response functions (IRFs) of various macroeconomic aggregates to a gradual increase in import prices ( $P_{F,t}^*$ ). In this scenario, import prices stabilise in the medium term, with a cumulative increase of 100 percent. The price of imported goods,

<sup>&</sup>lt;sup>7</sup>This is consistent with the view that elasticities of substitution are high in the medium- to long-run, if fragmentation is a permanent, phased-in process. However, results are robust to usual variation (Corsetti, Dedola, and Leduc (2009)) in these parameters. See Appendix D.

<sup>&</sup>lt;sup>8</sup>See Section 4.2.

 $P_{F,t}^*$ , will affect demand directly, both through the consumption basket  $C_{F,t}$  and through imported inputs  $(M_{F,t})$  in non-tradable goods production. Additionally, it indirectly affects demand through real wages.

An increase in imported inflation follows from the gradual increase in the price of foreign goods. This places upward pressure on CPI inflation, but is more than offset by the fall in domestic inflation, which declines due to the falls in both non-tradable and tradable inflation. This fall in domestic inflation is driven by the fall in consumption, which responds to the drop in households' permanent real income. Households partly compensate for the fall in real wages by increasing labour supply, which mitigates the impact of higher import prices on aggregate supply. Overall, the anticipation effect of lower real incomes leads to demand falling by more than supply, and consequently, to a fall in domestic inflation. Aggregate CPI inflation, which is a composite of domestic and imported goods inflation, falls on balance. This is reflected in a decrease in the natural real rate of interest, indicating that when demand materially adjusts in anticipation, the effect can be disinflationary. This prompts the central bank to ease policy, by lowering the nominal interest rate, in line with the Taylor-type rule characterising its reaction function.

**Front-loaded Fragmentation** Figure 5 shows the effect of a permanent and immediate increase in foreign prices  $P_{F,t}^*$ . This type of shock is intended to capture rapid 'fragmentation events' like price increases or tariff increases.

The increase in the price of foreign goods leads to a sharp increase in imported inflation, which quickly reverts to its steady-state level. Aggregate consumption falls on impact and stabilises at a lower steady state. Non-tradable inflation falls, following the fall in consumption demand. Domestic tradable inflation also decreases initially, following the fall in consumption. Tradable inflation rises sharply, reflecting the surge in imported-goods inflation. Altogether, this leads to a significant spike in aggregate CPI inflation. Non-tradable output falls temporarily as the higher price of the foreign input restricts supply, but then partially recovers as households increase labour supply to compensate for losses in real income. The economy enters a temporary period of stagflation, with prices increasing and aggregate demand decreasing. Unlike the gradual scenario, there is no change in the natural real rate. The Taylor rule followed by the central bank leads to an increase in the nominal rate in order to return inflation to target.

<sup>&</sup>lt;sup>9</sup>A caveat is in order: our analysis does not account for the use of fiscal proceeds from tariffs. One way to justify this would be to assume that proceeds from tariffs are used to stimulate supply and demand in equal amounts, without affecting the output gap or inflation. Alternatively, we can assume that import restrictions take the form of non-tariff barriers (which comprise the majority of trade restrictions), in which case there is no tax revenue to be rebated. More broadly, this exercise is intended to capture the realignment of trade, whereby geopolitics forces domestic firms to switch from low-cost to geopolitically friendly suppliers, leading to efficiency losses.

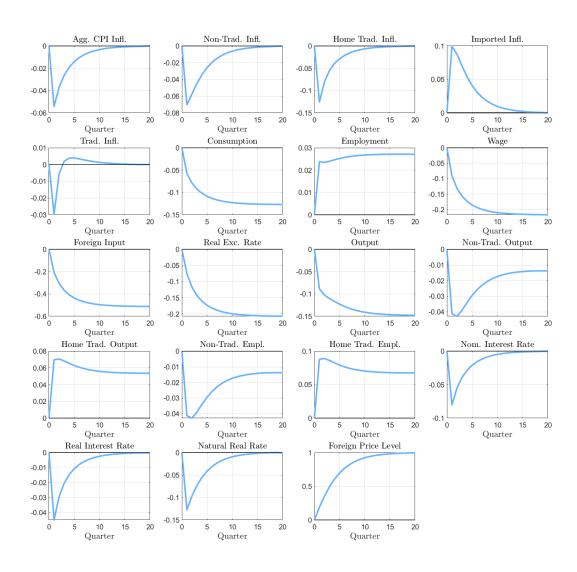


Figure 4: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* IRFs to a 100% gradual positive foreign price shock. The results are generated under a RANK calibration ( $\lambda = 0$ ). All the other parameters are calibrated according to Table 1.

Fall in Tradables Productivity  $A_{T,t}$  We consider an additional shock that can result from "trade fragmentation": a persistent and permanent decrease in the productivity of the tradable goods sector, which makes domestic production of tradable goods less competitive in the global market. In Figure 6, we show the response to a negative one standard deviation shock to total factor productivity in the tradable sector,  $A_{T,t}$ .

As in the standard New Keynesian model, this constraint on home tradable supply results in more employment per unit of output and an increase in inflation as marginal costs rise. With the gradual deterioration in TFP, real wages and consumption fall, reflecting the negative wealth effect.

Agg. CPI Infl Non-Trad. Infl. Imported Infl Home Trad. Infl 0.06 0.4 0.04 -0.1 -0.2 -0.04 0.02 0.2 -0.3 -0.06 -0.4 Quarter Quarter Quarter Quarter Trad. Infl. Consumption Employment Wage 0.04 0 0.2 -0.1 0.15 -0.05 -0.2 0.02 0.1 -0.3 0.01 -0.1 0.05 0 20 10 Quarter Quarter Quarter Quarter Foreign Input Real Exc. Rate Output Non-Trad. Output 0 0 -0.05 -0.1 -0.05 -0.1 -0.5 -0.2 -0.15 -0.1 -0.3 -0.2 -0.25 -0.4 -0.15 15 0 20 Quarter Quarter Quarter Quarter Home Trad. Output Non-Trad. Empl Home Trad. Empl Nom. Interest Rate 0.15 0.06 -0.05 0.1 0.04 0.05 -0.1 0.05 0.02 -0.15 10 10 10 10 15 Quarter Quarter Quarter Natural Real Rate Real Interest Rate Foreign Price Level 0.01 0.005 0.5 0.02 -0.005 15 20 Quarter Quarter Quarter

Figure 5: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* IRFs to a 100% front-loaded positive foreign price shock. The results are generated under a RANK calibration ( $\lambda = 0$ ). All the other parameters are calibrated according to Table 1.

Inflation in the non-tradable sector is negligible as both demand and marginal costs decrease in line with the decline in real wages. As a result, the fall in non-tradable inflation is not enough to outweigh the increase in home tradable inflation, leading to an increase in aggregate CPI inflation in this scenario. Monetary policy, which tracks CPI inflation, is initially contractionary, as this is needed to bring CPI inflation back to target, while the natural rate falls (reflecting lower domestic inflationary pressures), and gradually returns to steady state as the economy recovers.

Agg. CPI Infl Imported Infl Non-Trad. Infl Home Trad. Infl 0.015 0.005 0.03 0.03 0.01 0.02 0.02 0.005 0.01 0.01 -0.005 -0.005 15 20 20 10 Quarter Quarter Quarter Quarter Trad. Infl Consumption Employment Wage 0.06 0.03 0.04 0.02 -0.05 -0.04 0.01 0.02 -0.1 -0.08 20 10 20 20 Quarter Quarter Quarter Quarter Non-Trad. Output Foreign Input Real Exc. Rate Output 0.06 0.05 -0.05 0.04 -0.05 -0.1 0.02 -0.15 <sup>L</sup> -0.05 -0.1 15 20 Quarter Quarter Quarter Quarter Home Trad. Output Non-Trad. Empl. Home Trad. Empl Nom. Interest Rate 0.2 0.01 0.1 0.15 0.05 0.1 0.02 0.01 -0.05 0.05 20 10 Quarter Quarter Quarter Real Interest Rate Natural Real Rate Tradable TFP Level -0.05 0.005 -0. 0 20 15 Quarter Quarter

Figure 6: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

*Notes:* IRFs to a negative TFP shock in the tradable sector. The results are generated under a RANK calibration ( $\lambda = 0$ ). All the other parameters are calibrated according to Table 1.

To summarise, since all three scenarios consider shocks that constrain supply capacity, the supply-side effects are unambiguous. To capture general equilibrium effects, this section uses a simple framework to demonstrate how aggregate demand adjusts differently in various scenarios that model aspects of trade fragmentation. In the gradual fragmentation scenario, a steady increase in import prices reduces the purchasing power of labour income, through an increase in the price of imported consumption goods as well as through a fall in nominal wages. If this change is expected to be permanent, then households also expect a permanent fall in purchasing power, leading to a fall in consumption spending. Therefore, a fall in permanent labour income leads to a fall in demand, which affects the price-level re-

sponse to the initial increase in import prices. This scenario leads to stagnation, with lower real incomes and disinflationary pressures. In contrast, a front-loaded fragmentation scenario, which takes the form of a sharp permanent increase in import prices, may create a short-term tradeoff for policymakers, with weaker activity and higher CPI inflation. Finally, a fall in tradable sector productivity has, in principle, an ambiguous impact on CPI inflation, but it is moderately inflationary in our calibration.

These results suggest that the form in which fragmentation materialises, the extent to which it is anticipated by households, and households' ability to smooth consumption over time, all matter. The next section will consider the case where a proportion of households are unable to smooth consumption in response to changes in their permanent income.

#### 3.2 Baseline Model: TANK

Two important factors in gauging how inflation will respond to these trade-related shocks are the degree of forward-looking behaviour in demand and the extent to which households can effectively smooth consumption in the presence of a shock. This section considers a more general framework that allows for household heterogeneity on this front.

Relative to the previous section, the presence of constrained households introduces agents who cannot smooth consumption in response to the shock, although they can adjust their labour supply. The fall in real incomes affects these households directly, while the fall in labour demand and aggregate demand by unconstrained households affects them indirectly. As in the previous section, we consider three scenarios to show how the form of fragmentation will affect the demand-side adjustment.

The fall in permanent income highlighted in the previous section will be mitigated by the proportion of constrained households. While all households consume out of permanent income in a RANK model, only a proportion  $\lambda$  of households do so in a TANK model. The presence of constrained households lessens the adverse demand-side effect since they cannot cut consumption in anticipation of the shock. However, the fall in consumption from unconstrained households also affects constrained households, who consume out of labour income only and cannot smooth consumption over time.

**Gradual Fragmentation** To simulate a gradual shift towards a more restricted trade environment, Figure 7 plots the impulse response functions to an increase in the price of imported goods, which stabilises at a 100 percent higher level in the medium term. As in the RANK case, the price of imported goods,  $P_{F,t}^*$ , affects demand directly, both through the consumption basket (through  $C_{F,t}$ ) and through imported inputs in production ( $M_{F,t}$ ). Additionally, it indirectly affects demand through real wages.

The response of real and nominal variables in a TANK model is largely similar to the RANK setting (Figure 4). Unconstrained households, who can borrow to smooth consump-

tion, cut consumption by less. Comparing the two cases, the presence of constrained households mitigates the adverse effect of the shock on supply, as constrained households partly compensate for the loss in real wages by increasing labour supply. Hours worked and output fall by more in the non-tradable sector, reallocating to the tradable sector, where output and hours worked increases. Overall, output falls by less in the TANK case, as the presence of constrained households mitigates the impact of higher import prices on aggregate supply. While unconstrained households lower spending in response to lower permanent incomes, constrained households also cut consumption, given higher prices and the overall fall in domestic demand. Overall, the anticipation effect of lower real incomes on demand by unconstrained households and the effect of lower current income on demand by constrained households lead to demand falling more than supply, and consequently, to a fall in domestic inflation. As in the RANK model, the fall in domestic inflation more than offsets the persistent increase in imported inflation, leading to a fall in aggregate CPI inflation. The natural real rate of interest falls, suggesting that when demand adjusts, the effect can be disinflationary. The central bank's nominal interest rate falls in response, in line with the rule characterising the policy maker's reaction function.

Front-loaded Fragmentation Figure 8 shows the effect of a permanent and immediate increase in foreign prices  $P_{F,t}^*$ . This scenario leads to a sharp increase in imported inflation, which quickly reverts back to the steady-state level. The responses of nominal variables in a TANK model are similar to the RANK case. In the TANK model, aggregate consumption falls by more due to the larger response of the hand-to-mouth consumers. Consumption falls significantly on impact, before settling at a lower steady state level for both constrained and unconstrained consumers, and in the aggregate. Non-tradable output and employment fall temporarily as the higher price of the foreign input restricts supply, but both recover as households increase their labour supply to compensate for income losses. Relative to the RANK case, the TANK setting features a larger fall in non-tradable output and employment, which is reallocated to the tradable sector, where output and employment increase by more. As in the RANK case, aggregate inflation increases, driven by largely by the surge in imported inflation, and the economy enters a temporary period of stagflation. The policy rule followed by the central bank leads to an increase in the policy rate. Unlike the gradual fragmentation scenario, the natural real rate remains unchanged.

**Fall in Tradables Productivity** Finally, we consider a permanent and gradual decrease in the productivity of the tradable goods sector, which makes domestic production less competitive relative to foreign production. Figure 9 plots the responses of all variables to a one standard deviation negative shock to total factor productivity in the tradable sector,  $A_{T,t}$ .

