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Monetary policy along the yield curve: why can central banks affect long-term real rates?

Paul Beaudry,[1] Paolo Cavallino[2] and Tim Willems[3]

Abstract

This paper presents theory and evidence to advance the notion that very persistent policy-induced interest rate changes may have only weak effects on activity. This arises when consumption-savings decisions are not primarily driven by intertemporal substitution, but also by life-cycle forces. The small impact of persistent rate changes results when intertemporal substitution and asset valuation effects are offset by interest income effects, which affect asset demand. In our framework, knowing the exact location of r* becomes less critical to central banks, as interest rates can be kept away from this level for prolonged periods of time, allowing monetary policy to unconsciously drive trends in real rates. This perspective offers an explanation to a set of puzzles, including why long-term real rates often move quite closely with short-term policy rates.

Key words: Monetary policy, r-star, monetary transmission mechanism, retirement savings, unconventional monetary policy.

JEL classification: E21, E43, E44, E52, G51.

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1 Introduction

Changes in long-term real rates have been receiving considerable attention. This includes the secular decline observed in the decades prior to Covid, as well as recent changes in the opposite direction. The main class of explanations for these movements are also *real* in nature, such as productivity growth, demographics, income inequality, and changes in the demand and supply of safe assets. One factor that is often dismissed, is monetary policy – driven by the view that most long-term real economic outcomes are invariant to monetary policy beyond horizons long enough to allow prices to be reset.

From this perspective, it is puzzling that long-term real rates appear rather sensitive to changes in a central bank's policy rate. Cochrane and Piazzesi (2002), Hanson and Stein (2015), and Nakamura and Steinsson (2018) report evidence of such sensitivity in US data, while Hansen, McMahon, and Tong (2019) do so for the UK; earlier evidence by Skinner and Zettelmeyer (1995) reported similar findings for not only the US and UK, but also Germany and France.[1] An even greater challenge to the standard view is the striking observation that all of the post-1980 decline in long-term US rates is driven by movements occurring in a narrow 3-day window around FOMC meetings (Hillenbrand, 2023). One interpretation is that central banks have superior information on the real determinants of long-term rates and that its announcements convey this information. Such interpretation has the appealing property of being consistent with the standard view that long-term real rates are driven by real forces. However, it has the less attractive property of relying on central banks having substantial private information – or rare expertise – which is not directly available to markets. This, despite the latter having access to much of the same models and data, whilst also being populated by many former central bank employees.

An alternative, more direct interpretation is that central banks may be able to affect real rates over long stretches of time. The difficulty with this possibility is in explaining why very persistent rate changes rates would not have large effects on activity and therefore inflation.

Within the perspective of New Keynesian models, the main reason central banks are not thought to be able to affect long-term real rates, relates to their perceived strong impact on activity. In this class of models, the potency of monetary policy shocks is increasing in their persistence. Accordingly, if a central bank tried to maintain real rates away from their "natural" flexible-price level (referred to as r^*) for long periods of time,

[1] Cochrane and Piazzesi (2002, p.91) nicely summarize the standard view by noting: "Target changes seem to be accompanied by large changes in long-term interest rates (...) Can the Fed really raise the short rate 1 percent for five years or more, without leading to 1 percent lower inflation that would cancel any effect on longer yields?".

this would have ever-growing effects on activity and inflation. Recognizing this, monetary authorities will want to avoid such outcomes, or rapidly correct course when noticing their long-term stance is away from r^*. As a result, they become *de facto* constrained to keeping their long-run policy stance consistent with the real forces determining r^*.

But what if more persistent rate changes are *less* potent than temporary ones? Could reduced powers to affect activity in the long run imply greater control over long-term interest rates? Most importantly, are there reasons to question the notion that more persistent rate changes are more potent? This paper aims to shed light on these issues.

To help fix ideas, let the relationship between output and real interest rates be expressed as follows, in a manner consistent with (but more general than) the standard log-linearized New Keynesian model:

$$\hat{y}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \psi_j (r_{t+1+j} - r^*),$$

where $\hat{y}_t$ represents deviations in output from its natural level and $\mathbb{E}_t(r_{t+1+j} - r^*)$ captures expected deviations in real interest rates from the natural rate, r^*. Now suppose monetary policy is conducted so that the expected real rate deviates from r^* in a persistent fashion according to $(r_t - r^*) = \rho(r_{t-1} - r^*) + \epsilon_t$. Then, the impact effect on activity of a unit monetary shock (call that $\Psi(\rho)$) equals the persistence-weighted sum of horizon j-specific effects ψ_j, i.e., $\Psi(\rho) = \sum_{j=0}^{\infty} \psi_j \rho^j$. While the literature offers many estimates of $\Psi(\rho)$ for low values of ρ, knowing how $\Psi(\rho)$ behaves as ρ approaches 1 is much less clear and generally relies on theory – not empirics.[2]

In most infinitely-lived agent models, the potency of monetary policy, as governed by $\Psi(\rho)$, strengthens with the shock's persistence ρ due to the compounded power of intertemporal substitution.[3] However, when thinking about the impact of very persistent rate changes on consumption, forces other than intertemporal substitution are likely to play important roles. For example, persistent rate changes have substantial implications for working households' desire to accumulate (and hold on to) wealth, whilst also affecting the consumption possibilities of retirees. These life-cycle forces are generally absent from New Keynesian models because of their predominant use for short-term analyses, where

[2]When thinking of very persistent deviations of interest rates from r^*, these are most easily conceptualized as changes in the intercept of a Taylor rule (to the extent that they are not reflecting changes in the true r^*).

[3]For the baseline New Keynesian model, $\Psi(1) = -\infty$. This has raised issues like the Forward Guidance Puzzle and initiated approaches that give rise to a discounted Euler equation (Del Negro et al., 2013; McKay et al., 2016). However, even with a discounted Euler equation, the negative impact of a contractionary monetary shock on activity always strengthens with its persistence ρ, i.e., $\Psi'(\rho) < 0$.

ρ is implicitly low. However, it is $\Psi(1)$ that is important for understanding what happens if real rates were to deviate from r^* for long periods of time.

To better understand the potential effects of having monetary policy deviate persistently from r^*– that is for understanding the forces behind $\Phi(\rho)$ when ρ is close to 1 – this paper develops a Finitely-Lived Agent New Keynesian (FLANK) model. We show that such a model yields a rich description of the relation between the path of future interest rates and economic activity. The analysis highlights the role of three distinct channels by which changes in future real rates affect consumption. First, there is the standard channel of intertemporal substitution. Second, there is a valuation effect on existing assets. And third, there is a channel working through the demand for retirement savings and the spending of retirees. As we shall show, these different forces work in different directions – causing the net effect of interest rate changes at different points in the yield curve to potentially have different qualitative effects. In the knife-edge case where the three forces perfectly offset each other in the long run, $\Psi(1) = 0$ and a central bank is no longer constrained by an r^*. They could then become a main driver behind long-run real rates (within bounds) without this choice having any effects on aggregate output or inflation. The choice would nonetheless have important implications for asset valuations. In the more plausible case where the sum of the three forces is small, but not exactly zero, r^* is still not very relevant since interest rates can be kept away from r^* for long periods of time without major effects on activity or inflation. Knowing the exact location of r^* thus becomes of diminished relevance for monetary policy purposes when $\Psi(1)$ is small, as r^* would not put much of a constraint on long-term interest rates. One could say that r^* becomes quasi-irrelevant in this case, since the system becomes very "forgiving" towards a central bank working with a wrong view of r^*.[4] This highlights the central importance of $\Psi(1)$ for understanding what may drive real rates at longer horizons: is it the actual r^*, or are economic outcomes more shaped by its perceived value?

To see why persistent rate changes might not affect activity much (or even operate with the unconventional sign), note that most people enter their economic lives with a labor income stream that is of shorter duration than their expected consumption stream. That is, most people are naturally "short duration". This only strengthens as individuals age, since the duration of a worker's labor income stream shortens as retirement nears. In that case, lower interest rates make the present discounted value of liabilities (future consumption) go up *by more* than that of assets – making households feel poorer when

[4]It is important to stress that our analysis is done within the confines of a closed economy (see Cesa-Bianchi et al. (2023) and Obstfeld (2023) for open economy considerations). In this regard, our analysis is best thought of as applying to a rather large economy (like the US).

rates are low. While working agents typically accumulate financial assets of positive duration during their working lives, it is in practice difficult to match the long duration of pension-related liabilities – implying some negative duration gap remains.[5] As a result, households may be incentivized to *save more* when confronted with persistently lower rates – that is, to accumulate more units of assets to compensate for each unit now yielding less over time. Hence persistent rate cuts can cause the demand for assets to outstrip valuation effects – potentially *dampening* consumption demand instead of stimulating it.[6] In contrast, if the same household faced a temporary fall in interest rates, it would likely want to *consume more* because of standard intertemporal substitution. More generally, the impact of interest rate changes on activity will be shown to depend on their persistence, with the effects of short-term rates continuing to operate in the conventional direction (with higher rates being contractionary), but this sign may flip for longer-term rates.

The logic set out above is partial equilibrium in nature and therefore incomplete for our purposes. An important feature of our FLANK model is that it enables us to analyze under what conditions such partial intuition is maintained in general equilibrium. This will depend on a set of factors, including the expected duration of working and retirement phases, and the average duration of the aggregate asset portfolio. But a key parameter is the elasticity of intertemporal substitution (EIS). When $EIS \geq 1$, our FLANK model behaves similarly to standard infinitely-lived agent models, with the potency of monetary policy always increasing in its persistence. Central banks then cannot affect long-term real rates without creating strong inflation or deflation. In contrast, for $EIS < 1$ (a case for which there is strong empirical support[7]) a set of offsetting mechanisms arises, opening up the possibility that very persistent rate changes have little effect on activity. To the extent that the Phillips curve is locally rather flat, it follows that persistent rate changes could also do little to inflation.

[5]This is clear to pension funds (to whom many have outsourced the process of saving for retirement): pension funds often have negative duration gaps of about 10 years, which forced many to increase premiums during the zero-interest rate era, effectively asking for greater saving efforts from their members. See, e.g., https://macrosynergy.com/research/low-for-long-rates-pressure-on-pensions-and-insurances/. As a concrete example, ABP (the largest Dutch pension fund) issued a statement back in 2019 (www.abp.nl/content/dam/abp/nl/documents/persbericht%20premie-indexatie%202020.pdf) saying "Pensions are becoming increasingly expensive [...] With the current pension ambition and the expectation that interest rates will remain low for a long time, higher premiums will be needed."

[6]This is consistent with the empirical findings of Ring (2024), who finds that wealth taxation (which lowers the rate of return) tends to *increase* saving. Of note, Rajan (2013) already worried that the post-GFC era of low interest rates might not be expansionary because "savers put more money aside as interest rates fall in order to meet the savings they think they will need when they retire".

[7]See for example Yogo (2004) and Best et al. (2020) and Ring (2024), who all estimate the $EIS << 1$.

Related literature. Our paper relates to several contributions to the broader literature. First, we build on papers that have enriched the New Keynesian model with additional transmission mechanisms relating to agent-heterogeneity. A prominent example is the "TANK/HANK" literature, extending the standard model with liquidity-constrained "hand-to-mouth" consumers. This makes transmission run less through intertemporal substitution and more via general equilibrium effects (Kaplan et al., 2018). Our work also relates to Auclert (2019) who analyzes the impact of transitory rate changes – showing how the unhedged interest rate exposure, distinguishing solely between net assets that pay "today" versus "in the future", is sufficient with respect to the first-order response of consumption to shocks. When rate changes are persistent, the exact timing of cash flows starts to matter. In this context, Greenwald et al. (2023) develop a life-cycle model to understand how the observed decline in real rates has affected wealth inequality, also documenting how lower rates contract consumption possibilities for "the young" who have not yet accumulated many financial assets with positive duration, but have a long consumption stream to finance going forward.

Gertler's (1999) OLG framework, which we build upon, has also been used to analyze issues related to monetary policy by, among others, Sterk and Tenreyro (2016) and Galí (2021). Sterk and Tenreyro focus on a redistribution channel of monetary policy when prices are fully flexible, while Galí's work analyzes the conduct of monetary policy in the presence of bubble-driven fluctuations. Fujiwara and Teranishi (2008) use this type of model to examine the impact of demographics on r^*, whilst also investigating the distributional impact monetary policy may have on workers versus retirees. Bielecki et al. (2022) offer a more general OLG framework to analyze the heterogeneous impact monetary policy can have across generations; Eggertsson et al. (2019) use an OLG model to formalize thinking about "secular stagnation". Our paper, in contrast, focuses on the impact that a retirement savings motive has on the monetary transmission mechanism and the resulting powers of central banks.

Our work also relates to papers which question whether lower interest rates are always expansionary. Bilbiie (2008) features "inverted aggregate demand logic" stemming from limited asset market participation. In Mian et al. (2021) monetary stimulus promotes debt accumulation, which – while being stimulative in the short run – ultimately starts forming a drag on the economy, as savers have lower MPCs in their model. Abadi et al. (2023), Eggertsson et al. (forthcoming), and Cavallino and Sandri (2023) also present frameworks in which rate cuts can be contractionary, due to an adverse impact on the banking sector or capital flows. Daniel et al. (2021) obtain such an effect by introducing agents who "live off income". In contrast, our model emphasizes that the link between activity and interest rates may vary along the yield curve. Of note, there is also the "neo-Fisherian" literature which explores the possibility that a persistent increase in rates might help to raise inflation (Schmitt-Grohé and Uribe, 2014; Cochrane, 2018).

Finally, our model links to the literature investigating the ability of monetary policy to affect long-term real rates. Papers like Nakamura and Steinsson (2018), Hansen, McMahon and Tong (2019), and Hillenbrand (2023) explain this via a central bank information effect, while Rungcharoenkitkul and Winkler (2023) allow for two-sided learning (with markets not just learning from the central bank, but the reverse occurring as well). Hanson and Stein (2005) allude to the impact of monetary policy on term premia, while Beaudry et al. (2024) develop a model featuring r^*-multiplicity (with monetary policy being able to affect which equilibrium gets to prevail). The point of our paper is not to deny that these factors play a role, but rather to highlight an alternative mechanism – based on the "quasi-irrelevance of r^*" – which is different in spirit and implications.

Outline. In Section 2 we present a set of data patterns aimed at motivating our introduction of life-cycle forces into a New Keynesian setup. Section 3 presents our FLANK model. Section 4 discusses the model's implications for monetary policy, emphasizing why we can simultaneously have short-lived interest rate cuts being expansionary while persistent cuts have very little effect. This section makes explicit the forces determining $\Psi(\rho)$, and especially $\Psi(1)$, and offers a set of calibrations. Section 5 uses high-frequency identification to examine whether monetary policy shocks at the longer end of the yield curve may have weaker (or even perverse) effects on activity in comparison to those occurring at the short end. Section 6 shines further light on why precise knowledge of r^* may be considered quasi-irrelevant for monetary policymaking in our setting. Section 7 discusses some of the model's assumptions and relevant extensions, after which Section 8 concludes.

2 Motivating evidence

A key implication of the life-cycle forces we focus on, is that households' consumption decisions will be influenced by their need/desire to save for retirement. This force can be seen as creating a target for desired wealth holdings – an insight associated with Modigliani's life-cycle hypothesis framework, see e.g. Modigliani and Sterling (1983). The value of such target will depend on many factors, including longevity, income, intertemporal substitution, and time preferences. But, very importantly, such a target level is also likely to depend on the expected path for interest rates.