Home tradable employment increases relative to home tradable output. Inflation in this sector also increases, due to the rise in the marginal cost of production. Comparing the RANK and TANK models highlights how the tradable TFP shock affects labour income

Agg. CPI Infl. Trad. Infl Non-Trad. Infl. Home Trad. Infl Imported Infl 0.0 -0.02 -0.02 -0.05 -0.01 0.05 -0.04 -0.04 -0.1 -0.02 -0.06 -0.03 20 Quarter Quarter Quarter Quarter Quarter ConsumptionUnconstrained Cons Constrained Cons Unconstrained Empl. Constrained Empl. -0.05 -0.05 -0.05 0.02 0.05 -0.1 -0 -0. 0.01 20 Quarter Quarter Quarter Quarter Non-Trad. Output Home Trad. Output Non-Trad. Empl Home Trad. Empl. 0.1 0.15 -0.01 -0.01 -0.05 -0.02 -0.02 0.05 -0. -0.03 -0.03 0.05 -0.15 -0.04 -0.04 20 Quarter Quarter Quarter Quarter Quarter Foreign Input Real Interest Rate Natural Real Rate Wage Nom. Interest Rate -0.05-0.01 -0.2 -0.1 -0.02 -0.15 -0.03 -0.4 -0.2 -0.04 Ω 10 Quarter Quarter Real Exc. Rate Foreign Price Level -0.05-0.1 RANK -0.15 TANK -0.2 10 Quarter Quarter

Figure 7: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* The RANK case ( $\lambda = 0$ ) is shown in the solid blue lines, while the TANK case ( $\lambda = 0.3$ ) is shown in the black dashed lines. All other parameters are calibrated according to Table 1.

and firm profits differently from an import price shock. Constrained households, who only consume out of labour income, experience a much smaller drop in consumption than unconstrained households, whose consumption also depends on firm profits. While constrained employment increases only moderately, unconstrained employment rises to a much greater extent. Overall, there is a less adverse fall in consumption, as labour incomes are supported by higher hours of work. Non-tradable output and employment fall by a smaller extent, relative to the RANK model. As a result, aggregate output falls by less overall. Nevertheless, this scenario is still moderately inflationary, as non-tradable inflation falls by a small amount, failing to outweigh the spike in tradables inflation.

Agg. CPI Infl. Trad. Infl. Non-Trad. Infl. Home Trad. Infl Imported Infl 0.2 -0.02 0.04 0.15 -0.1 0.4 -0.04 0.1 -0.2 0.02 0.2 -0.06 -0.3 0.05 20 Quarter Quarter Quarter Quarter Quarter ConsumptionUnconstrained Cons Constrained Cons Unconstrained Empl. Constrained Empl. 0.04 0.15 -0. -0.1 -0.1 0.02 -0.15 -0.15 -0.2 0.05 0.01 -0.2 -0.2 20 20 10 20 20 Quarter Quarter Quarter Quarter Quarter Output Non-Trad. Output Home Trad. Output Non-Trad. Empl Home Trad. Empl 0.2 0.15 -0.05 -0.05 -0.1 0.15 0. -0.15 0.1 0.05 -0.2 0.05 -0.25 10 20 20 20 20 Quarter Quarter Quarter Quarter Quarter Wage Foreign Input Real Interest Rate Nom. Interest Rate Natural Real Rate 0.0 0.06 -0.1 0.005 0.04 -0.2 -0.5 0.0 -0.3 0.02 -0.005 -0.4 -0.01 10 10 Quarter Quarter Real Exc. Rate Foreign Price Level -0.1 -0.2 0.5 RANK TANK -0.3 20 10 Quarter Quarter

Figure 8: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* The RANK case ( $\lambda = 0$ ) is shown in the solid blue lines, while the TANK case ( $\lambda = 0.3$ ) is shown in the black dashed lines. All other parameters are calibrated according to Table 1.

In summary, the form in which fragmentation takes place will matter for the balance of supply and demand. The sudden implementation of tariffs will have different effects compared to a gradual implementation or an adverse shock to tradables productivity. The presence of hand-to-mouth households mitigates the adverse impact of the fragmentation scenarios on aggregate supply. The demand-side impact features competing effects: while fewer forward-looking agents reduce their consumption in anticipation of lower future incomes, this still affects the demand of hand-to-mouth households, who have a higher marginal

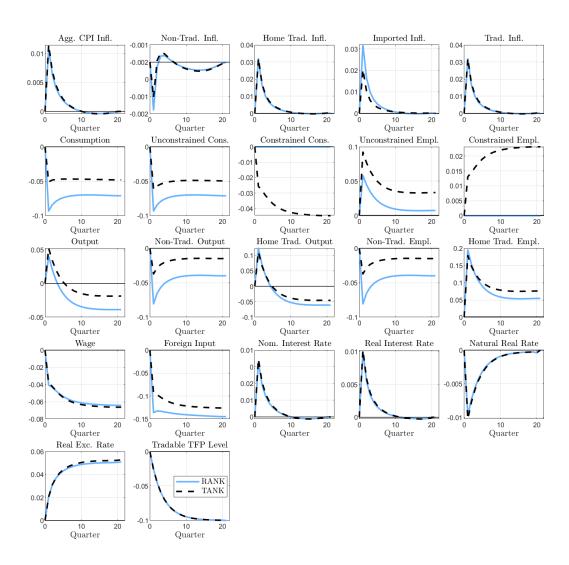


Figure 9: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

*Notes:* The RANK case ( $\lambda = 0$ ) is shown in the solid blue lines, while the TANK case ( $\lambda = 0.3$ ) is shown in the black dashed lines. All other parameters are calibrated according to Table 1.

propensity to consume out of labour income.<sup>10</sup> Finally, the TANK model highlights how aggregate demand can depend on the distributional effects of the various shocks. This is most salient with the adverse TFP shock, which has a much less negative effect on labour income relative to an import price shock. Nevertheless, the net inflationary impact and the appro-

<sup>&</sup>lt;sup>10</sup>Including investment would increase the importance of forward-looking behaviour for our results. As firms are more forward-looking than households, a gradual increase in import prices or a deterioration in productivity would lead to a fall in investment, deepening the contraction in demand and intensifying disinflationary pressures. Similarly, including uncertainty around the price path will tend to reinforce the anticipation effect, depressing demand more in response to the gradual shocks to import prices or productivity.

priate monetary policy response in each of the fragmentation scenarios remain the same across the RANK and TANK models.

#### 4 Robustness Exercises

This section examines two important dimensions of the model: the degree of trade openness and the share of imported inputs in production.

#### 4.1 Different Degrees of Openness

This section explores the impact of the initial level of trade openness on the macroeconomic response to fragmentation. We therefore consider the same set of fragmentation scenarios while varying the degree of home bias, denoted by  $1-\theta$ . A lower value of  $\theta$  indicates a less open economy: the consumption basket and price index are less affected by foreign prices and consumption relies more on domestic production. This adjustment allows us to capture variation in the degree of exposure to trade shocks and how the policy response may vary with the level of openness.

More open economies are more directly exposed to fluctuations in foreign prices; absent diversification strategies, this would directly affect their consumption and production responses.<sup>11</sup> In the scenarios involving permanent increases in foreign prices, whether gradual or front-loaded, the more open economy experiences a more pronounced decline in the natural rate of interest. All the results we discussed in the previous sections are exacerbated in more open economies. However, in the case of a negative TFP shock, output falls more in a closed economy, as more open economies can mitigate the impact by diversifying away from domestic shocks through trade with foreign suppliers (and buyers).

**Gradual Fragmentation** Figure 10 shows that the open economies ( $\theta = 0.6$  and  $\theta = 0.8$ ) respond differently from the relatively closed economy ( $\theta = 0.1$ ), in a gradual fragmentation scenario. Aggregate inflation has the same behaviour in all three cases; however, the underlying mechanisms differ quantitatively.

A useful benchmark to start with is the relatively closed economy case ( $\theta = 0.1$ , red dotted lines). Here, imported inputs are used mainly as an intermediate input for the production of non-tradable goods and the consumption basket of domestic households consists mainly of domestically produced tradable and non-tradable goods. As the price of imported intermediate inputs increases, and as the real exchange rate depreciates, imported inflation increases and the use of the imported input falls. Non-tradable employment moves in line

<sup>&</sup>lt;sup>11</sup>Caselli, Koren, Lisicky, and Tenreyro (2020) discuss conditions under which diversification of international buyers and suppliers can reduce volatility in more open economies.

with non-tradable output, which falls slightly before returning to steady state. Employment for both types of households increases and real wages fall. The adverse effect on labour income leads to a fall in consumption for both types of households. The fall in aggregate consumption leads to downward pressure on domestic inflation and aggregate CPI inflation.

Next, consider the open economies ( $\theta=0.6$  and  $\theta=0.8$ , dashed black lines and solid blue lines respectively), where imported goods are not only used as intermediate inputs in the production of non-tradable goods but also comprise part of the consumption basket (as foreign tradable goods). The additional fall in non-tradable goods inflation relative to the case of  $\theta=0.1$  is due to the fall in real income and domestic consumption, which now exerts additional downward pressure on domestic inflation. The fall in home tradables inflation counters the sharp rise in imported inflation, leading to a fall in tradables inflation overall. More openness also leads to a larger domestic adjustment in response to the import price shock. The import price shock leads households to substitute towards relatively cheaper domestically produced goods and production factors, which moderates the fall in output. While there is a moderate and permanent fall in non-tradable employment and output, the opposite is true for the domestic tradable sector: employment and output both increase here. A greater exposure to foreign shocks is reflected in a larger fall in the natural rate of interest in the more open economies.

**Front-loaded Fragmentation** In the front-loaded fragmentation scenario, openness also plays a significant role in the response to changes in the price of imported goods. Notably, in the more open economy, non-tradable inflation exhibits a more pronounced decline immediately following the shock. However, overall, the spike in aggregate CPI inflation is larger in the more open economy, as a result of the sharper increase in imported inflation.

Consider the case of a relatively closed economy, where the import price shock mainly affects intermediate inputs in the production of non-tradable goods ( $\theta = 0.1$ ). As the price of foreign inputs rises, there is a decrease in demand for these inputs, leading to a moderate fall in non-tradable output, non-tradable employment, and real wages. As a result, aggregate consumption also falls moderately.

The same dynamics are more pronounced in the open economies ( $\theta = 0.6$  and  $\theta = 0.8$ , dashed black lines and solid blue lines, respectively), as imported goods also affect the consumption basket. The import price shock therefore adversely affects consumption, and to a greater degree in the more open economy. The shock also affects the domestic economy as households substitute towards relatively cheaper domestically produced goods.<sup>12</sup> Output and employment increase in the tradable sector, while the output and employment in the non-tradable sector recover quickly, yet only incompletely, from the shock. The fall in

<sup>&</sup>lt;sup>12</sup>See Section D for a discussion of the role of elasticities of substitution between tradable and non-tradable goods.

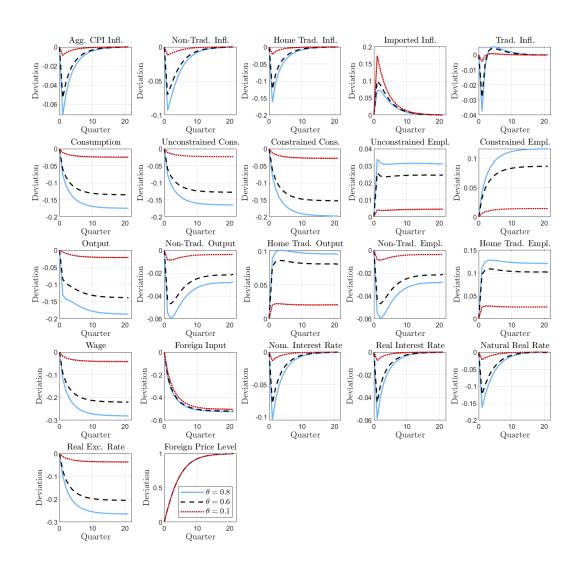


Figure 10: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* The share of foreign tradables in the domestic household's consumption basket is denoted by  $\theta$ . We show simulations for a relatively closed economy ( $\theta = 0.1$ ) with a red dotted line, a moderately open economy ( $\theta = 0.6$ ) with a black dashed line, and a very open economy ( $\theta = 0.8$ ) with a blue line.

consumption, which would occur as a result of more expensive foreign goods, is partially mitigated by the increase in domestic demand.

**Fall in Tradables Productivity** Figure 12 shows the impulse response functions for a one-standard deviation fall in TFP for the tradable sector. As before, consider first the  $\theta = 0.1$  specification (red dotted lines). In this case, imported inputs are mainly used in the production of non-tradable goods. An adverse productivity shock in the tradable sector leads to a fall in tradable output. Each unit of tradable output now requires more labour to

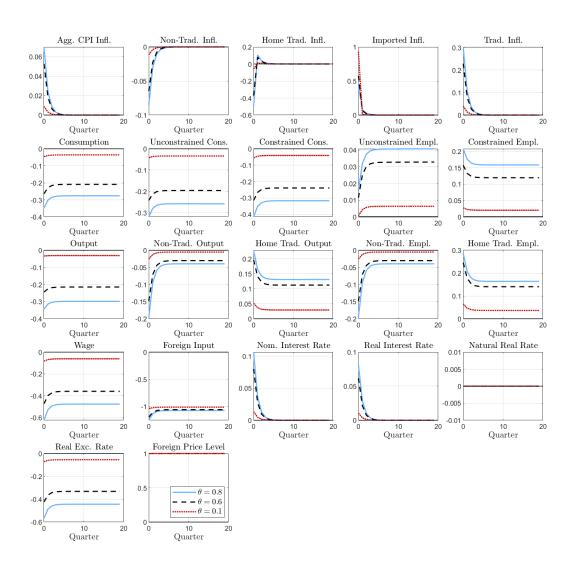


Figure 11: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* The share of foreign tradables in the domestic household's consumption basket is denoted by  $\theta$ . We show simulations for a relatively closed economy ( $\theta = 0.1$ ) with a red dotted line, a moderately open economy ( $\theta = 0.6$ ) with a black dashed line, and a very open economy ( $\theta = 0.8$ ) with a blue line.

produce, and hours worked increase gradually due to higher labour demand. The adverse TFP shock leads to an increase in marginal costs in the tradable sector, which initially leads to upward pressure on tradable inflation. The fall in real wages leads to a fall in consumption. This is largely driven by a fall in the consumption of unconstrained households, who receive firm profits. Constrained households, who consume only out of labour income, experience a smaller fall in consumption. This shock also leads to a larger increase in the labour supply of unconstrained households relative to constrained households.