In the presence of life-cycle forces, it is therefore not wealth *per se* that should drive consumption; what matters as well, is the expected flow of income that is expected to be generated by those wealth holdings (its expected annuity value or, for short, its flow value). For example, when interest rates are lowered, this tends to increase measured wealth holdings through valuation effects. However, it is not immediately clear that this should boost consumption, as desired wealth holdings may increase simultaneously. This is especially relevant if a reduction in interest rates is viewed as persistent, since this reduces the flow value of wealth – thereby possibly creating an increased desire to accumulate assets, to compensate for the lower interest income per unit held. Without controlling for interest rate effects on asset demand, the link between consumption and wealth may therefore be very weak. In contrast, once one controls for interest rate movements, consumption and adjusted wealth should comove positively – as people will want to spend their asset holdings in excess of their desired (targeted) levels.

The potential relevance of this logic, which differentiates between pure wealth and rate-adjusted wealth, can be seen in Figure 1. Panel (a) plots the relationship between the natural log of detrended U.S. real consumption per capita ($\ln C_t$) and the natural log of detrended beginning-of-period real U.S. wealth holdings per capita ($\ln W_{t-1}$) over 1982Q1-2019Q4.[8] As can be seen, there is very little relationship between the two, with their correlation amounting to an insignificant 0.056. At face value, this may give the impression that there is no link between fluctuations in wealth and consumption.

An important reason the link between consumption and wealth may be so weak is that "raw" wealth might not be the right measure to capture people's consumption possibilities, since it neglects the flow-aspect (the holder of \$500,000 can afford to consume more when those assets yield 5% per annum, as opposed to only 0.5%). Under this logic, consumption should be driven by something closer to *the product of* the interest rate and wealth holdings, as that captures both dimensions (stock and flow). To look at this

[8]All data are at the quarterly frequency and available from FRED starting 1982Q1. The consumption series has code PCE; the wealth series has code TABSHNO. Price deflation is done using the CPI (CPIAUCSL), while per-capita amounts are obtained through division by POPTHM. Consumption and wealth are made stationary by linear detrending using the average growth rate of U.S. real GDP per capita over the pre-GFC period (a quarterly rate of 0.54%). Over the entire period, real GDP grew at a quarterly rate of 0.4%. Using this slightly lower detrending factor gives similar results, but with the correlation between consumption and raw wealth even becoming slightly negative (-0.064). We choose to detrend using the slightly higher pre-GFC growth rate, as the forces emphasized in our paper suggest that the post-GFC period may have had low growth because of the inability of monetary policy to push the economy towards its potential.

possibility, Panel (b) of Figure 1 plots the two same variables as Panel (a), except that now wealth is multiplied by a long-term real interest rate, i.e., we are now looking at the correlation between consumption and $\ln(r_t^{LT} W_{t-1})$, where both are detrended by the common underlying growth rate.[9] This simple adjustment has a striking effect on the correlation: it jumps to 0.850, is very significant, and not driven by outliers. In other words, the data suggest that consumption is much more closely related to a wealth-flow concept than to "raw" wealth.

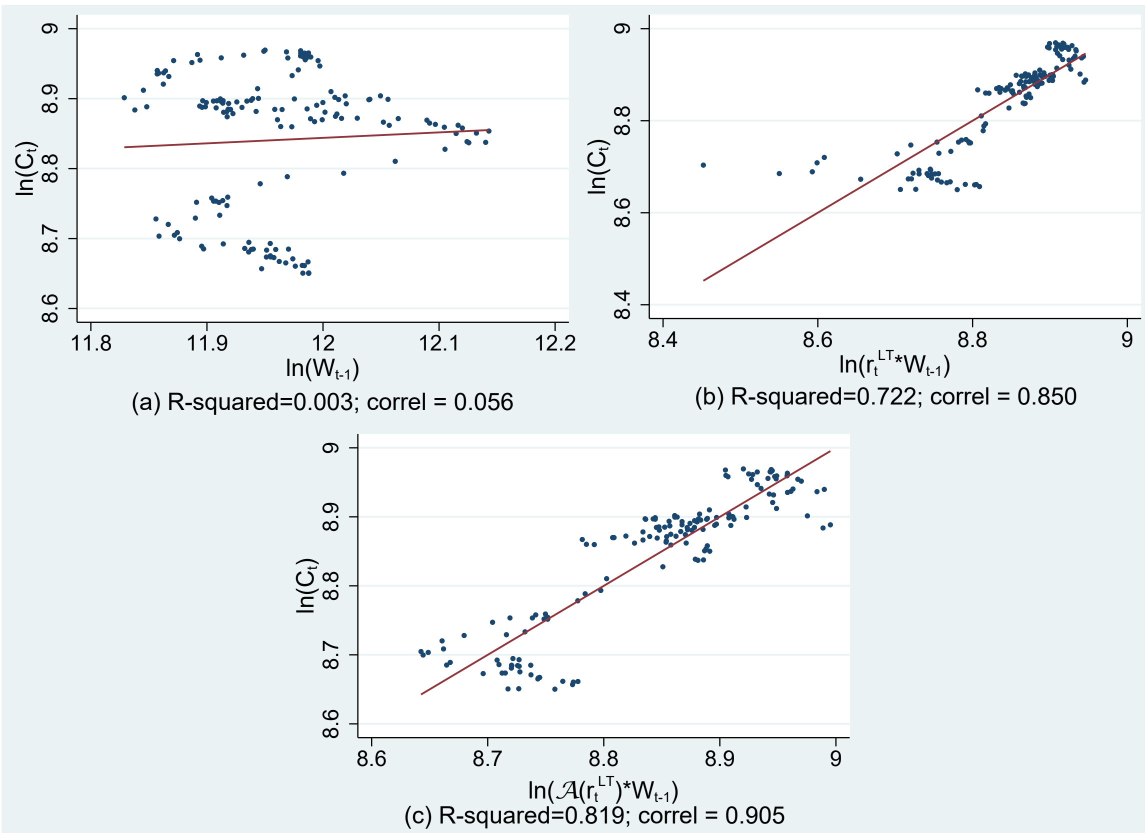

Figure 1: Scatter plot illustrating the correlation between detrended U.S. real consumption levels and detrended real wealth holdings. Panel (a) features no adjustment for the level of interest rates; Panel (b) looks at the simple product $r^{LT}W$; Panel (c) performs the rate-adjustment in the way prescribed by our model, via application of (1). Quarterly data from 1982Q1-2019Q4.

The adjustment to wealth leading to Panel (b) is very coarse – a simple product of r^{LT} and W. In the life-cycle model we will develop in Section 3, the relevant adjustment factor for wealth is more involved and can be approximated by:

[9] This real rate is taken as the ex-ante 10-year real rate, available from FRED via code REAINTRATREARAT10Y.

$$\mathcal{A}(r_t^{LT}) = 1 - [\beta(1-\delta_2)]^{\frac{1}{\sigma}}(1+r_t^{LT})^{\frac{1-\sigma}{\sigma}}, \tag{1}$$

where β is the household's discount factor, δ_2^{-1} is the expected length of retirement, σ^{-1} is the elasticity of intertemporal substitution (EIS), and r_t^{LT} is the long-term (net) rate of return on safe bonds. Note that when the EIS goes to 0, (1) is approximately equal to r_t^{LT}. So, if one subscribes to the view that the EIS is very low, this offers a rationalization for the coarse adjustment factor used in Panel (b).

To further foreshadow implications of our analysis, Panel (c) of Figure 1 again plots U.S. consumption against an adjusted wealth term – as we had done in Panel (b) – but now the adjustment factor follows (1), i.e., it takes the form prescribed by our theory. We use the following parameter values: $\sigma = 4$, $\beta = 0.99$, $\delta_2 = 0.0125$ (yielding an average retirement period of 80 quarters, i.e., 20 years) and maintain the 10-year real rate as our measure of r_t^{LT}. The relation between consumption and this theoretically-grounded measure of rate-adjusted wealth is even stronger than in Panel (b), with the correlation coefficient now rising to 0.905. This further confirms the potential importance of life-cycle considerations for consumption-saving decisions, with consumption dynamics much more closely related to rate-adjusted wealth than to raw wealth.