In a more open economy ( $\theta = 0.6$  and  $\theta = 0.8$ ), the preference for home-produced trad-

ables is weaker, and the presence of foreign-produced tradables mitigates the adverse effect of the domestic productivity shock. The shock affects unconstrained households to a greater extent in an open economy, as consumption falls by more and employment increases by more for these households. Total consumption falls by more in the open economies as a result. The labour supply of both types of households, but particularly the unconstrained households, also helps to mitigate the adverse effect of this shock: employment in the tradable sector increases, partially offsetting losses in productivity. The fall in productivity leads to a fall in the real wage. As in the  $\theta=0.1$  case, the adverse productivity shock in the tradable sector has some spillovers for the non-tradable sector, with non-tradable employment and output falling sharply but recovering quickly. While the effect on output is less adverse in the more open economies, tradables inflation and aggregate CPI inflation are roughly similar regardless of the degree of openness.

#### 4.2 Higher share of imported inputs in production ( $\kappa$ )

In this section, we consider our fragmentation scenarios under a higher calibration for  $\kappa$ , which captures the share of imported inputs used in the production of non-tradables. Relative to the baseline calibration of  $\kappa \approx 0$ , a higher  $\kappa$  reflects a tighter constraint on domestic supply capacity as import prices increase.<sup>13</sup>

**Gradual Fragmentation** Figure 13 presents the gradual fragmentation scenario for our baseline calibration (dashed black lines) and a higher  $\kappa$  (solid blue lines). As in the baseline calibration, the use of the foreign input falls in response to an increase in import prices. The effect on domestic supply is reflected in the non-tradable sector, as output falls to a greater extent, stabilising at a permanently lower level relative to the baseline calibration. However, in an economy where production is more dependent on imported inputs, an increase in the price of the foreign input leads to substitution towards the cheaper production input, labour. As a result, employment in the non-tradable sector increases following an initial drop, and stabilises at a permanently higher level. The domestic adjustment in response to this shock extends to the tradable sector, where output and employment increase to a greater extent. With greater dependence on imported inputs in non-tradable production, the reallocation from non-tradables to tradables in response to the import price shock is therefore more pronounced than in the baseline model, as the domestic tradable sector not

<sup>&</sup>lt;sup>13</sup>Non-tradable goods are produced using a Cobb-Douglas function combining labour and an imported inputs, which keeps the labour share of total factor expenditures constant even as import prices rise. The adverse effects on aggregate demand can be even more pronounced under CES production with low substitutability between inputs, as higher import prices reduce the share of income accruing to domestic factors of production, weakening aggregate demand and dampening inflationary pressures. This effect is even stronger in a TANK framework, which features households who depend heavily on labour income. See Auclert, Monnery, Rognlie, and Straub (2023), Chan, Diz, and Kanngiesser (2024), and Gnocato (2025).

Agg. CPI Infl. Non-Trad. Infl Home Trad. Infl Imported Infl Trad. Infl. -0.002 0.01 Deviation 0.001 Deviation 20.0 10.0 Deviation 0.03 10.0 0.005 -0.001 0.005 -0.002 20 20 Quarter Quarter Quarter Quarter Quarter Consumption Unconstrained Cons Constrained Cons Unconstrained Empl. Constrained Empl 0.02 -0.0 Deviation 70.0-70.0-Deviation Deviation Deviation Deviation 0.015 -0.02 -0.0 0.0 0.01 -0.03 -0.06 -0.04 10 20 10 10 Quarter Quarter Quarter Quarter Quarter Non-Trad. Empl. Output Non-Trad. Output Home Trad. Output Home Trad. Empl 0.2 0.05 -0.0 -0.0 Deviation Deviation Deviation Deviation -0.0 viat -0.02 0. De -0.03 -0.03 -0.05 20 Quarter Quarter Quarter Quarter Quarter Wage Real Interest Rate Natural Real Rate Foreign Input Nom. Interest Rate 0.01 -0.02 Deviation Deviation Deviation Deviation Deviation 0.02 -0.005 -0.04 0.005 -0. 0.0 -0.06 -0.01 -0.08 10 10 Quarter 20 10 Quarter 20 10 Quarter 20 Quarter Quarter Tradable TFP Level Real Exc. Rate Deviation 20.0 Deviation  $-\theta = 0.6$ -0.05 -0.1<sub>0</sub> 10 Quarter 20 10 Quarter 20

Figure 12: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

*Notes:* The share of foreign tradables in the domestic household's consumption basket is denoted by  $\theta$ . We show simulations for a relatively closed economy ( $\theta = 0.1$ ) with a red dotted line, a moderately open economy ( $\theta = 0.6$ ) with a black dashed line, and a very open economy ( $\theta = 0.8$ ) with a blue line.

only becomes more competitive with respect to foreign tradables, but also more competitive relative to the non-tradable sector. <sup>14</sup>

<sup>&</sup>lt;sup>14</sup>This exercise highlights how the impact of the import price shock varies across sectors, as only the non-tradable sector uses imported inputs in production. As a shock that affects the relative price of imports to exports and thereby the competitiveness of the non-tradable sector vis-à-vis the domestic tradable sector, it prompts a reallocation of resources. Our positive implications are similar to Guerrieri, Lorenzoni, Straub, and Werning (2021), who study the effect of asymmetric shocks in a two-sector model with downward wage rigidities and costly labor reallocation. Even with perfect labour mobility across sectors, our import price shock operates like a cost-push shock, leading to inflation in the tradable sector and unemployment in the

In the gradual scenario, the increase in import prices affects demand through lower real disposable income for constrained households and lower real permanent income for unconstrained households. Households compensate for the fall in income by increasing labour supply. This effect outweighs the increase in labour demand from factor substitution. As a result, real wages fall for both types of households and to a greater extent relative to the baseline model. While labour supply increases, consumption also falls to a greater degree for both types of households given the more adverse effect on the terms-of-trade. Relative to the baseline model, non-tradable inflation falls by more despite the increased dependence on imported inputs in production. This is attributable to a fall in labour costs (which offsets the increase in imported input prices) and a larger fall in domestic demand relative to the baseline model.

**Front-loaded Fragmentation** In our baseline calibration, this scenario yielded a stagflationary dynamic for policymakers, with a fall in output and a sharp increase in aggregate CPI inflation. Relative to the baseline calibration, a higher  $\kappa$  leads to a less severe trade-off for policymakers, with a larger fall in output and a more moderate increase in inflation (see Figure 14).

As in the gradual scenario, the effect of higher  $\kappa$  on domestic supply is reflected in the non-tradable sector, where output falls to a greater extent, stabilising at a permanently lower level relative to the baseline calibration. As labour becomes a relatively cheaper production input, employment in the non-tradable sector increases, settling at a permanently higher level. There is a moderate degree of reallocation to the domestic tradable sector, with output and employment increasing, but to a similar extent as in the baseline calibration.

The increase in import prices has an adverse effect on consumption, through lower real incomes for both types of households. This fall in consumption is even larger in the case of higher  $\kappa$ , as real wages fall by more. Labour supply increases for both types of households. Relative to the baseline model, non-tradable inflation falls by less, as production is more dependent in imported inputs whose prices increase immediately in the front-loaded scenario. This increase in marginal costs is only slightly offset by a larger fall in real wages relative to the baseline scenario. Therefore, as in the baseline calibration, the fall in non-tradables inflation in the front-loaded scenario is not enough to offset the large spike in tradables inflation, and aggregate CPI inflation increases as a result, leading to a stagflationary scenario for policymakers.

**Fall in Tradables Productivity**  $A_{T,t}$  The parameter we vary in this exercise,  $\kappa$ , only affects the non-tradable sector. However, there are important transmission mechanisms through the real exchange rate. In the baseline calibration ( $\kappa \approx 0$ , black dashed lines), a permanent

non-tradable sector, which is the only sector with nominal rigidities. See Section 6 for the case where wage rigidities are also present.

Agg. CPI Infl. Trad. Infl. Non-Trad. Infl Home Trad. Infl Imported Infl Deviation 40.0-Deviation Deviation -0.02 -0. 0.05 -0.04 -0.06 20 20 20 Quarter Quarter Quarter Quarter Quarter Constrained Empl. Consumption Unconstrained Cons Constrained Cons Unconstrained Empl. -0.05 Deviation Deviation Deviation Deviation 0. -0. -0. -0. 0.05 -0.15 -0.15 Quarter Quarter Quarter Quarter Quarter Non-Trad. Empl. Non-Trad. Output Home Trad. Output Home Trad. Empl. 0. 0.1 0.05 -0.05 Deviation Deviation Deviation Deviation -0.0 0. -0.1 20 Quarter Quarter Quarter Quarter Quarter Wage Natural Real Rate Foreign Input Deviation Deviation Deviation Deviation Deviation -0.05 -0.2 -0.4 -0. -0. -0.15 10 Quarter 10 20 10 20 10 Quarter Quarter Real Exc. Rate Foreign Price Level Deviation  $\kappa = 0$ -0.3 0 10 Quarter 20 Quarter

Figure 13: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* IRFs to a 100% gradual positive foreign price shock. The results are generated under a TANK calibration where  $\kappa=0.3$  is shown in the solid blue lines, while  $\kappa=0$  is shown in the black dashed lines. All the other parameters are calibrated according to Table 1.

adverse tradables TFP shock leads to a real exchange rate depreciation. There is a fall in consumption and an increase in labour supply for both types of households. The depreciation also shifts demand to from foreign tradables to home-tradables. While output and employment initially fall in the non-tradable sector, they recover as demand switches towards this sector as well.

Next, consider the case where non-tradables production uses a higher share of imported inputs ( $\kappa = 0.3$ , blue solid lines). Weaker productivity in the tradable sector leads to a more moderate real depreciation, yielding a smaller increase in marginal costs for non-tradable

Agg. CPI Infl. Trad. Infl. Non-Trad. Infl. Imported Infl Home Trad. Infl 0.2 0.15 0.04 0.4 -0.04 -0.2 0.1 0.02 0.2 -0.06 0.05 20 20 Quarter Quarter Quarter Quarter Quarter ConsumptionUnconstrained Cons Constrained Cons Unconstrained Empl. Constrained Empl -0. -0. -0.2 -0.2 -0 : 20 10 20 10 10 20 Quarter Quarter Quarter Quarter Quarter Non-Trad. Empl Non-Trad. Output Home Trad. Output Home Trad. Empl 0.2 0.1 0.1 0.15 0. -0.2 0.1 0.05 0.05 -0.3 20 20 Quarter Quarter Quarter Quarter Quarter Wage Foreign Input Nom. Interest Rate Natural Real Rate Real Interest Rate 0.01 0.06 -0.1 0.005 0.04 -0.2 0.05 -0.3 0.02 -0.4 -0.005 -0.5 -0.01 <sup>L</sup> 10 10 10 10 Quarter Real Exc. Rate Foreign Price Level -0.1 -0.2 -0.3 -0.4  $-\kappa = 0$ Quarter Quarter

Figure 14: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* IRFs to a 100% gradual positive foreign price shock. The results are generated under a TANK calibration where  $\kappa=0.3$  is shown in the solid blue lines, while  $\kappa=0$  is shown in the black dashed lines. All the other parameters are calibrated according to Table 1.

firms. Over time, and relative to the baseline calibration, this sector produces less, with more labour, and less foreign inputs. The real wage falls by more as the non-tradable sector does not absorb as much labour as in the baseline calibration. This weighs on consumption, particularly of constrained households who consume only out of labour income. Home tradables inflation is weaker due to lower demand and lower labour costs, despite the adverse shock to productivity. This scenario, which led to a small increase in aggregate CPI inflation in the baseline calibration, is even less inflationary in the case where imported inputs are a greater share of non-tradables production.

Agg. CPI Infl. Non-Trad. Infl Trad. Infl. Home Trad. Infl Imported Infl 0.02 0.01 0.03 0.03 0.015 0.005 0.02 0.02 0.01 -0.01 20 20 20 20 20 Quarter Quarter Quarter Quarter Quarter Consumption Unconstrained Cons Constrained Cons Unconstrained Empl Constrained Empl Deviation Deviation 10.0 Deviation Deviation -0.02 -0.02 -0.02 0.05 -0.04 -0.04 -0.06 10 20 10 20 Quarter Quarter Quarter Quarter Quarter Non-Trad. Empl. Output Non-Trad. Output Home Trad. Output Home Trad. Empl 0.2 0.1 -0.0 Deviation Deviation Deviation Deviation -0.01 -0.0 0. -0.02 -0.03 -0.03 -0.05 20 Quarter Quarter Quarter Quarter Wage Real Interest Rate Natural Real Rate Foreign Input Nom. Interest Rate 0.01 -0.02 Deviation Deviation Deviation Deviation 0.005 -0.0 -0.00 -0. -0.06 -0 005 -0.01 L -0.08 10 Quarter 10 20 10 20 10 20 10 20 20 Quarter Quarter Quarter Real Exc. Rate Tradable TFP Level Deviation -0.05 -0.1<sub>0</sub> 20 10 Quarter 20

Figure 15: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

Notes: IRFs to a 100% gradual positive foreign price shock. The results are generated under a TANK calibration where  $\kappa = 0.3$  is shown in the solid blue lines, while  $\kappa = 0$  is shown in the black dashed lines. All the other parameters are calibrated according to Table 1.

Quarter

In summary, higher  $\kappa$ , which captures a greater degree of dependence of production on imported inputs, does not change the qualitative interpretations and policy implications we draw from the various fragmentation scenarios. This calibration represents a more severe constraint on domestic supply as import prices increase, exacerbating the relative price dynamics of the import price shocks. However, the demand-side implications remain largely unchanged: factor substitution (towards labour) is not enough to stimulate aggregate demand. The fall in real wages prompts an increase in labour supply and a fall in consumption. Consequently, the conclusions from our baseline model hold: the fall in consumption leads to disinflationary pressures and stagnation in the gradual scenario. In the front-loaded scenario, the fall in consumption and downward pressure on non-tradables inflation is not sufficient to outweigh the sharp increase in tradables inflation, resulting in a tradeoff for policymakers. The adverse TFP scenario is still moderately inflationary, though less so. A greater share of imported inputs in non-tradables production limits the ability of this sector to absorb labour following an adverse TFP shock in the tradable sector. This leads to lower marginal costs in the domestic sectors, which weakens aggregate CPI inflation.