Table 1 offers a supplementary perspective on the same theme. First, Column (1) estimates the regression behind Panel (a) of Figure 1. It confirms that, when using a raw wealth metric, wealth does not seem to be driving consumption.

Next, Column (2) replaces raw wealth by rate-adjusted wealth (using the adjustment factor (1), as prescribed by our model). The estimated coefficient on this adjusted wealth term shoots up to 0.290 and is highly significant, echoing the high correlation in Panel (c) of Figure 1.

In Column (3) of Table 1, we split this adjusted wealth term into its two components: raw wealth, W, and the rate-adjustment term, $\mathcal{A}$. Our theory predicts that these two terms should enter with the same coefficient, as households should be indifferent as to whether they are benefitting from a higher stock W or superior flow as captured via $\mathcal{A}$. Column (3) supports this prediction. The coefficient on raw wealth is 0.279, while that on the adjustment factor is 0.290, with the two not being significantly different from each other (p-value = 0.822). It is worth noting that the coefficient on the adjustment factor presented in Column (3) indicates that, holding raw wealth constant, long-term real rates are positively correlated with consumption. From a wealth-flow interpretation, this should not be surprising as higher long-run returns expand consumption possibilities for any given level of wealth.

Table 1: OLS regression analyzing impact of (adjusted) wealth and real interest rates on consumption

Dependent variable: natural log of detrended real PCE per capita				
	(1)	(2)	(3)	(4)
$\ln W_{t-1}$	0.078 (1.01)		0.279^{***} (6.29)	
$\ln\left(\mathcal{A}(r_t^{LT})W_{t-1}\right)$		0.290^{***} (23.35)		0.362^{***} (13.93)
$\ln \mathcal{A}(r_t^{LT})$			0.290^{***} (22.86)	
$1 + r_t^{ST}$				-1.106^{***} (-3.35)
constant	7.903^{***} (8.47)	6.709^{***} (72.68)	6.835^{***} (12.66)	7.304^{***} (38.45)
R^2	0.0031	0.819	0.819	0.834
observations	151	151	151	151

Notes: OLS estimates at the quarterly frequency. Numbers in parentheses represent t-statistics, calculated using robust standard errors. * denotes significance at the 10% level, ** implies significance at the 5% level, *** indicates significance at the 1% level.

In Column (4), we add a measure of *short-term* real interest rates to the exercise.[10] We now see that, holding wealth constant, consumption is *negatively* correlated with short-term real rates yet *positively* correlated with long-term real rates. The negative correlation with short-term real rates is consistent with standard intertemporal substitution, while the positive correlation with long-term real rates is consistent with life-cycle considerations. Although we do not claim that such a simple regression cleanly identifies causal effects, the resulting pattern of opposing signs is nonetheless intriguing and suggesting that short-run interest rates may have a qualitatively different effect on consumption compared to long-run rates.

In the next section we will extend a standard New Keynesian model to include life-cycle considerations – showing how and why such forces can cause agents to react, in opposite ways, to changes in short- versus long-term rates. In particular, we will show that such a model implies behavior that is consistent with all three correlations implied by Column (4) of Table 1, with important implications for the conduct of monetary policy.

[10] This real rate was obtained by subtracting 1-year expected inflation (FRED code: EXPINF1YR) from the effective Federal funds rate.

3 A life-cycle model for monetary policy

This section describes our model.[11] Since we adopt a common production setup – with monopolistically competitive firms facing price adjustment costs – and we combine this with life-cycle consumption-savings decisions, one can refer to this type of model as a "FLANK", for Finitely-Lived Agent New Keynesian model. Throughout the analysis, we will maintain the assumption that all households are optimizers. This assumption may not be very realistic given the substantial evidence supporting the presence of household which are possibly characterized as "hand-to-mouth". As we discuss in Section 7, we do not think this modelling choice hinders the model's main insights even if we agree that life cycle optimizers may represent only a fraction of the population.

Environment. There is a measure one of households, subject to a life-cycle dynamic as in Gertler (1999, which – in turn – built on Yaari (1965) and Blanchard (1985)). Each household starts life in a work state and transits out with Poisson probability δ_1 (either due to being sent to retire, or because of a health shock preventing the household from continuing work). At this transition, the household enters the retirement state during which it faces a per-period Poisson probability of dying equal to $\delta_2 \geq \delta_1$. Deceased households are immediately replaced by new, working households, implying that the fraction of working households in the population is constant at $\vartheta = \frac{\delta_2}{\delta_1+\delta_2}$.

Retired Households. Household level decisions are best understood backwards. In the retirement state, a household derives income from its financial wealth. This wealth reflects both past savings and a possible lump-sum public pension payment. Retired households invest their wealth in a portfolio of short- and long-term bonds. Short-term bonds are one-period assets whose nominal return, i_t, is set by the central bank. Their real return is $r_{t+1} \equiv i_t/\mathbb{E}_t\pi_{t+1}$, where π_t denotes the gross inflation rate. Following Woodford (2001), we model long-term bonds as real perpetuities with coupons that decay geometrically at rate μ. This implies that a bond issued in period t pays $(1-\mu)^h$ units of consumption $h+1$ periods later. Note that the bond's duration is decreasing in μ (and that setting $\mu = 1$ reduces this bond to a one-period instrument). The return on the long-term bond is:

$$r^b_{t+1} = \frac{1 + (1-\mu)\, q_{t+1}}{q_t},$$

where q_t is the price of the long-term bond. The optimization problem faced by a retired

[11]The real side of the model shares many features with the continuous time model in Beaudry et al. (2024). Our model departs from Beaudry et al. (2024) in that it is set in discrete time, is stochastic, allows for long-term debt, and is embedded in a New Keynesian setup.

household j with CRRA-preferences (where $1/\sigma$ is the EIS) reads:

$$V_t^r\left(\tilde{a}_t^j\right) = \max_{c_t^j, \alpha_t^j, \tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} + \left(1-\delta_2\right)\beta_t \mathbb{E}_t\left[V_{t+1}^r\left(\tilde{a}_{t+1}^j\right)\right] \right\},$$

$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j\left(\tilde{a}_t^j - c_t^j\right), \tag{2}$$

$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right)\alpha_t^j \tag{3}$$

where c_t^j is consumption, $\alpha_t^j \equiv \left(q_t b_t^j\right)/a_t^j$ is the share of wealth invested in long-term bonds and $\tilde{a}_t^j \equiv r_t^j a_{t-1}^j$ is the beginning-of-period t stock of wealth held by household j, such that the real rate of return r_t^j gets to work on whatever is left after period-$(t-1)$ consumption has been financed, i.e., on $a_{t-1}^j = \tilde{a}_{t-1}^j - c_{t-1}^j$. Finally, $\beta_t \equiv \beta e^{\varepsilon_t^\beta}$, where ε_t^β is a demand shifter. Optimal consumption will satisfy:

$$\left(c_t^j\right)^{-\sigma} = \left(1-\delta_2\right)\beta_t \mathbb{E}_t\left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} r_{t+1}\right], \tag{4}$$

with the portfolio optimality condition:

$$0 = \mathbb{E}_t\left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j}\left(r_{t+1}^b - r_{t+1}\right)\right]. \tag{5}$$

At the same time, the envelope theorem implies that:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(c_t^j\right)^{-\sigma}, \tag{6}$$

so that (5) boils down to:

$$0 = \mathbb{E}_t\left[\left(c_{t+1}^j\right)^{-\sigma}\left(r_{t+1}^b - r_{t+1}\right)\right].$$

If we furthermore combine the above with the guess that $V_t^r\left(\tilde{a}_t^j\right) \equiv \frac{\left(\tilde{a}_t^j\right)^{1-\sigma}}{1-\sigma}\Gamma_t^j$, with Γ_t^j conjectured to be a function of the future path of r_t^j and independent of $\tilde{a}_t^j$, this gives:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(\tilde{a}_t^j\right)^{-\sigma}\Gamma_t^j. \tag{7}$$