### 5 Welfare

In this section, we study the welfare optimal response to the range of fragmentation scenarios in our model, following Drechsel, McLeay, Tenreyro, and Turri (2025).

#### 5.1 Social Planner's Solution

We characterise the social planner's solution in a small open economy. To maximise household utility, the social planner chooses the composition of consumption  $(C_{H,t}, C_{F,t}, C_{N,t})$ , input intensity in non-tradable production  $(M_{F,t})$ , resource allocation across sectors  $(N_{N,t}, N_{H,t})$ , and the external asset position  $(B_t^*)$ , taking as given preferences, production, resource constraints, and international prices,

$$\max_{\substack{\{C_{H,t},C_{F,t},C_{N,t},\\N_{N,t},N_{H,t},M_{F,t},B_t^*\}_{t=0}^{\infty}}} \sum_{t=0}^{\infty} \beta^t \left[ (1-\varsigma)(\theta \log C_{F,t} + (1-\theta) \log C_{H,t}) + \varsigma \log C_{N,t} - \frac{(N_{H,t} + N_{N,t})^{1+\phi}}{1+\phi} \right]$$

subject to

$$A_{N,t}M_{F,t}^{\kappa}N_{N,t}^{1-\kappa} + A_{H,t}N_{H,t}^{1-\zeta} = C_{H,t} + C_{N,t} + C_{H,t}^{*}$$
(19)

$$P_{F,t}^* C_{F,t} - P_{H,t}^* C_{H,t}^* + P_{F,t}^* M_{F,t} = (1 + i_{t-1}^*) B_{t-1}^* - \frac{\chi}{2} (B_t^* - \bar{B}^*)^2 - B_t^*$$
 (20)

$$C_{H,t}^* = \frac{C_{H,t}}{C_{F,t}} \frac{\theta}{(1-\theta)} \theta^* C_t^*. \tag{21}$$

The first constraint is a resource constraint, the second captures the trade balance, while the third describes foreign demand for home-produced goods. The solution to the planner's problem is sketched in Appendix B.

Comparing efficient and decentralised outcomes highlights the distortionary effects of price rigidity in the non-tradable sector, an inefficient use of foreign bonds to smooth shocks due to the cost of changing bond positions, and a suboptimal policy response to the shocks. Abstracting from frictions in the decentralised economy, the social planner is able to find a more optimal allocation of inputs within and across sectors and a more optimal choice of household consumption and employment. In this analysis, we focus focus on a RANK model under Cole-Obstfeld preferences ( $\sigma = \mu = \iota = 1$ ).

**Gradual Fragmentation** Figure 16 shows the response of the social planner to a gradual increase in import prices. The import price shock affects the optimal intensity of imported inputs in the non-tradable sector, which becomes more labour-intensive instead. As the marginal product of labour falls in this sector, labour reallocates towards the home tradable sector. This shock is effectively an adverse terms-of-trade shock, increasing the relative price of foreign tradables to home tradables. As a result, there is also a change in the composition of domestic consumption, which switches from foreign tradables to home tradables.

The most notable difference with respect to the decentralised equilibrium is a small expansion, not a sharp contraction in the non-tradable sector, and a much smaller expansion in the home-tradable sector. This comparison suggests that although some reallocation between sectors is efficient, nominal rigidities in the non-tradable sector result in too much reallocation: in the decentralised equilibrium, there is a sharp fall in quantities (output and employment) in the non-tradable sector, and an increase in labour in the home tradable sector. In addition, the labour intensity in the home-tradable sector is much higher in the decentralised equilibrium. There are diminishing returns in this sector, as output increases less than hours. In the decentralised equilibrium, this excess contraction in output and employment in the non-tradable sector may also be attributable to a policy rule targeting aggregate CPI inflation.

The outcomes in the efficient allocation suggest that the social planner relies less on a large reallocation of resources from non-tradables to tradables and more on accumulating foreign bonds to smooth the shock intertemporally in response to import prices that increase gradually and permanently. The social planner is able to achieve a better trade-off between consumption and employment, by tolerating a slightly larger fall in consumption over time in exchange for a much smaller increase in employment.

**Front-loaded Fragmentation** In Figure 17, we show the response of the social planner to a front-loaded increase in import prices. Qualitatively, we see some of the same patterns from the gradual case: there is a fall in imported consumption goods and imported inputs for non-tradable production, a more intensive use of labour in the non-tradable sector, and an increase in consumption of home tradables relative to foreign tradables. Unlike the gradual scenario, these adjustments take place immediately and there is a mild contraction in the non-tradable sector.

As in the decentralised equilibrium (Figure 5), the domestic adjustment in response to this shock involves a reallocation of resources towards the home tradable sector. However, this reallocation occurs to a much smaller extent than in the decentralised case, as the social planner does not face nominal rigidities in the non-tradable sector. Consequently, the decline in output and employment in the non-tradable sector is much more moderate relative to the decentralised equilibrium.

Finally, as in the gradual case, the social planner is able to trade off a slightly larger fall in consumption (driven by a large fall in consumption of foreign tradables) for a much smaller

Home Trad. Consumption Home Trad. Employment Home Trad. Output 0.01 0.01 0.008 0.008 0.008 0.006 0.006 0.006 0.004 0.004 0.004 0.002 0.002 0.002 10 20 Quarter Non-Trad. Consumption Non-Trad. Trad. Employment Non-Trad. Trad. Output 0 0.01 0.01 -0.002 0.008 -0.004 0.006 -0.006 0.004 0.01 0.002 0.005 -0.01 20 15 Quarter Quarter Quarter Foreign Consumption Consumption Employment 0 0 0.01 0.008 -0.05 0.006 0.004 -0.1 -0.4 0.002 -0.15 -0.6 20 20 20 Quarter Quarter Foreign Input Foreign Price Level 8.0 -0.1 -0.2 0.6 -0.3 0.4 0.2 -0.4 20 10 20 Quarter

Figure 16: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* Social Planner response to a gradual shock in  $P_{F,t}^*$ . The results are generated under  $\sigma = \mu = \iota = 1$ ,  $\lambda = 0$  and  $\theta^* = 0.01$ . All the other parameters are calibrated according to Table 1.

increase in aggregate employment.

**Tradables Productivity** Finally, we consider the response of the social planner to a gradual and permanent deterioration in home tradables productivity in Figure 18. As expected, the home tradable sector contracts as consumption switches from home tradables to foreign tradables.

In the decentralised equilibrium (Figure 6), the non-tradable sector contracted, while labour intensity in the home tradable sector increased. In the efficient allocation, there is an expansion in non-tradables production and a contraction in home tradables production

Home Trad. Consumption Home Trad. Employment Home Trad. Output 0.01 0.01 0.008 0.01 0.008 0.006 0.006 0.004 0.005 0.004 0.002 0.002 20 Non-Trad. Consumption Non-Trad. Trad. Employment Non-Trad. Trad. Output 0 -0.002 -0.0001 -0.001 -0.004 -0.0002 -0.006 -0.0003 -0.003 -0.01 -0.0004 20 20 20 Quarter Quarter Quarter Foreign Consumption Consumption Employment 0 0.01 0.008 -0.4 0.006 -0.1 0.004 -0.6 -0.15 0.002 -0.8 20 20 Quarter Quarter Foreign Price Level Foreign Input 8.0 -0.2 0.6 -0.4 0.4 0.2 -0.8 10 20 15 20 5 15

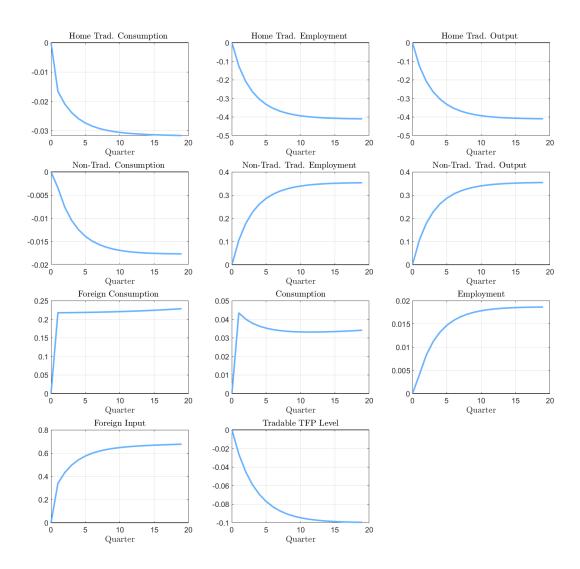
Figure 17: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* Social Planner response to a front-loaded shock in  $P_{F,t}^*$ . The results are generated under  $\sigma = \mu = \iota = 1$ ,  $\lambda = 0$  and  $\theta^* = 0.01$ . All the other parameters are calibrated according to Table 1.

with no increase in labour intensity. This suggests the reallocation from home tradables to non-tradables is inefficiently low in the decentralised equilibrium due to nominal rigidities in the non-tradable sector.

Finally, most notably, consumption increases in the efficient allocation, while it decreases in the decentralised equilibrium. The increase in consumption is driven by an increase in the consumption of foreign tradables. In the efficient allocation, employment increases gradually to a higher steady-state level. As imports of foreign tradable consumption goods and foreign inputs in expanding non-tradables production increase, while home tradables consumption (by both foreign and domestic agents) falls.

Figure 18: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.



*Notes:* Social Planner Response to a gradual shock in  $A_{H,t}$ . The results are generated under  $\sigma = \mu = \iota = 1$ ,  $\lambda = 0$  and  $\theta^* = 0.01$ . All the other parameters are calibrated according to Table 1.

To summarise, this section considered the efficient allocation across the three fragmentation scenarios. This exercise highlights the following inefficiencies in the decentralised equilibrium. First, sticky non-tradables prices prevent optimal domestic reallocation between sectors. Second, the cost of adjusting bond positions results in a suboptimal intertemporal response to shocks. A social planner chooses different allocations for the composition of consumption, the intensity of imported production inputs, the allocation of labour across sectors, and external assets to attain better outcomes for aggregate consumption and employment. The efficient allocation suggests that in the decentralised equilibrium, there is generally too much reallocation to the home-tradable sector in the scenarios with import price shocks, and too little reallocation to the non-tradable sector home-tradable sector in

case of an adverse shock to TFP in the home tradable sector.

#### 6 Extensions

In this section, we extend the RANK model introduced in Section 3.1 to incorporate nominal wage rigidities.

#### 6.1 Households

Relative to the RANK model in Section 3.1, the key difference we introduce is that the labour supply chosen by households is provided to a union in return for a nominal wage  $W_t$ . The union allocates the workers into categories indexed by  $j \in [0,1]$  and sells these units of labour varieties,  $N_t(j)$ , at wage  $W_t(j)$ . Due to imperfect substitutability among labour types, the union can act as a monopolist over each variety.

**Labour Packers** Varieties  $N_t(j)$  are in turn combined by labour "packers" according to a CES production function,

$$N_t = \left[ \int_0^1 N_t(j)^{1-\frac{1}{\epsilon_w}} dj \right]^{\frac{1}{1-\frac{1}{\epsilon_w}}}.$$

 $N_t(j)$  denotes the demand for a specific labour variety j, while  $N_t$  denotes aggregate labour demand. The elasticity of substitution between labour varieties is  $\epsilon_w$ . After labour packers assemble the labour bundle, they supply it to firms at wage  $W_t$  to firms, which is then used in the production process. Labour packers maximise the following objective,

$$\max_{N_t(j)} \left\{ W_t N_t - \int_0^1 N_t(j) W_t(j) dj \right\},\,$$

subject to the CES production function. The first-order condition,

$$W_t N_t^{\frac{1}{\epsilon_w}} N_t(j)^{-\frac{1}{\epsilon_w}} - W_t(j) = 0,$$

leads to following expression for labour demand,

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t.$$

As the labour packers are perfectly competitive, we can use the zero-profit condition to calculate the wage index,

$$W_t N_t - \int_0^1 N_t(j) W_t(j) dj = 0 \Longrightarrow$$
 $W_t N_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t W_t(j) dj,$ 
 $W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{\frac{1}{1-\epsilon_w}}.$ 

**Labour Unions** The labour union maximises

$$\max_{W_t(j)} \delta_t N_t^d W_t(j) - \frac{N_t^{1+\phi}}{1+\phi}$$

subject to

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t^d.$$

This is equivalent to the maximisation of household utility subject to the budget constraint and the demand for labour. The first order condition is given by

$$\delta_t \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} (1 - \epsilon_w) N_t^d + \epsilon_w \left[ \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t^d \right]^{\phi} \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w - 1} \frac{N_t^d}{W_t} = 0,$$

which yields

$$W_t(j) = \frac{\epsilon_w}{\epsilon_w - 1} \frac{(N_t^d)^{\phi}}{C_t^{\sigma}},$$

where  $\frac{\epsilon_w}{\epsilon_w-1}=\mathcal{M}^w$  and  $mc_t^w=\frac{(N_t^d)^\phi}{C_t^\sigma}$ . This characterizes the flexible wage setting. To introduce nominal and real wage stickiness, we assume that with probability  $\theta_w$ , a union cannot reoptimise its wage and is instead bound to a wage that is indexed to a composite price index from the previous period,

$$W_t(j) = \begin{cases} W_{t-1}(j) \left( (\Pi_w^{ss})^{1-\xi_w} (\Pi_{t-1}^w)^{\xi_w} \right) & \text{with prob } \theta_w \\ W_t^*(j) & \text{with prob } 1 - \theta_w \end{cases}.$$

Therefore,

$$W_{t+s}(j) = W_t^*(j) \left(\Pi_{ss}^W\right)^{s(1-\xi_w)} \left(\prod_{g=0}^{s-1} \left(\left(\Pi_{t+g}^W\right)^{\xi_w}\right)\right) = W_t^*(j) \left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)} \left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right].$$

Given this demand constraint and assuming that a union j always meets the demand for its labour at the current wage, labour unions solve the following optimisation problem,

$$\max_{W_t^*(j)} E_t \sum_{s=0}^{\infty} (\theta_w)^s \Lambda_{t,t+s} P_{t+s} \left[ \left( \frac{W_t^*(j)}{P_{t+s}} - m c_{t+s}^W \right) \left( \frac{W_t^*(j)}{W_{t+s}} \right)^{-\frac{M_w}{M_{w-1}}} N_{t+s} \right]. \tag{22}$$

Taking the derivative with respect to  $W_t^*(j)$  delivers the familiar wage inflation schedule,

$$\begin{split} \frac{f_{t}^{W,1}}{f_{t}^{W,2}}\mathcal{M}_{w} &= w_{t}^{*} = \frac{W_{t}^{*}}{W_{t}} = \left(\frac{1 - \theta_{w} \left(\zeta_{t}^{W}\right)^{\frac{1}{M_{w}-1}}}{1 - \theta_{w}}\right)^{1 - \mathcal{M}_{w}}, \\ f_{t}^{W,1} &= N_{t} \frac{mc_{t}^{W}}{w_{t}} + \theta_{w} E_{t} \left[\left(\Lambda_{t,t+1}\right) \left(\frac{\Pi_{t+1}^{W}}{\Pi_{t+1}^{CPI}}\right) \left(\frac{\Pi_{t+1}^{W}}{\Pi_{ss}^{W}}\right)^{\frac{M_{w}}{M_{w}-1}} f_{t+1}^{W,1}\right], \\ f_{t}^{W,2} &= N_{t} + \theta_{w} \beta E_{t} \left[\left(\Lambda_{t,t+1}\right) \left(\frac{\Pi_{t+1}^{W}}{\Pi_{t+1}^{CPI}}\right) \left(\frac{\Pi_{t+1}^{W}}{\Pi_{ss}^{W}}\right)^{\frac{1}{M_{w}-1}} f_{t+1}^{W,2}\right], \\ \zeta_{t}^{W} &= \Pi_{t}^{W} / \Pi_{ss}^{W}, \\ w_{t} &= \Pi_{t}^{W} / \Pi_{t} w_{t-1}, \\ \mathcal{D}_{t}^{W} &= (1 - \theta_{w}) \left(\frac{1 - \theta_{w} \left(\zeta_{t}^{W}\right)^{\frac{1}{M_{w}-1}}}{1 - \theta_{w}}\right)^{\mathcal{M}_{w}} + \theta_{w} \left(\zeta_{t}^{W}\right)^{\frac{M_{w}}{M_{w}-1}} \mathcal{D}_{t-1}^{W}, \end{split}$$

where  $D_t^W$  is wage dispersion. We calibrate  $\theta_w = 0.92$ ,  $\xi_w = 0$ ,  $\epsilon_w = 11$  as in Schmitt-Grohé and Uribe (2006) and Chan, Diz, and Kanngiesser (2024).

**Gradual Fragmentation** Figure 19 presents the gradual fragmentation scenario for a calibration with high wage stickiness ( $\theta_w = 0.92$ ). While the fall in real wages is more gradual with respect to the baseline model, a salient difference is the behaviour of employment, which falls by more in the non-tradable sector and no longer increases in the home tradable sector. Output in both sectors falls, following a similar trajectory to employment. As a result, aggregate output falls to a greater extent, owing to the fall in home tradables output, which no longer increases as in the baseline calibration.

Consumption falls by more in the short-run, but its trajectory is largely similar to that in the baseline model. Although employment falls, the more moderate decrease in real wages provides some support for consumption. Marginal costs are also affected by the more modest decline in real wages, which leads to a more moderate fall in non-tradables inflation. However, this is outweighed by a sharper fall in home-tradables inflation as demand decreases. As a result, aggregate CPI falls by more and this scenario leads to stagnation, as in our baseline calibration.

The demand-side effect of this gradual import price shock is broadly similar, as wage indexation slows the fall in real wages but leads to large falls in employment. Wage indexation therefore *moderates* the disinflationary pressures present in the baseline calibration: despite the larger fall in output, there is a roughly similar fall in aggregate CPI and a slight overshoot in the medium term. This scenario remains one of stagnation, as aggregate CPI inflation falls alongside a larger fall in aggregate output.

Agg. CPI Infl Non-Trad. Infl. Home Trad. Infl. Imported Infl. 0.06 -0.02 -0.05 -0.02 0.04 -0.04 -0.1 -0.04 0.02 -0.06 15 20 15 Quarter Quarter Quarter Quarter Trad. Infl Wage Infl Consumption Employment n -0.05 -0.05 -0.1 -0.1 -0.04 -0.2 0 -0.1 20 20 10 10 Quarter Quarter Quarter Quarter Wage Foreign Input Real Exc. Rate Output -0.2 -0.1 -0.1 -0.2 -0.4 -0.2 Quarter Quarter Quarter Quarter Non-Trad. Output Home Trad. Output Non-Trad. Empl. Home Trad. Empl 0.1 0 -0.05 -0.1 -0.1 -0.2 -0.2 -0.1 20 0 Quarter Nom. Interest Rate Real Interest Rate Natural Real Rate Foreign Price Level -0.02 -0.05 0.5 -0.04 -0.1 -0.1 15 15 15 Quarter Quarter Quarter Quarter

Figure 19: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:* IRFs to a 100% gradual positive foreign price shock. The results are generated under a RANK calibration with  $\theta_w = 0.92$ . All the other parameters are calibrated according to Table 1.

Front-loaded Fragmentation In our baseline calibration with flexible wages (Figure 5), this scenario led to a stagflationary outcome and a tradeoff for policymakers. With wage stickiness, this tradeoff is more severe, as output falls to a much greater extent, alongside aggregate CPI outturns that are larger and more persistent. As in the gradual scenario, nominal wage rigidities mitigate the fall in real wages and relative to the baseline calibration, real wages fall by less initially. In contrast to the increase in employment under flexible wages, employment falls but converges slightly above steady state. At the sectoral level, this is attributable to the large fall in home tradables employment (which increased in the flexible wage case). Non-tradable employment falls by more in the case of sticky wages. Aggregate

and sectoral output follow the same patterns.

The demand-side implications remain the same for this scenario: the more moderate fall in real wages counteracts the large fall in employment, preserving a similar fall in real incomes for households as in the baseline model. As a result, the fall in consumption is roughly similar in magnitude to the baseline calibration.

The trajectory for aggregate CPI inflation reflects to a large extent the behaviour of non-tradable inflation, which now increases and remains persistently above steady state. Despite the fall in consumption, non-tradable inflation increases with higher wage stickiness, as the fall in real wages is much smaller and no longer sufficient to offset the increase in imported input costs. As a result, aggregate CPI is slightly higher on impact and more persistent relative to the baseline calibration.

Fall in Tradables Productivity Figure 21 presents the responses to an adverse productivity shock in the tradable goods sector with higher wage stickiness. As in the other two scenarios, nominal wage rigidities mitigate the fall in wages that would otherwise occur under flexible wages, resulting in a more moderate decline in the real wage. The increase in marginal costs leads to a prolonged contraction in tradable output, while the increase in tradable employment is more modest compared to the flexible-wage case. The most notable difference with respect to the baseline calibration is the fall in aggregate employment; as the increase in tradable employment is much smaller with sticky wages, it no longer outweighs the larger fall in non-tradable employment. Aggregate consumption falls by roughly the same amount in the sticky-wage case, as real wages decline more moderately but employment also falls. Outcomes for aggregate CPI inflation and its subcomponents are roughly similar in magnitude to the baseline calibration.

To summarise, higher nominal wage rigidity does not change the qualitative interpretations and policy implications we draw from the various fragmentation scenarios. The demand-side effects in the various scenarios are largely similar, as wage indexation slows the fall in real wages but leads to a large decline in employment. As a result, wage indexation moderates the disinflationary effect in the gradual scenario: despite the larger fall in output, there is a roughly similar fall in aggregate CPI and a slight overshoot in the medium term. However, this scenario remains one of stagnation, as aggregate CPI inflation falls alongside a larger fall in aggregate output. In the front-loaded scenario, wage stickiness exacerbates the trade-off in the stagflationary scenario: there is a significant fall in output while the increase in aggregate CPI inflation is slightly higher on impact and more persistent. This is attributable to the behaviour of non-tradables inflation, which increases persistently as wage stickiness prevents a larger fall in real wages. Finally, the adverse TFP scenario is still characterised by moderate inflationary pressures. Wage stickiness mitigates the increase in tradable sector employment, while the demand-side impact is roughly similar as the more modest decline in real wages is offset by the fall in aggregate employment.

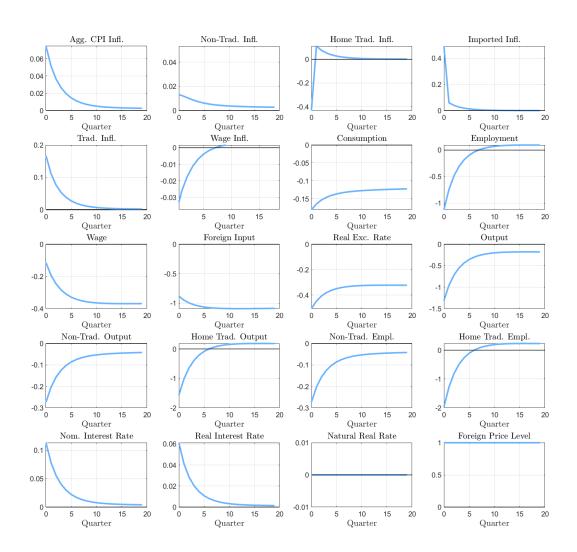


Figure 20: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:* IRFs to a 100% front-loaded positive foreign price shock. The results are generated under a RANK calibration with  $\theta_w = 0.92$ . All the other parameters are calibrated according to Table 1.

# 7 Conclusion

There is a broad consensus that a realignment of trading patterns is taking place. While trade fragmentation is likely to result in higher imported goods prices and lower real incomes, we show that the inflationary impact and the appropriate monetary policy response depend crucially on how demand adjusts to lower incomes, which in turn, depends on the form that fragmentation takes.

We study the macroeconomic effects of fragmentation using a two-sector, open economy New Keynesian model featuring imperfect risk sharing and heterogeneous households. We

Agg. CPI Infl Non-Trad. Infl Home Trad. Infl Imported Infl 0.03 0.015 0.001 0.01 0.02 0.01 0.00 0.002 0.01 0.005 0.001 0 20 15 20 15 20 Quarter Quarter Quarter Quarter Trad. Infl Wage Infl. Consumption Employment 0.03 0.02 -0.01 -0.05 -0.005 -0.02 0.01 -0.03 -0. -0.01 -0.04 15 20 20 10 15 20 Quarter Quarter Quarter Quarter Wage Real Exc. Rate Output 0.06 0 -0.01 -0.02 -0.05 0.04 -0.02 -0. 0.02 -0.03 -0.06 -0.15 0 -0.04 20 20 20 0 Quarter Quarter Quarter Quarter Non-Trad. Output Home Trad. Output Non-Trad. Empl. Home Trad. Empl. 0.06 -0.02 0.04 -0.04 -0.05 0.02 -0.06 10 20 10 10 Quarter Quarter Quarter Tradable TFP Level Nom. Interest Rate Real Interest Rate Natural Real Rate 0.02 0.015 0.01 0.005 -0.005 0.005 15 20 20 15 Quarter Quarter Quarter Quarter

Figure 21: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

Notes: IRFs to a negative TFP shock in the tradable sector. The results are generated under a RANK calibration with  $\theta_w = 0.92$ . All the other parameters are calibrated according to Table 1.

consider different fragmentation scenarios by varying the speed at which foreign prices adjust to a permanently higher level, as well as a negative shock to tradable sector productivity. The scenarios we consider capture the usual supply-side channels whereby higher input prices affect inflation, but emphasise how the overall effect on inflation depends on the adjustment of demand to lower real incomes. The balance between supply and demand differs in each of the scenarios, which has implications for inflation, and hence, monetary policy.

Our results suggest that trade fragmentation is not necessarily inflationary. In our first scenario, while the gradual and permanent increase in foreign prices yields a persistent increase in imported good price inflation, the pass-through to aggregate inflation is counteracted by a fall in domestic inflation. The key mechanism for this result is that forward-looking households reduce their spending in anticipation of more restrictive future supply, as they respond to lower permanent income by smoothing consumption. The natural rate of interest decreases, suggesting that, when we allow for demand to adjust, the overall effect is not inflationary. The economy enters a period of stagnation, with lower real incomes, lower demand and lower inflationary pressures. Adding investment and uncertainty to the model should exacerbate the negative impact on demand and its disiflationary effects. In our second scenario, a front-loaded and persistent increase in the price of imported goods leads to a temporary period of stagflation, with lower consumption and higher short-term inflationary pressures. Finally, a fall in the productivity of domestic tradable goods production is moderately inflationary. This scenario leads to higher marginal costs in the domestic tradable sector. However, the demand-side effects are different from the import price shocks, as labour income is less adversely affected.

To understand the various channels shaping the response of demand and supply under fragmentation, we compare our baseline results to a RANK model: the degree of forward-looking behaviour, the extent of demand spillovers between domestic agents, and the differential impact of the shock on labour and profit income are all important determinants of the aggregate demand response.

We also consider extensions with varying degrees of home bias, greater dependence on imported inputs in production, wage rigidities, price flexibility, and non-unitary elasticities of substitution. We find that macroeconomic outcomes in response to an increase in imported good prices are more adverse in open economies, as they are more exposed to foreign shocks. However, in the case of purely domestic shocks to tradables TFP, openness mitigates the supply-side impact by allowing for diversification away from home tradables.

We also show that our results are robust to a higher share of imported inputs in domestic production and greater wage rigidities, which both capture additional domestic supply-side constraints. In the case of greater reliance on imported inputs in production, our various fragmentation shocks lead to a rise in employment, either through factor substitution or adverse TFP. However, this only stimulates the demand-side moderately through labour income. In the case where imported inputs are a greater share of non-tradables production, the level of labour demand is already lower than our baseline calibration.