By combining (6) and (7) we obtain:

$$\left(c_t^j\right)^{-\sigma} = \left(\tilde{a}_t^j\right)^{-\sigma}\Gamma_t^j \Leftrightarrow c_t^j = \tilde{a}_t^j\left(\Gamma_t^j\right)^{-\frac{1}{\sigma}}, \tag{8}$$

which we can plug into (2) to yield:

$$\tilde{a}_{t+1}^{j} = r_{t+1}^{j} \tilde{a}_{t}^{j} \left[1 - \left(\Gamma_{t}^{j} \right)^{-\frac{1}{\sigma}} \right]. \tag{9}$$

Finally, plugging (7), (8), and (9) into (4) gives a non-linear difference equation for Γ_t:

$$\left[\left(\Gamma_{t}^{j} \right)^{\frac{1}{\sigma}} - 1 \right]^{\sigma} = (1 - \delta_2)\, \beta_t \mathbb{E}_t \left[r_{t+1} \Gamma_{t+1}^{j} \left(r_{t+1}^{j} \right)^{-\sigma} \right]. \tag{10}$$

This verifies our guess that Γ_t^j is independent of $\tilde{a}_t^j$, confirming that it is only a function of future expected rates of return and demand shocks.

Using the above, we can write the utility of retirees as $V^r \left(\tilde{a}_t^j, \Gamma_t^j \right) = (1 - \sigma)^{-1} \left(\tilde{a}_t^j \right)^{1-\sigma} \Gamma_t^j$, where V^r thus depends both on the stock of assets with which the household enters retirement ($\tilde{a}_t^j$) as well as on the entire future path of interest rates working over that stock (captured by Γ_t). For a given value of assets $\tilde{a}_t^j$, retired households are better off when rates are expected to be high, as this offers them a superior stream of interest revenues.

Let $c_t^r \equiv \int_{\mathbf{R}_{r,t}} c_t^j dj / (1 - \vartheta)$ be the consumption of the representative retired agent and define $a_t^r \equiv \int_{\mathbf{R}_{r,t}} a_t^j dj / (1 - \vartheta)$ as its financial wealth, where $\mathbf{R}_{r,t}$ denotes the set of households in the retired state at time t. Given that all retired households choose the same portfolio of assets, that is $\alpha_t^j = \alpha_t^r$, for all $j \in \mathbf{R}_{r,t}$, this also implies $\Gamma_t^j = \Gamma_t$, for all $j \in \mathbf{R}_{r,t}$. Therefore:

$$c_t^r = a_t^r \left[\left(\Gamma_t^r \right)^{\frac{1}{\sigma}} - 1 \right]^{-1},$$

and a_t^r evolves as:

$$a_{t+1}^r = \left[(1 - \delta_2)\, a_t^r r_{t+1}^r + \delta_2 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right) \right] \left[1 - \left(\Gamma_{t+1} \right)^{-\frac{1}{\sigma}} \right].$$

where τ_t^r is the lump-sum transfer received by households upon retirement. This transfer can be thought as a public pension transfer that is paid once to the household upon retiring, and thereafter managed by the household.

Working Households. Next, consider a working household. It receives a real wage w_t for any labor input ℓ_t it provides, plus transfers from good-producing firms and transfers from/to the government. A working household faces a δ_1 probability of moving into retirement next period. A working household's decision problem can be written as:

$$
\begin{aligned}
V_t^w\left(\tilde{a}_t^j\right) &= \max_{c_t^j,\ell_t^j,\alpha_t^j,\tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(\ell_t^j\right)^{1+\varphi}}{1+\varphi} + \beta_t \mathbb{E}_t \left[\left(1-\delta_1\right) V_{t+1}^w \left(\tilde{a}_{t+1}^j\right) + \delta_1 V_{t+1}^r \left(\tilde{a}_{t+1}^j + \tau_{t+1}^r\right)\right] \right\}, \\
s.t.\ \tilde{a}_{t+1}^j &= r_{t+1}^j \left(\tilde{a}_t^j - c_t^j + \ell_t^j w_t + z_t^j + \tau_t^w + \tau_t^n\right), \\
r_{t+1}^j &= r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right) \alpha_t^j
\end{aligned}
$$

where z_t^j represents dividends received from good-producing firms. τ_t^w and τ_t^n both represent tax/transfer schemes. τ_t^w is a tax that is used by the government to pay expenditures and interest on debt. τ_t^n is tax or transfer scheme that is used by the government to ensure that the inheritance received by newly born households allows them to resemble existing working households – implying that we can treat working households as a representative agent. The optimality conditions give rise to the following Euler equation:

$$
\left(c_t^j\right)^{-\sigma} = \beta_t \left\{ \left(1-\delta_1\right) \mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(\tilde{a}_{t+1}^j + \tau_{t+1}^n\right)^{-\sigma} \Gamma_{t+1} r_{t+1} \right] \right\}, \tag{11}
$$

supplemented by the portfolio decision and the labor supply schedule:

$$
\begin{aligned}
0 &= \mathbb{E}_t \left[\left\{ \left(1-\delta_1\right) \left(c_{t+1}^j\right)^{-\sigma} + \delta_1 \frac{dV_{t+1}^r \left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} \right\} \left(r_{t+1}^b - r_{t+1}\right) \right], \\
w_t &= \chi \left(c_t^j\right)^{\sigma} \left(\ell_t^j\right)^{\varphi}.
\end{aligned}
$$

Note how the Euler equation for working households (11) features two terms on the RHS: the first term is familiar from standard models without retirement and implies that a lower interest rate, *ceteris paribus*, decreases the household's desire to save; this is standard intertemporal substitution. The second term on the RHS of (11), however, stems from the introduction of the prospect of retirement and shows how consumption is driven by wealth ($\tilde{a}_{t+1}^j$) adjusted for the expected path of interest rates (as captured by $\Gamma_{t+1} r_{t+1}$).

Since the assets of new and existing working households are equalized via the transfer scheme τ_t^n, working households can be treated as homogeneous. Let c_t^w denote the consumption of the representative working household and a_t^w its end-of-period financial wealth. Then, c_t^w solves:

$$
\left(c_t^w\right)^{-\sigma} = \beta_t \left\{ \left(1-\delta_1\right) \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w\right)^{-\sigma} \Gamma_{t+1} r_{t+1} \right] \right\},
$$

where a_t^w evolves as:

$$a_{t+1}^w = (1-\delta_1)\, a_t^w r_{t+1}^w + \delta_1 a_t^r r_{t+1}^r - c_{t+1}^w + \ell_{t+1} w_{t+1} + z_{t+1} + \tau_{t+1}^w.$$

Good-producing firms. Each working household $j \in \mathbf{R}_{w,t}$ owns and manages a firm that produces a differentiated good using the technology $y_t^j = A\ell_t^j$. Upon retirement, households liquidate their firms which are replaced by new ones owned by new working household.[12] Firms are monopolistically competitive and set prices subject to a quadratic adjustment cost *a la* Rotemberg (1982). Let P_t^j be the price chosen by firm j at time t and $\pi_t^j \equiv P_t^j / P_{t-1}^j$ be its growth rate. Then, the firm pays adjustment cost $\Theta\left(\pi_t^j\right) = y_t^j \frac{\theta}{2}\left(\pi_t^j - \bar{\pi}\right)^2$, where $\bar{\pi}$ is the inflation target and θ governs the cost of adjusting prices. The resulting Phillips curve takes the standard form (which, to a first-order approximation, has the same reduced form as the one under Calvo-pricing; Roberts (1995)):

$$(\pi_t - \bar{\pi})\, \pi_t = \lambda \left(\frac{\epsilon}{\epsilon - 1} mc_t - 1\right) + \mathbb{E}_t \left[\Lambda_{t,t+1}^w (\pi_{t+1} - \bar{\pi})\, \pi_{t+1} \frac{y_{t+1}}{y_t}\right],$$

where $\lambda \equiv (\epsilon - 1)/\theta$ represents the slope of the Phillips curve and ϵ is the elasticity of substitution between product varieties,[13] $y_t = \int_{\mathbf{R}_{w,t}} y_t^j dj$ denotes aggregate output, while $\Lambda_{t,t+1}^w$ is the stochastic discount factor of the representative working household:

$$\Lambda_{t,t+1}^w = \beta_t \frac{(1-\delta_1)\left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w\right)^{-\sigma} \Gamma_{t+1}}{\left(c_t^w\right)^{-\sigma}}.$$

This captures the familiar notion that households place more weight on the future when their marginal utility is high, but it features the additional forces stemming from retirement preoccupations. In particular, households now place more weight on the future when they hold fewer assets a_t^w or when the interest rate path is lower (as captured via Γ).