Our extension with wage rigidities lead to more moderate disinflation in the gradual fragmentation case and more persistent inflation in the front-loaded case. While wage stickiness moderates the fall in real wages, it also leads to a fall in labour demand. The demand-side effect of our various shocks are therefore preserved, as real wages fall more moderately but employment also falls more steeply. Relatedly, allowing prices to adjust more frequently increases disinflationary pressures in the gradual scenario, while reducing the trade-off for policymakers in the front-loaded case.

Finally, increased substitutability between home and foreign tradables, and lower substi-

tutability between tradables and non-tradables leads to a less adverse terms-of-trade shock for the domestic economy. This specification seems to matter most for the gradual scenario, where it leads to less stagnation relative to the baseline calibration. Overall, these extensions highlight the robustness of our main conclusions across the different fragmentation scenarios as well as the underlying mechanisms.

Finally, we study a social planner's response to the shocks in our different fragmentation scenarios. Comparing the efficient allocation to the decentralised equilibrium isolates the sources of inefficiencies in the economy's adjustment to the various fragmentation shocks. Sticky prices in the non-tradable sector create a wedge between prices and marginal cost, while the cost of adjusting bond positions results in a suboptimal intertemporal response to shocks. In the case of import price shocks, there is an overallocation of resources to the home tradable sector while in the case of an adverse shock to home tradables, there is an underallocation of resources to the non-tradable sector. In the case of a gradual increase in import prices, foreign assets are underused to smooth through shocks. The extent of domestic reallocation and external adjustment in the response to shocks is generally inefficient in the decentralised equilibrium, resulting in excess of hours worked. In all cases, by choosing a different composition for consumption, production input intensity, and labour allocation, the social planner is able to trade off a slightly bigger fall in aggregate consumption for a much smaller increase in employment.

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## A Model

#### A.1 First Order Conditions - Unconstrained Households

Unconstrained households maximise their expected lifetime utility by choosing a sequence  $\{C_t^U, N_t^U, b_t, b_t^*\}_{t=0}^{\infty}$  subject to the series of budget constraints (2). For unconstrained households, the first-order conditions with respect to  $C_t^U, N_t^U, b_t, b_t^*$  are respectively given by:

$$(C_t^U)^{-\sigma} = \delta_t$$

$$\kappa_\ell(N_t^U)^\phi = \delta_t \frac{W_t}{P_t}$$

$$\delta_t = \beta \mathbb{E}_t \left[ \frac{(1+i_t)}{(1+\pi_{t+1})} \delta_{t+1} \right]$$

$$\delta_t[S_t + S_t \chi(b_t^* - b^*)] = \beta \mathbb{E}_t \left[ \frac{(1+i_t^*)}{(1+\pi_{t+1}^*)} S_{t+1} \delta_{t+1} \right]$$

where  $\delta_t$  is the Lagrange multiplier on the budget constraint.

#### A.2 Non-Tradable Firms Problem

Firm i in the non-tradable sector maximises profits

$$\max_{M_{F,t}(i),N_{N,t}(i),P_{N,t}(i)} P_{N,t}(i) Y_{N,t}(i) - (1-\tau) \left( P_{F,t} M_{F,t}(i) - W_t N_{N,t}(i) \right) - AC_t(i)$$

subject to its production technology (12) and the demand function for its product (11). This problem yields the following first-order conditions

$$(1 - \tau)W_t = MC_t(i)(1 - \kappa)\frac{Y_{N,t}(i)P_{N,t}(i)}{N_{N,t}(i)}$$
(A.1)

$$(1 - \tau)P_{F,t} = MC_t(i)\kappa \frac{Y_{N,t}(i)P_{N,t}(i)}{M_{F,t}(i)}$$
(A.2)

where  $MC_t(i)$  is the Lagrange multiplier on the technology constraint. We can interpret this multiplier as the shadow cost of producing an additional unit of good  $Y_{N,t}(i)$ , that is, the marginal cost. Substituting equations (A.1) and (A.2) into the production function we obtain demand functions for the two production inputs,

$$N_{N,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}} \left[ \frac{1 - \kappa}{\kappa} \frac{P_{F,t}}{W_t} \right]^{\kappa}, \quad M_{F,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}} \left[ \frac{\kappa}{1 - \kappa} \frac{W_t}{P_{F,t}} \right]^{1 - \kappa}$$

The total factor cost function is equal to

$$TC_{t}(i) (W_{t}, Y_{N,t}(i), P_{F,t}, A_{N,t}) = (1 - \tau) (W_{t}N_{N,t} + P_{F,t}M_{F,t})$$

$$= (1 - \tau) \left(\frac{Y_{N,t}(i)}{A_{t}}W_{t}^{1-\kappa}P_{F,t}^{\kappa}\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right]\right)$$
(A.3)

Differentiation of the total cost function yields marginal cost,

$$MC_{N,t}(i) = (1 - \tau) \frac{W_t^{1-\kappa} P_{F,t}^{\kappa}}{A_t} \left[ \left( \frac{\kappa}{(1 - \kappa)} \right)^{1-\kappa} + \left( \frac{(1 - \kappa)}{\kappa} \right)^{\kappa} \right]$$
(A.4)

Since equation (A.4) is solely a function of factor prices and productivity,  $MC_{N,t}(i) = MC_{N,t} \, \forall i$ . We can rewrite nominal marginal cost in real terms as follows,

$$MC_{N,t}^{r}(i) = \frac{MC_{N,t}}{P_t} = \frac{(1-\tau)}{A_t} \left(\frac{W_t}{P_t}\right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t}\right)^{\kappa} \left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right]$$
(A.5)

#### A.3 Tradable Firms Problem

Tradable sector production is given by

$$Y_{H,t} = A_{H,t} N_{H,t}^{1-\zeta} (A.6)$$

where  $A_{H,t} = (A_{H,t}^{ss})^{1-\rho_h} A_{H,t-1}^{\rho_h} \epsilon_{H,t}$ , and  $\rho_h \in (0,1]$ . Therefore, the problem of the tradable firm is to maximise profits

$$\max_{N_{H,t}} P_{H,t} Y_{H,t} - W_t N_{H,t}$$

subject to its production technology (15). This yields

$$\frac{W_t}{P_{H,t}} = (1 - \zeta)A_{H,t}N_{H,t}^{-\zeta} \implies W_t N_{H,t} = (1 - \zeta)Y_{H,t}P_{H,t}$$
(A.7)

which gives us profits  $\Psi_{H,t} = (\zeta) P_{H,t} Y_{H,t}$ .

### A.4 Exogenous processes

$$P_{F,t}^* = (P_{F,t}^{*ss})^{1-\rho_F} (P_{F,t-1}^*)^{\rho_F} \exp(\nu_{F,t}) \quad \rho_F \in [0,1]$$
(A.8)

$$A_{N,t} = (A_N^{ss})^{1-\rho_N} A_{N,t-1}^{\rho_N} \exp(\nu_{N,t}), \quad \rho_N \in (0,1]$$
(A.9)

$$A_{H,t} = (A_{H,t}^{ss})^{1-\rho_H} A_{H,t-1}^{\rho_H} \exp(\nu_{H,t}), \quad \rho_H \in (0,1]$$
(A.10)

$$P_{H,t}^* = \left(P_{H,t}^{*ss}\right)^{1-\rho_{H^*}} \left(P_{H,t-1}^*\right)^{\rho_{H^*}} \exp(\nu_{H^*,t}) \quad \rho_{H^*} \in (0,1]$$
(A.11)

$$C_t^* = (C_t^{*ss})^{1-\rho_{C^*}} \left(C_{t-1}^*\right)^{\rho_{C^*}} \exp(\nu_{C^*,t}) \quad \rho_{C^*} \in (0,1]$$
(A.12)

$$r_t^* = (r_t^{*ss})^{1-\rho_{r^*}} (r_{t-1}^*)^{\rho_{r^*}} \exp(\nu_{r^*,t}) \quad \rho_{r^*} \in (0,1]$$
(A.13)

# A.5 Equilibrium Conditions

We can derive demand functions for  $Y_{H,t}$  and  $Y_{N,t}$  in order to obtain the market clearing conditions. Tradables are by definition consumed both domestically and in the rest of the

world,

$$C_{H,t} = (1 - \theta) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} C_{T,t} = (1 - \theta)(1 - \varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_{t}}{P_{T,t}}\right)^{\iota} C_{t}$$

$$C_{H,t}^{*} = \theta^{*} \left(\frac{P_{t}}{P_{H,t}}\right)^{\mu} \mathcal{S}_{t}^{\mu} C_{t}^{*}$$

$$Y_{H,t} = (1 - \theta)(1 - \varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_{t}}{P_{T,t}}\right)^{\iota} C_{t} + \theta^{*} \left(\frac{P_{t}}{P_{H,t}}\right)^{\mu} \mathcal{S}_{t}^{\mu} C_{t}^{*}$$

where  $C_{H,t}^*$ , the foreign tradable demand, is derived by assuming symmetric preferences in the rest of the world. For non-tradables, instead, we calculate domestic demand as follows,

$$C_{N,t} = \varsigma \left(\frac{P_t}{P_{N,t}}\right)^t C_t \tag{A.14}$$

Therefore, we can express domestic aggregate output as follows,

$$Y_{t} = \varsigma \left(\frac{P_{t}}{P_{N,t}}\right)^{\iota} C_{t} + (1 - \theta)(1 - \varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_{t}}{P_{T,t}}\right)^{\iota} C_{t} + \theta^{*} \left(\frac{P_{t}}{P_{H,t}}\right)^{\mu} \mathcal{S}_{t}^{\mu} C_{t}^{*}$$
(A.15)

### A.6 Natural Level of Output

To derive  $Y_t^n$ , we set the real marginal cost to the inverse of the desired markup,

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}$$

Using the real marginal cost in equation (A.5), we obtain

$$\frac{\epsilon - 1}{\epsilon} = \frac{(1 - \tau)}{A_{N,t}} \left(\frac{W_t}{P_t}\right)^{1 - \kappa} \left(\frac{P_{F,t}}{P_t}\right)^{\kappa} \left[ \left(\frac{\kappa}{1 - \kappa}\right)^{1 - \kappa} + \left(\frac{1 - \kappa}{\kappa}\right)^{\kappa} \right]$$
(A.16)

Under perfectly competitive labour markets, we can write the labour supply condition as follows<sup>15</sup>

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\phi}$$

Therefore,

$$\frac{\epsilon - 1}{\epsilon} = \frac{(1 - \tau)}{A_{N,t}} \left( (C_t^n)^{\sigma} (N_t^n)^{\phi} \right)^{1 - \kappa} \left( \frac{P_{F,t}}{P_t} \right)^{\kappa} \left[ \left( \frac{\kappa}{(1 - \kappa)} \right)^{1 - \kappa} + \left( \frac{(1 - \kappa)}{\kappa} \right)^{\kappa} \right]$$
(A.17)

Next, we express  $C_t$  in terms of  $C_t^*$ . Using the household's first-order condition on foreign bond holding, and assuming that the same conditions hold in the foreign economy,

$$\frac{1}{(1+i_t^*)} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{(1+\pi_{t+1}^*)} \right]. \tag{A.18}$$

<sup>&</sup>lt;sup>15</sup>Galí, López-Salido, and Vallés (2007).

If  $\Pi = \Pi^* = 1$  in steady state, then  $r = r^* = \frac{1}{\beta}$ . We can therefore write the international risk-sharing condition,

$$\mathbb{E}_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^{\sigma} \right] = \mathbb{E}_t \left[ \left( \frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \frac{S_t}{S_{t+1}} \right] \left[ 1 + \chi(b_t^* - b^*) \right]$$
(A.19)

which we can rewrite as

$$\mathcal{D}_t^{rac{1}{\sigma}} = rac{C_t}{C_t^* \mathcal{S}_t} \qquad \mathcal{D}_t^{rac{1}{\sigma}} = (1 + \chi(b_t^* - b^*))$$

Using the goods market clearing equation and the assumption that international markets are incomplete, we can write

$$Y_{t} = \varsigma \left(\frac{P_{t}}{P_{N,t}}\right)^{\iota} C_{t} + (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_{t}}{P_{T,t}}\right)^{\iota} C_{t} + \theta^{*} \left(\frac{P_{t}}{P_{H,t}}\right)^{\mu} \mathcal{S}_{t}^{\mu} C_{t}^{*}$$

$$Y_{t} = \left[\varsigma \left(\frac{P_{t}}{P_{N,t}}\right)^{\iota} + (1-\theta)(1-\varsigma) \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\mu} \left(\frac{P_{t}}{P_{T,t}}\right)^{\iota} + \theta^{*} \left(\frac{P_{t}}{P_{H,t}}\right)^{\mu} \frac{\mathcal{S}_{t}^{\mu-1}}{\mathcal{D}_{t}^{\frac{1}{\sigma}}}\right] C_{t}$$

$$C_{t} = \frac{1}{\Sigma_{\varsigma\theta,t}} Y_{t}$$

where  $\Sigma_{\varsigma\theta,t} = \left[\varsigma\left(\frac{P_t}{P_{N,t}}\right)^t + (1-\theta)(1-\varsigma)\left(\frac{P_{T,t}}{P_{H,t}}\right)^\mu\left(\frac{P_t}{P_{T,t}}\right)^t + \theta^*\left(\frac{P_t}{P_{H,t}}\right)^\mu\frac{S_t^{\mu-1}}{\mathcal{D}_t^{\frac{1}{\theta}}}\right]$ . The relationship between domestic consumption and aggregate domestic production is described by  $\Sigma_{\varsigma\theta,t}$ . In an open economy, these two variables do not move in lockstep; instead, their dynamics are influenced by the degree of openness to trade. A higher  $\theta$  widens the wedge between consumption and production, attributable to the increased share of tradable goods destined for export.

Since  $N_t^n = N_{N,t}^n + N_{H,t}^n$ , we can write

$$N_{t}^{n} = \left(\frac{Y_{H,t}^{n}}{A_{H,t}}\right)^{\frac{1}{1-\zeta}} + Y_{N,t}^{n} \left(\frac{P_{F,t}}{P_{N,t}}\right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}}$$

Substituting back in equation (A.17), we obtain

$$Y_t^n = \Sigma_{\varsigma\theta,t} \left\{ \left( A_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}} \right) \left[ N_{H,t}^n + N_{NT,t}^n \right]^{-\phi} \right\}^{\frac{1}{\sigma}}$$
(A.20)

Equation (A.20) describes the natural level of output, which can vary with technology in both sectors, relative prices of input and foreign demand. Finally, the natural real interest rate is the risk-free real interest rate consistent with the Euler equation when output is at its

natural level at all times,

$$(C_{t+1}^n)^{\sigma} = \beta(1 + r_t^n)(C_t^n)^{\sigma}$$
  
 $(1 + r_t^n) = \frac{1}{\beta} \left( \left( \frac{Y_{t+1}^n}{Y_t^n} \right) \frac{\Sigma_{\zeta\theta,t}}{\Sigma_{\zeta\theta,t+1}} \right)^{\sigma},$ 

where we have used  $C_t = \frac{1}{\Sigma_{c\theta,t}} [Y_t]$ .

## **B** Efficient Allocation

Assuming Cole-Obstfeld conditions, we can simplify the following expressions

$$\begin{split} P_{t} &= P_{T,t}^{1-\varsigma} P_{N,t}^{\varsigma} \\ P_{T,t} &= P_{H,t}^{1-\theta} P_{F,t}^{\theta} \\ C_{T,t} &= \frac{C_{H,t}^{1-\theta} C_{F,t}^{\theta}}{\theta^{\theta} (1-\theta)^{(1-\theta)}} \\ C_{t} &= \frac{C_{N,t}^{\varsigma} C_{T,t}^{1-\varsigma}}{\varsigma^{\varsigma} (1-\varsigma)^{1-\varsigma}} = \frac{C_{N,t}^{\varsigma}}{\varsigma^{\varsigma} (1-\varsigma)^{1-\varsigma}} \left( \frac{C_{H,t}^{1-\theta} C_{F,t}^{\theta}}{\theta^{\theta} (1-\theta)^{(1-\theta)}} \right)^{1-\varsigma} \end{split}$$

where  $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ .

$$\max_{\substack{\{C_{H,t},C_{F,t},C_{N,t},\\N_{N,t},N_{H,t},M_{F,t},B_t^*\}_{t=0}^{\infty}}} \sum_{t=0}^{\infty} \beta^t \left[ (1-\varsigma)(\theta \log C_{F,t} + (1-\theta) \log C_{H,t}) + \varsigma \log C_{N,t} - \frac{(N_{H,t} + N_{N,t})^{1+\phi}}{1+\phi} \right]$$

subject to:

$$A_{N,t}M_{F,t}^{\kappa}N_{N,t}^{1-\kappa} + A_{H,t}N_{H,t}^{1-\zeta} = C_{H,t} + C_{N,t} + C_{H,t}^{*}$$

$$P_{F,t}^{*}C_{F,t} = P_{H,t}^{*}C_{H,t}^{*} - P_{F,t}^{*}M_{F,t} + (1+i_{t-1}^{*})B_{t-1}^{*} - \frac{\chi}{2}(B_{t}^{*} - \bar{B}^{*})^{2} - B_{t}^{*}$$
(A.1)
$$(A.2)$$

$$C_{H,t}^* = \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^* C_t^*$$
(A.3)

The first constraint captures a resource constraint: output from the two sectors in the domestic economy is either consumed domestically or exported (as foreign consumption of domestic tradables). The second constraint describes the trade balance: the value of domestic consumption of foreign goods is equal to the net foreign asset position plus the revenue from exporting tradables for foreign consumption, minus the cost of imported intermediate inputs. Finally, the third constraint captures foreign demand for home produced goods.

The system of demand functions is given by

$$C_{H,t} = (1 - \theta) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} C_{T,t}$$

$$C_{F,t} = \theta \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} C_{T,t}$$

$$C_{N,t} = \varsigma \left(\frac{P_{N,t}}{P_t}\right)^{-1} C_t$$

$$C_{T,t} = (1 - \varsigma) \left(\frac{P_{T,t}}{P_t}\right)^{-1} C_t$$

$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\epsilon} C_{N,t}.$$

The terms of trade are defined as the price of imports in terms of the price of domestic goods.

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}.$$

The relevant price index for  $P_{F,t}$ ,  $P_{H,t}$  in our case is  $P_t = P_{N,t}^{\varsigma} P_{H,t}^{(1-\varsigma)(1-\theta)} P_{F,t}^{(1-\varsigma)\theta}$ . So we can write:

$$\begin{split} \frac{P_{H,t}}{P_t} &= \frac{P_{H,t}}{P_{N,t}^{\varsigma} P_{H,t}^{(1-\varsigma)(1-\theta)} P_{F,t}^{(1-\varsigma)\theta}} = \mathcal{T}_t^{-\theta(1-\varsigma)} \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\varsigma} \\ \frac{P_{F,t}}{P_t} &= \frac{P_{F,t}}{P_{N,t}^{\varsigma} P_{H,t}^{(1-\varsigma)(1-\theta)} P_{F,t}^{(1-\varsigma)\theta}} = \mathcal{T}_t^{1-\theta(1-\varsigma)} \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\varsigma}. \end{split}$$

This means that we can rewrite the system of demand as

$$C_{H,t} = (1 - \theta) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-1} (1 - \varsigma) \left(\frac{P_{T,t}}{P_t}\right)^{-1} C_t = (1 - \theta) \left(\frac{P_t}{P_{H,t}}\right) (1 - \varsigma) C_t$$

$$= (1 - \theta) (1 - \varsigma) C_t \mathcal{T}_t^{\theta(1 - \varsigma)} \left(\frac{P_{N,t}}{P_{H,t}}\right)^{\varsigma}$$

$$C_{F,t} = \theta \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-1} (1 - \varsigma) \left(\frac{P_{T,t}}{P_t}\right)^{-1} C_t = \theta \left(\frac{P_t}{P_{F,t}}\right) (1 - \varsigma) C_t = \theta (1 - \varsigma) C_t \mathcal{T}_t^{\theta(1 - \varsigma) - 1} \left(\frac{P_{N,t}}{P_{H,t}}\right)^{\varsigma}$$

The law of one price requires that  $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ ,  $P_{H,t} = \mathcal{E}_t P_{H,t}^*$ . For the rest of the world, we assume that  $\alpha^* \to 0$ ,  $\alpha^* C_t^* > 0$ . We also assume that the rest of the world does not have any non-traded goods in its consumption basket,  $\varsigma^* \to 0$ . Therefore, we can write:

$$\mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\mathcal{E}_t (P_{F,t}^*)}{P_t} = \frac{P_{F,t}}{P_t} = \mathcal{T}_t^{1-\theta(1-\varsigma)} \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\varsigma}$$

As the foreign demand for the home-produced good is:

$$C_{H,t}^* = \mathcal{T}_t \theta^* C_t^*$$

We can use the system of demand to rewrite  $\mathcal{T}_t$  by taking the ratio of  $C_{H,t}$ ,  $C_{F,t}$ :

$$\frac{C_{H,t}}{C_{F,t}} = \frac{(1-\theta)(1-\varsigma)}{\theta(1-\varsigma)C_t} C_t \mathcal{T}_t^{\theta(1-\varsigma)} \left(\frac{P_{N,t}}{P_{H,t}}\right)^{\varsigma} \mathcal{T}_t^{1-\theta(1-\varsigma)} \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\varsigma} \\
\frac{C_{H,t}}{C_{F,t}} = \frac{(1-\theta)(1-\varsigma)}{\theta(1-\varsigma)} \mathcal{T}_t \\
\mathcal{T}_t = \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)}$$

Finally, we obtain

$$C_{H,t}^* = \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^* C_t^*$$
(A.4)

### **B.1** Lagrangian

$$\begin{split} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^{t} \Big\{ (1-\varsigma)(\theta \log C_{F,t} + (1-\theta) \log C_{H,t}) + \varsigma \log C_{N,t} - \frac{(N_{H,t} + N_{N,t})^{1+\phi}}{1+\phi} \\ &+ \vartheta_{t} \Big[ A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{1-\kappa} + A_{H,t} N_{H,t}^{1-\zeta} - C_{H,t} - C_{N,t} - \frac{C_{H,t}}{C_{F,t}} \frac{\theta (1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^{*} C_{t}^{*} \Big] \\ &+ \omega_{t} \Big[ P_{H,t}^{*} \frac{C_{H,t}}{C_{F,t}} \frac{\theta (1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^{*} C_{t}^{*} - P_{F,t}^{*} M_{F,t} - P_{F,t}^{*} C_{F,t} \\ &+ (1+i_{t-1}^{*}) B_{t-1}^{*} - \frac{\chi}{2} (B_{t}^{*} - \bar{B}^{*})^{2} - B_{t}^{*} \Big] \Big\} \end{split}$$

Let us take first order conditions:

wrt to 
$$C_{H,t}: \frac{(1-\theta)(1-\varsigma)}{C_{H,t}} + \vartheta_t \left[ -1 - \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^*}{C_{F,t}} \right] + \omega_t \left[ \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^*}{C_{F,t}} P_{H,t}^* \right] = 0$$
wrt to  $C_{F,t}: \frac{\theta(1-\varsigma)}{C_{F,t}} + \vartheta_t \left[ \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^* C_{H,t}}{C_{F,t}^2} \right] + \omega_t \left[ -\frac{\theta}{(1-\theta)} \frac{\theta^* C_t^* C_{H,t}}{C_{F,t}^2} P_{H,t}^* - P_{F,t}^* \right] = 0$ 
wrt to  $C_{N,t}: \frac{\varsigma}{C_{N,t}} + \vartheta_t [-1] = 0$ 
wrt to  $N_{N,t}: -(N_{H,t} + N_{N,t})^{\phi} + \vartheta_t [A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{-\kappa} (1-\kappa)] = 0$ 
wrt to  $N_{H,t}: -(N_{H,t} + N_{N,t})^{\phi} + \vartheta_t [A_{H,t} N_{H,t}^{-\zeta} (1-\zeta)] = 0$ 
wrt to  $M_{F,t}: \vartheta_t \left[ A_{N,t} M_{F,t}^{\kappa-1} N_{N,t}^{1-\kappa} \kappa \right] + \omega_t [-P_{F,t}^*] = 0$ 
wrt to  $B_t^*: \omega_t [-\chi (B_t^* - \bar{B}^*) - 1] + \beta \omega_{t+1} [(1+i_t^*)] = 0$ 

Let us rewrite:

$$\begin{split} \frac{(1-\theta)(1-\varsigma)}{C_{H,t}} &= \vartheta_t \left[ 1 + \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^*}{C_{F,t}} \right] - \omega_t \left[ \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^*}{C_{F,t}} P_{H,t}^* \right] \\ & \frac{\theta(1-\varsigma)}{C_{F,t}} = \omega_t \left[ \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^* C_{H,t}}{C_{F,t}^2} P_{H,t}^* + P_{F,t}^* \right] - \vartheta_t \left[ \frac{\theta}{(1-\theta)} \frac{\theta^* C_t^* C_{H,t}}{C_{F,t}^2} \right] \\ & \frac{\varsigma}{C_{N,t}} = \vartheta_t \\ & (N_{H,t} + N_{N,t})^{\phi} = \vartheta_t [A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{-\kappa} (1-\kappa)] \\ & (N_{H,t} + N_{N,t})^{\phi} = \vartheta_t [A_{H,t} N_{H,t}^{-\zeta} (1-\zeta)] \\ & \vartheta_t \left[ A_{N,t} M_{F,t}^{\kappa-1} N_{N,t}^{1-\kappa} \kappa \right] = \omega_t P_{F,t}^* \\ & \beta \omega_{t+1} [(1+i_t^*)] = \omega_t [\chi(B_t^* - \bar{B}^*) + 1] \\ & A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{1-\kappa} + A_{H,t} N_{H,t}^{1-\zeta} = C_{H,t} + C_{N,t} + \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^* C_t^* \\ & P_{F,t}^* C_{F,t} = P_{H,t}^* \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^* C_t^* - P_{F,t}^* M_{F,t} + (1+i_{t-1}^*) B_{t-1}^* - \frac{\chi}{2} (B_t^* - \bar{B}^*)^2 \end{split}$$

From here we can solve for the steady state through these equations:

$$\begin{split} N_{N,t} &= \left(\frac{A_{N,t} M_{F,t}^{\kappa}}{A_{H,t}} \frac{(1-\kappa)}{(1-\zeta)} N_{H,t}^{\zeta}\right)^{\frac{1}{\kappa}} \\ C_{N,t} &= \frac{\varsigma}{(N_{H,t} + N_{N,t})^{\phi}} [A_{H,t} N_{H,t}^{-\zeta} (1-\zeta)] \\ \frac{\varsigma}{C_{N,t}} &= \vartheta_{t} \\ \frac{\vartheta_{t}}{P_{F,t}^{*}} \left[A_{N,t} M_{F,t}^{\kappa-1} N_{N,t}^{1-\kappa} \kappa\right] = \omega_{t} \\ \frac{(1-\theta)(1-\varsigma)}{\vartheta_{t} \left[1 + \frac{\theta}{(1-\theta)} \frac{\vartheta^{*}C_{t}^{*}}{C_{F,t}}\right] - \omega_{t} \left[\frac{\theta}{(1-\theta)} \frac{\vartheta^{*}C_{t}^{*}}{C_{F,t}} P_{H,t}^{*}\right]} = C_{H,t} \\ C_{F,t} &= \frac{C_{H,t}}{A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{1-\kappa} + A_{H,t} N_{H,t}^{1-\zeta} - C_{H,t} - C_{N,t}} \frac{\theta}{(1-\theta)} \theta^{*} C_{t}^{*} \\ N_{H,t} &= \vartheta_{t} [A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{-\kappa} (1-\kappa)] - N_{N,t} \quad \phi = 1 \\ P_{F,t}^{*} M_{F,t} &= P_{H,t}^{*} \frac{C_{H,t}}{C_{F,t}} \frac{\theta(1-\varsigma)}{(1-\theta)(1-\varsigma)} \theta^{*} C_{t}^{*} - P_{F,t}^{*} C_{F,t} + (1+i_{t-1}^{*}) B_{t-1}^{*} - \frac{\chi}{2} (B_{ty}^{*} - \bar{B}^{*})^{2} - B_{t}^{*} \end{split}$$

# C Role of Price Rigidity

Under Rotemberg pricing, our model does not capture the increased frequency of price adjustments following the recent sizable and front-loaded increase in energy prices (Cavallo, Lippi, and Miyahara (2024), Morales-Jiménez and Stevens (2024), Montag and Villar Vallenas (2025), etc.). Our main calibration uses an empirically realistic average price duration

of three quarters (9 months). However, to better understand the role of price rigidities, we run our specification with an implied duration of 1.25 quarters (slightly less than 4 months).