The real marginal cost of production is $mc_t = (1-\tau_t)\, w_t / A$, where τ_t is a wage subsidy financed through lump-sum taxes levied directly on good-producing firms. We use this subsidy to undo the steady-state markup and to eliminate the impact of labor supply wealth effects on inflation, such that $mc_t = \frac{\epsilon-1}{\epsilon}\chi\left(\frac{y_t}{\vartheta A}\right)^{1+\varphi}$. Since all firms are identical, the real dividend generated by each firm is $z_t = \frac{y_t}{\vartheta}\left[1 - \frac{\theta}{2}(\pi_t - \bar{\pi})^2\right] - \ell_t w_t$.

[12] Firms have no physical capital. Hence, their liquidation value is zero. Alternatively, we could assume that retiring households sell their good-producing firms to new households. This would strengthen the "asset valuation channel" (described later) as a rate cut would then not only increase bond prices, but also stock prices.

[13] Households consume a CES aggregate of all varieties: $c_t^j = \left[\int_{\mathbf{R}_w} c_t^j(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$.

Government. The budget constraint of the government reads:

$$s_t^g + q_t b_t^g = q_{t-1} b_{t-1}^g r_t^b + s_{t-1}^g r_t + \vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r,$$

where s_t^g and b_t^g are the supply of short- and long-term government bonds, respectively. Without loss of generality, we take the limit for $s_t^g \downarrow 0$ and assume $b_t^g = b^g$, for all $t \geq 0$. This implies that tax policy must satisfy $\vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r = -b^g \left(1 - \mu q_t\right).$

The central bank conducts monetary policy according to the following Taylor rule:

$$i_t = r^* \bar{\pi} \left(\frac{\mathbb{E}_t \left[\pi_{t+1}\right]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t^i}, \tag{12}$$

where $\phi > 0$ governs the central bank's responsiveness to expected inflation-deviations from target ($\bar{\pi}$), r^* is the steady-state real interest rate, and ε_t is a monetary policy shock.

Market clearing . Market clearing requires that:

$$\begin{aligned}
\vartheta c_t^w + (1-\vartheta) c_t^r &= y_t \left[1 - \frac{\theta}{2} \left(\pi_t - \bar{\pi}\right)^2 \right], \\
\vartheta a_t^w + (1-\vartheta) a_t^r &= q_t b^g, \\
\vartheta b_t^w + (1-\vartheta) b_t^r &= b^g,
\end{aligned}$$

where $b_t^r \equiv \int_{\mathbf{R}_{r,t}} b_t^j dj / (1-\vartheta)$ and $b_t^w \equiv \int_{\mathbf{R}_{w,t}} b_t^j dj / \vartheta$ are the long-term bond holdings of the representative retiree and the representative worker, respectively.

Exogenous Processes. We allow the model to be hit by two types of shocks: first, a standard monetary policy shock "ε_t^i" to the Taylor rule (12) and, second, a demand shock to β, ε_t^β. The exogenous variables ε_t^i and ε_t^β are assumed to follow AR(1) processes:

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \epsilon_t^i, \tag{13}$$

$$\varepsilon_t^\beta = \rho_\beta \varepsilon_{t-1}^\beta + \sigma_\beta \epsilon_t^\beta, \tag{14}$$

with the innovations "ϵ^i" and "ϵ^β" following a standard-normal distribution (σ_i and σ_β scale the shocks' standard deviations).

We furthermore assume that the inflation target is zero ($\bar{\pi} = 1$). The equilibrium and steady state equations of our full model can be found in Appendix A.

4 Model properties: analytical and quantitative

In order to highlight how the life-cycle forces associated with retirement risk affect monetary policy, we simplify our model to derive a set of analytical results that help clarify the main mechanisms at play. Our simplifying assumptions lead to a compact system that is not much more difficult to handle than the standard 3-equation New Keynesian model, while simultaneously capturing a set of forces stemming from life-cycle considerations. We then derive a "term structure representation" of the Euler equation, which shows how interest rates at different horizons affect activity differently. This ultimately enables us to discuss when and why our framework implies that the potency of monetary policy may be decreasing in the persistence with which it is conducted, which – we will argue – has important implications for the powers stemming from monetary policy.

4.1 Simplifying the model

To provide a model which can be easily compared with a standard New Keynesian model, we assume that the transfer received by households upon retirement, τ^r, is designed to keep the distribution of financial wealth between workers and retirees constant at its steady-state level.[14] Keeping the share of wealth held by retirees versus working households constant is useful for presentational purposes, enabling us to obtain analytical solutions, while we shall later illustrate that it is not driving the model's implications (neither qualitatively nor quantitatively). In addition, we set the level of government debt, b^g, so that the steady-state real interest rate ("r^*") equals $1/\beta$. This assumption ensures that the system of log-linearized equilibrium conditions derived below nests the standard representative agent New Keynesian ("RANK") model, which results when setting $\delta_1 = 0$ (implying there are no retirees, as every household remains in its working state *ad infinitum*). Finally, we impose $\delta_2 < \mu$, so that the expected duration of retirement is larger than the duration of the assets held by households – implying that households have a negative duration gap (recall the discussion around footnote 5).

With these simplifications, the log-linearized equilibrium can be expressed as:

$$\hat{y}_t = (1-\gamma)\,\hat{c}_t^w + \gamma\hat{c}_t^r \tag{15}$$

[14] To simplify the algebra, we also assume that the time-varying nature of the transfer is unexpected, meaning that working households do not anticipate receiving a transfer that varies with the state of the economy. This assumption is not necessary for our main results, but it does make the presentation more transparent.

$$\hat{c}_t^r = \hat{q}_t + \frac{\sigma - 1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{\Gamma}_{t+1} - \frac{1}{\sigma}\varepsilon_t^\beta \tag{16}$$

$$\hat{c}_t^w = (1 - \delta_1)\left(\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \frac{\sigma - 1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{\Gamma}_{t+1}\right) - \frac{1}{\sigma}\varepsilon_t^\beta \tag{17}$$

$$\hat{\Gamma}_t = \beta\left(1 - \delta_2\right)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} - (\sigma - 1)\,\mathbb{E}_t\hat{r}_{t+1} + \varepsilon_t^\beta\right] \tag{18}$$

$$\hat{q}_t = \beta\left(1 - \mu\right)\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \tag{19}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{20}$$

with

$$\mathbb{E}_t\hat{r}_{t+1} = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho$$
$$\hat{i}_t = \varrho + (1 + \phi)\,\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon_t^i$$

where $\varrho \equiv \log r^*$, $\kappa \equiv \lambda(1 + \varphi)$, and $\gamma \equiv \delta_1/[1 + \delta_1 - (1 - \delta_2)^{\frac{1+\sigma}{\sigma}}]$ is the steady state consumption share of retirees. Hats denote deviations from steady state (except for $\hat{i}_t$, which denotes the log of i_t).

From (17) one can see how the workers' Euler equation incorporates both the standard force of intertemporal substitution, as captured by the first term on the RHS, and a second term which captures wealth-related factors associated with retirement preoccupations. As the probability of entering the retirement state (δ_1) goes up, the weight on wealth-related factors increases relative to the role of inter-temporal substitution. In this sense, the greater are life-cycle forces, the more wealth factors will be at the center of consumption decisions and the monetary transmission mechanism. Moreover, as the EIS is reduced, the wealth-related factors become more dominant in the determination of consumption.