This change mainly affects the non-tradable sector, as only non-tradable firms face nominal rigidities. Indeed, in all three scenarios, we find that non-tradable inflation falls relatively more than in our baseline calibration. In line with the empirical evidence post-2021, a higher frequency of price adjustment matters particularly in the front-loaded case, where non-tradable inflation falls by nearly twice as much as in the baseline calibration (Figure 8). This leads to a smaller increase in aggregate CPI and therefore in the nominal interest rate. Our main conclusions remain are robust to the higher frequency of price change.

Gradual Fragmentation In the case where import prices rise gradually, the differences relative to the baseline calibration are most salient in the short-run. With more flexible prices, non-tradables output and employment fall by less, while prices in the non-tradable sector adjust, and fall, by more in response to the import price shock. Meanwhile, output and employment in the domestic tradable sector increase by less as well. Consequently, total output falls by less in the short run as well. Overall, and comparing with the baseline calibration, price rigidities had mitigated the fall in non-tradables prices. With more flexible prices, the more moderate increase in supply capacity in the domestic tradable sector also leads to a smaller fall in tradables inflation. As non-tradables inflation falls by more, while domestic tradables inflation falls by less, the response of aggregate CPI inflation is similar to the baseline calibration.

Front-loaded Fragmentation The front-loaded nature of the import price shock in this scenario exacerbates the dynamics in the gradual fragmentation scenario. Relative to the baseline calibration, more flexible prices allow non-tradables inflation to fall to a greater extent, while moderating the fall in non-tradables output and employment. Home tradables output and employment increase by less as well. The much larger fall in non-tradables inflation moderates the spike in aggregate CPI. As a result, real wages do not fall by as much as the baseline calibration. Consequently, both constrained and unconstrained household consumption do not fall by as much as the baseline calibration. As output falls by less and aggregate CPI inflation increases by less, this scenario is less stagflationary under more flexible prices.

**Fall in Tradables Productivity** The most salient difference in this extension relative to the baseline calibration (Figure 9) is in the behaviour of the RANK responses. <sup>16</sup> In the case where tradables productivity deteriorates to a permanently lower level, more flexible prices

<sup>&</sup>lt;sup>16</sup>The difference in the TANK responses in this extension and the baseline calibration is negligible. The TFP shock affects the tradable sector, where prices are determined in global markets. Firm profits are received from both sectors, and accrue to unconstrained households. However, only firm profits in the non-tradable sector are affected by increased price flexibility.

Agg. CPI Infl. Non-Trad. Infl. Imported Infl. <10<sup>-3</sup> Trad. Infl. Home Trad. Infl -0.02 0.1 -0.04 0.05 -0.06 20 Quarter Quarter Quarter Quarter Quarter ConsumptionUnconstrained Cons Constrained Cons Unconstrained Empl. Constrained Empl. 0.03 -0.05 -0.05 -0.05 0.02 -0.1 -0. Quarter Quarter Quarter Quarter Quarter Non-Trad. Output Home Trad. Output Non-Trad. Empl Home Trad. Empl 0.1 -0.05 -0.01 -0.01 -0. -0.015 -0.02 20 10 Quarter Quarter Quarter Quarter Quarter Wage Foreign Input Nom. Interest Rate Real Interest Rate Natural Real Rate -0.05 -0.01 -0. -0.02 -0.4 -0. -0.15 -0.03 -0.2 -0.15 L -0.6 0 10 Quarter 10 20 Quarter Real Exc. Rate Foreign Price Level -0.05 -0.1 0.5 RANK TANK -0.15 -0.2 10 Quarter 10 Quarter

Figure 22: IRFs to a Gradual Increase in Foreign Price Level.

*Notes:*  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line, with implied price duration of 1.25 quarters. All the other parameters are calibrated according to Table 1.

also lead to a much more moderate fall in non-tradables output and employment. The margin of adjustment is in prices instead, where they fall by more in the non-tradable sector with flexible prices. The fall in non-tradables inflation is greater than the baseline calibration, but still minor, leaving aggregate CPI inflation similar in magnitude.

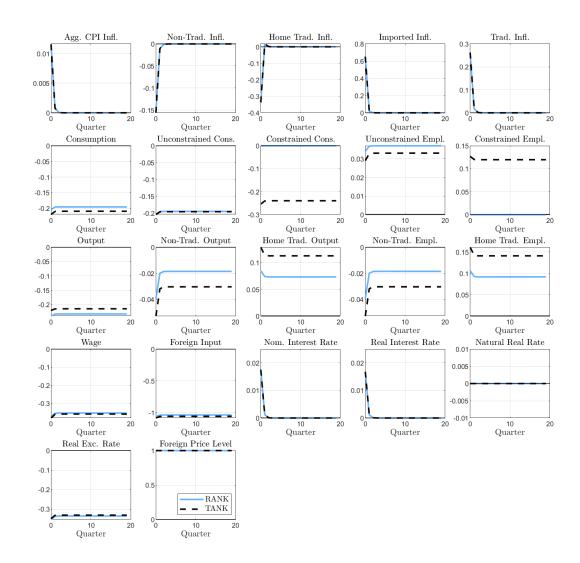


Figure 23: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes:*  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line, with implied price duration of 1.25 quarters. All the other parameters are calibrated according to Table 1.

# D Non-unitary Elasticities of Substitution

In this section, we present our main results for a higher elasticity of substitution between domestic and foreign tradable goods ( $\mu=6$ ) and lower elasticity of substitution between tradable and non-tradable goods ( $\iota=0.55$ ), following Corsetti, Dedola, and Leduc (2009). Keeping the remaining parameters unchanged, this exercise shows that our main results are not dependent on the assumption of unitary elasticity of substitution between non-tradables and tradables and between foreign and domestic tradables.

Agg. CPI Infl. Non-Trad. Infl. Home Trad. Infl Trad. Infl. Imported Infl 0.01 0.01 0.04 0.04 0.04 0.005 0.03 0.03 0.02 0.02 0.02 -0.005 -0.01 20 20 20 Quarter Quarter Quarter Quarter Quarter Constrained Empl Consumption Unconstrained Cons Constrained Cons Unconstrained Empl. 0. -0.01 -0.02 -0.02 -0.02 0.05 0.01 -0.04 -0.03 0.005 20 10 20 20 Quarter Quarter Quarter Quarter Quarter Non-Trad. Empl. Non-Trad. Output Home Trad. Output Home Trad. Empl 0.05 0.1 -0.0 -0.01 0.15 0.05 -0.02 -0.03 -0.03 0.05 20 0 Quarter Quarter Quarter Quarter Foreign Input Nom. Interest Rate Real Interest Rate Natural Real Rate Wage 0.0 0.01 -0.02 0.005 0.00 -0. -0.06 -0.005 -0.005 -0.01 -0.01 -0.01 -0.08 20 10 20 10 20 20 Quarter Quarter Quarter Real Exc. Rate Tradable TFP Level RANK -0.05 TANK 0.02 -0.1 0

Figure 24: IRFs to a Negative Gradual and Permanent Shock to Tradable TFP.

*Notes*:  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line, with implied price duration of 1.25 quarters. All the other parameters are calibrated according to Table 1.

10 Quarter

10 Quarter

20

Gradual Fragmentation Under this calibration, the gradual scenario still leads to stagnation, as aggregate CPI inflation falls alongside output. However, the outcomes are less adverse than in the baseline calibration. The import price shock still leads to a fall in nontradable output and employment, with a short-run impact comparable to the baseline case. Comparing the two calibrations suggests that the medium-term recovery in the non-tradable sector is in part driven by substitution from tradables to non-tradables. Instead, in this calibration, output and employment in the the non-tradable sector remains depressed in the medium-term.

Increased substitutability between home and foreign tradables, and lower substitutabil-

ity between tradables and non-tradables implies that this is a less adverse terms-of-trade shock for the domestic economy. Despite the more moderate fall in aggregate CPI inflation, real wages do not fall by as much. Consequently, aggregate consumption does not fall by as much, as the drop in unconstrained and constrained household consumption is less severe. Moreover, the increase in employment by both types of households is much smaller in this calibration. Relative to the baseline calibration, aggregate CPI inflation falls by less, as both non-tradables and domestic tradables inflation both fall to a much smaller extent. Along with the smaller fall in the natural real rate of interest, these responses suggest that the demand-side impact of this shock is smaller.

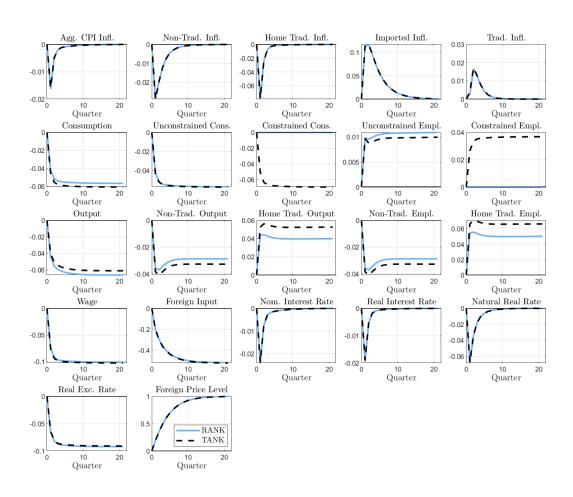


Figure 25: IRFs to a Gradual Increase in Foreign Price Level.

*Notes*:  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line. The elasticity of substitution between tradable goods ( $\mu$ ) is set to 6. The elasticity of substitution and between tradable and non-tradable goods ( $\iota$ ) is set to 0.55. All the other parameters are calibrated according to Table 1.

Front-loaded Fragmentation This calibration seems to be less important for the case of a front-loaded import price shock, as the outcome is still stagflationary with a trade-off of the same magnitude as in the baseline calibration. The only difference from the baseline calibration appears to be a sectoral rebalancing: there is a larger fall in non-tradable output and employment on impact and over the medium-term, which is reallocated to the hometradable sector. Aggregate output and employment remain unchanged, and the demand-side effect is nearly identical to the baseline calibration.

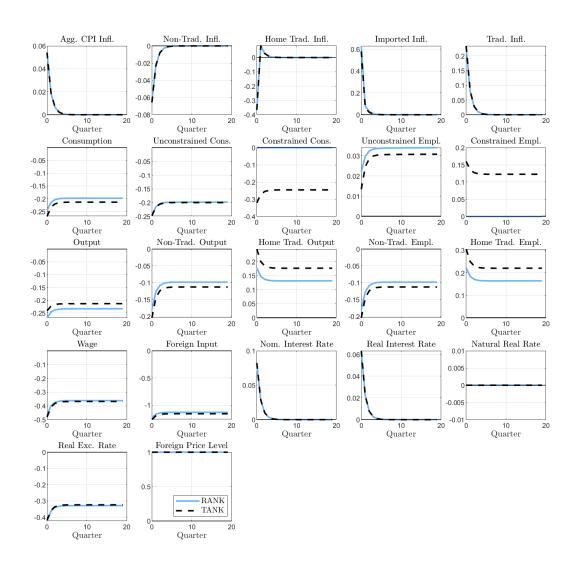
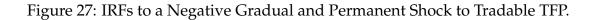
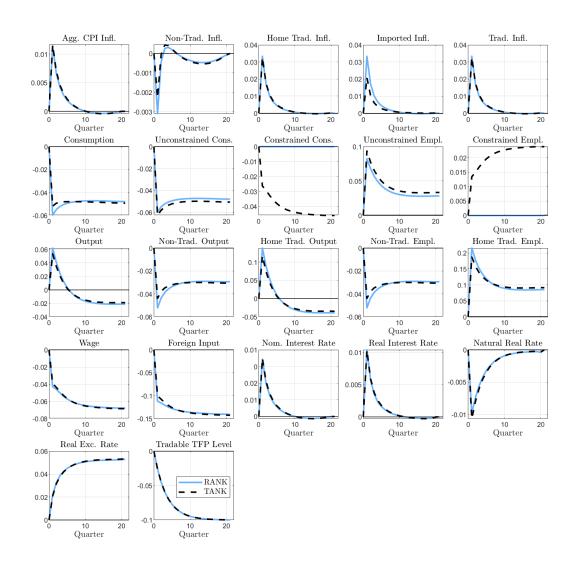


Figure 26: IRFs to a Front-loaded Increase in Foreign Price Level.

*Notes*:  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line. The elasticity of substitution between tradable goods ( $\mu$ ) is set to 6. The elasticity of substitution and between tradable and non-tradable goods ( $\iota$ ) is set to 0.55. All the other parameters are calibrated according to Table 1.

**Fall in Tradables Productivity** In this calibration, an adverse shock to tradables TFP leads to a fall in home tradables output over time, and an increase in the amount of labour employed in this sector. Relative to the baseline calibration, output and employment in the non-tradable sector falls by less. Aggregate output falls by less as a result. In this calibration, as non-tradables are less substitutable with tradables consumption, the increase in employment supports domestic consumption, which falls by less relative to the baseline calibration. This scenario remains moderately inflationary.





*Notes*:  $\lambda = 0$  in blue,  $\lambda = 0.3$  in black dashed line. The elasticity of substitution between tradable goods ( $\mu$ ) is set to 6. The elasticity of substitution and between tradable and non-tradable goods ( $\iota$ ) is set to 0.55. All the other parameters are calibrated according to Table 1.