From (16) and (17), one can see that wealth-related factors consist of two distinct parts: a direct valuation effect in blue and an effect from the demand for retirement savings in red (which is inversely related to the interest income that past savings are expected to generate going forward). Let's discuss these in turn, starting with the former. As (19) shows, a higher expected path for real rates depresses the price q of the long-term bond contemporaneously. Via the blue terms in equations (16) and (17) this exerts a negative effect on consumption demand. We call this the "asset valuation channel". It works as a pure wealth effect, with rate hikes weighing on economic activity.

But at the same time, the red terms indicate that if $\sigma > 1$ a higher real rate also exerts a countervailing force *increasing* consumption. The reason is that, for a fixed value of assets, a higher interest rate implies that these assets will deliver a greater flow income to the owning household. This greater flow income lowers the need to hold as many assets for retirement when $\sigma > 1$, thus lowering asset demand, thereby stimulating demand for goods. To the extent that the increase in the interest rate is expected to persist, equation (18) – which summarizes the expected path of future interest rates – shows that this gets captured through a lower $\mathbb{E}_t\hat{\Gamma}_{t+1}$, giving this channel a boost. One can see this red term as an "asset demand channel", and one should keep in mind that it reflects how the expected stream of interest income affects the incentive to save for retirement.

4.2 How the effect of interest rates on activity varies along the yield curve

To see these effects in a slightly different light, it is helpful to recognize that both q_t and Γ_{t+1} can be expressed as function of current and future interest rates – giving rise to a term structure representation for the Euler equation. In particular, disregarding ε_t^β for the moment, the workers' Euler equation can be written as:

$$\hat{c}_t^w = (1-\delta_1)\,\mathbb{E}_t\hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1 \sum_{j=1}^{\infty} \beta^j \left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \mathbb{E}_t r_{t+1+j} \qquad (21)$$

This formulation of the Euler equation can be seen as incorporating several special cases present in the literature. For $\delta_1 = 0$, we obtain the standard RANK formulation. If $\sigma = 1$ and $\delta_1 > 0$, we have a formulation that is equivalent to putting assets directly into the utility function. Finally, if we have $\sigma = 1$, $\delta_1 > 0$, and $\mu = 1$, then we have a discounted Euler equation. Note that if $\sigma \leq 1$, then interest rates *at all future horizons* enter this Euler equation with a negative sign. Hence, in such a case, interest rate policy always works in the conventional way. Moreover, the more a rate decrease (increase) is viewed as being persistent, the more it will stimulate (contract) consumption demand.

In contrast, when $\sigma > 1$ ($EIS < 1$), monetary policy can affect the economy very differently depending on whether it only affects short-term rates, or if interest rates further out in the term structure are affected as well. In the remainder of this paper, we will focus our discussion on the case where $EIS < 1$ (which, according to studies like Yogo (2005), Best et al. (2020), and Ring (2024), is the most empirically plausible case).

The first aspect to note from (21) is that an increase in the short-term rate r_{t+1} will always contract consumption demand (and vice versa for a cut). However, the effects of future rates on y_t will depend on the sign of $\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}}-(1-\mu)^j\right]$. This term captures the competition between the valuation effects induced by interest rates effect, versus the induced effects on asset demand (stemming from the desire to save for retirement).[15] From equation (21) we see that – when holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant – when σ is sufficiently high and/or the interest rate considered is sufficiently far out into the future, a higher rate favors *more* consumption in the present. In other words, equation (21) indicates that the partial effect of increasing interest rates on current consumption (holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant) will tend to change sign, from negative to positive as one looks further in the future and the EIS is sufficiently low.[16] This arises as valuation effects only affect long-term assets, and these diminish further out in the future when $\mu > 0$ (which implies that the duration in assets is finite). Importantly, such sign-switching cannot arise under a discounted Euler equation formulation (more on this around our discussion of Proposition 2 below).

However, (21) only provides a partial answer to the effects of interest rates on activity since it is holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant and it ignores retirees' consumption. Before deriving the explicit expressions for the impact of future rates on current activity, we need to ensure that the equilibrium of the system (15)-(19) is well defined, i.e. stable and unique. Recall that monetary policy is governed by the parameter ϕ, which expresses the degree to which expected real interest rates are increased in response to expected inflation. The Taylor principle would suggest that ϕ may need to be strictly greater than zero. However in our setup, as expressed in Proposition 1, the model maintains determinacy even if $\phi = 0$.

Proposition 1. *With $\theta > 0$ (sticky prices), a constant real rate policy ($\phi = 0$) is sufficient to deliver determinacy.*

Proofs of all propositions are in Appendix B. In light of Proposition 1, the rest of the paper will set $\phi = 0$ to ensure determinacy while simultaneously allowing us to discuss the effects of different real rate paths on activity (and see Appendix C for a visual representation of the model's determinacy region). Once we solve (16) and (17) forward, the impact of future rates on current activity and inflation can be expressed as:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi^y_j \mathbb{E}_t \hat{r}_{t+1+j} \tag{22}$$

[15]Note that asset duration is governed by $(1-\mu)$. The duration of pension-related liabilities is increasing in $(1-\delta_2)$, as the expected duration of the retirement state is decreasing in the death probability δ_2.

[16]This can be seen from the fact that $\beta^j[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}}-(1-\mu)^j]$ will be positive for high enough j as long as $\sigma > 1$ and $(1-\mu) < 1$, that is, under the condition that not all bonds are consols.

$$\hat{\pi}_t = \sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} \tag{23}$$

with $\psi_0^y = -\frac{1}{\sigma}$, $\psi_0^\pi = -\frac{\kappa}{\sigma}$,

$$\psi_j^y = (1-\delta_1)\psi_{j-1}^y + \xi_j^\psi,$$
$$\psi_j^\pi = \beta\psi_{j-1}^\pi + \kappa\psi_j^y,$$

and

$$\xi_j^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} - \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j.$$

Here, each coefficient ψ_j^y represents the isolated impact that the real rate at horizon j has on output in the present (with ψ_j^π representing the equivalent concept for inflation). Note that it is always the case that an increase in the nearest term rate $\mathbb{E}_t\hat{r}_{t+1}$ depresses current activity, as this effect is driven solely by intertemporal substitution ($\psi_0^y = -\frac{1}{\sigma} < 0$). However, the effect of interest rates further out into the future becomes ambiguous as the three forces are at play: intertemporal substitution, valuation effects, and effects on asset demand. Before deriving some of the properties of the ψ_j's when $\delta_1 > 0$ (i.e., when life cycle forces are present), it is worth recalling that our model collapses to the RANK model when $\delta_1 = 0$. In that case, $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^\pi = \kappa\frac{1-\beta^{j+1}}{1-\beta}\psi_j^y$ for all $j \geq 0$. This implies that near-term interest rates always have the exact same effect on output as rates further out into the term structure (with this effect always being equal to $-\frac{1}{\sigma}$).

In contrast, as noted in Proposition 2, especially part (c), when $\delta_1 > 0$, the sign of ψ_j^y becomes dependent on the EIS. If the EIS is sufficiently large, interest rates at all horizons will have conventional effects on y_t and π_t as intertemporal substitution remains the dominant force. However, when the EIS is sufficiently small, interest rates further out in future will have an effect that is *opposite* in sign to that associated with short-term rates since asset demand effects (driven by an interest income effect) will dominate.

Proposition 2. *For $\delta_1 > 0$ (i.e., when introducing retirement risk, giving rise to our "FLANK" model), we have that:*

(a) The ability of interest rates to affect activity and inflation in the conventional direction (i.e., with contractionary shocks lowering activity and inflation, and vice versa)

is weakened relative to RANK: $\psi_j^y > -\frac{1}{\sigma}$ *and* $\psi_j^\pi > -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$*, for all* $j \geq 1$*;*

(b) In the limit, taking the horizon j *to infinity,* $\mathbb{E}_t \hat{r}_{t+1+j}$ *ceases to affect activity and inflation in the present:* $\lim_{j\to\infty} \psi_j^y = 0$ *and* $\lim_{j\to\infty} \psi_j^\pi = 0$*;*

(c) At every horizon $j \geq 1$*,* ψ_j^y *and* ψ_j^π *are increasing in* σ*; they eventually become positive as* σ *is increased;*

(d) The ability of interest rate policy to affect activity and inflation in the conventional direction is increasing in retirees' death probability (δ_2) *and increasing in the duration of available assets (i.e., decreasing in* μ*) for all* $j \geq 1$*.*

The main takeaway from Proposition 2 is that, with life-cycle forces, the effect that interest rates have on activity can vary along the yield curve – both quantitatively and qualitatively (with FLANK allowing for the possibility that higher near-term rates are contractionary, whereas higher rates further out into the term structure can be expansionary at the same time). Parts (a) and (b) of this proposition are shared by models featuring a "discounted" Euler equation (McKay et al., 2017). Parts (c) and (d) are specific to our FLANK model. Part (c) of the proposition implies that, in FLANK, interest rates further out in the yield curve may have *opposite* effects to that of near-term rates (with higher long-term rates *boosting* activity; a prediction we will test in Section 5). This is something that can neither arise in a RANK setup, nor under a discounted Euler equation.

Part (d) of Proposition 2 provides additional insight on the determination of the ψ_js. It shows that interest rate policy loses potency (in the conventional direction) as households' longevity increases (lower δ_2). The reason is that this increases the duration of households' liabilities – with them having to finance a longer consumption stream in retirement, where households rely on asset income – meaning that low interest rates in the future (which are normally expansionary) incite more savings by working households and slower asset depletion by retirees.

Similarly, part (d) implies that interest rate policy loses potency in the conventional direction when the duration of households' assets decreases (higher μ). The reason is that this weakens the asset valuation effect, which works in the conventional direction (with lower rates being expansionary). This part of our proposition is relevant when thinking about the potential role of QE in affecting the monetary transmission mechanism. Since QE acts like an asset swap, with the central bank replacing high-duration assets (longer-term government bonds) with overnight central bank reserves carrying zero duration, QE can be seen as the central bank pushing up μ (lowering the share of longer-term bonds held by the public[17]), which renders conventional monetary policy (conducted via the interest rate) less potent.[18]

It important to emphasize that Part (c) of Proposition 2 is central to our key results which are to follow, as it opens the door to the possibility that persistent rate changes may have qualitatively different effects compared to more temporary ones.

4.3 The effect of interest rate persistence on potency and direction

We are now in a position to discuss how the potency of monetary policy shocks can change with their persistence. To explore this issue, consider a shock ε_t^i to the interest rate rule that follows an AR(1) process with with autocorrelation parameter ρ_i (as specified in (13)). These assumptions imply that the policy shock induces a time path for the real interest rate given by $\mathbb{E}_t \hat{r}_{t+1+j} = \mathbb{E}_t \varepsilon_{t+j}^i = (\rho_i)^j \epsilon_t^i$. The impact responses of output and inflation to such monetary policy shock are then given by:

$$\hat{y}_t = \Psi^y(\rho_i)\epsilon_t^i, \tag{24}$$

$$\hat{\pi}_t = \frac{\kappa}{1-\rho_i\beta}\Psi^y(\rho_i)\epsilon_t^i, \tag{25}$$

where

$$\begin{aligned}\Psi^y(\rho_i) &\equiv \sum_{j=0}^{\infty} \psi_j^y \rho_i^j \\ &= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} + \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\frac{\sigma-1}{\sigma}}{1-\rho_i\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\rho_i\beta(1-\mu)}\right]\end{aligned} \tag{26}$$

[17]At this stage it is important to note that our Blanchard-Yaari-Gertler setup implies that Ricardian Equivalence does not hold; because of this breakdown, the maturity structure of assets held by the public starts to matter. For $\delta_1 = 0$, Ricardian Equivalence holds and μ no longer matters for (22) and (23).

[18]Concerns related to this aspect of our model have recently come to the fore. As noted in Bloomberg (2023): "UK households are on aggregate about £10 billion ($12.7 billion) a year better off as a result of a jump in interest rates [...] At current rates, savers collectively are earning £24 billion more a year than in November 2021 [...] Respondents to GfK's June consumer confidence barometer said their personal finance situation had improved sharply last month, despite the surge in mortgage rates [...] The data suggests interest rates may not be as effective a monetary policy tool as they were in 2008".

captures the effect of a monetary policy shock ϵ_t^i with persistence ρ_i on current output. Since this simplified version of our model features no state variables, we have that $\hat{y}_t = \rho_i^t \hat{y}_0$ and $\hat{\pi}_t = \rho_i^t \hat{\pi}_0$ – implying that results continue to apply at all horizons $t \geq 0$.

Equations (24) and (25) carry several interesting implications about how changing the persistence of monetary shocks affects their efficacy in terms of affecting output and inflation. If either $\delta_1 = 0$ (no retirement preoccupations) or $\sigma \leq 1$, then more persistent monetary policy shocks always have greater potency than temporary changes. In particular, when persistence ρ_i goes to 1, the potency of monetary shocks becomes very large, and goes to infinity if $\delta_1 = 0$ (i.e., for the RANK model). It is because of this potency that it is generally thought that monetary policy cannot keep real interest rates away from their flexible-price counterpart r^* for long periods without having major effects on inflation. However, in the presence of a retirement savings motive ($\delta_1 > 0$) and if $\sigma > 1$, the link between the persistence of monetary shocks and their effect on the economy becomes more involved.

While it is clear from (24) and (25) that the link between the persistence of monetary shocks and their effects on the economy depend on many parameters, Proposition 4 emphasizes the role played by the EIS $(1/\sigma)$. In particular, it emphasizes the existence of two threshold levels for σ for which the relationship between monetary shock persistence and their effect on the economy changes qualitatively.

Proposition 3. *For $\delta_1 = 0$, $\Psi^y(\rho_i) < 0$ for all $\rho_i \in [0, 1]$, $\partial \Psi^y(\rho_i)/\partial \rho_i < 0$, and $\lim_{\rho_i \to 1} \Psi^y(\rho_i) = -\infty$.*

Proposition 4. *If $\delta_1 > 0$, then $\lim_{\rho_i \to 1} \Psi^y(\rho_i)$ is finite and $\exists \sigma^*, \sigma^{**}$ with $\sigma^{**} > \sigma^*$, such that for very persistent monetary policy shocks (ρ_i close to 1):*

(a) If $\sigma < \sigma^*$, then $\Psi^y(\rho_i) < 0$ and $\partial \Psi^y(\rho_i)/\partial \rho_i < 0$, meaning that more persistent shocks have a stronger effect on activity in the conventional direction (i.e., with contractionary shocks lowering activity and vice versa);

(b) If $\sigma > \sigma^*$, then $\partial \Psi^y(\rho_i)/\partial \rho_i > 0$, meaning that increases in shock persistence DECREASE *the shock's effect on activity in the conventional direction;*

(c) If $\sigma > \sigma^{**}$, then $\Psi^y(\rho_i) > 0$, meaning that sufficiently persistent monetary policy shocks can affect activity in the unconventional direction.

The main aspect to note from Proposition 4(b) is that, when σ is high enough in FLANK, a more persistent monetary shock will be *less* potent than a more temporary one – making for a stark contrast with RANK (covered by Proposition 3).[19] This arises because the effects of monetary shocks on consumption are not just driven by intertemporal substitution in FLANK. Instead, they are also driven by how the rate change affects the desire to accumulate, and hold on to, assets (to ensure income in retirement). The latter depends on whether the lower (higher) rates are incentivizing households to hold more (less) wealth and whether valuation effects are sufficiently large to offset any changes in their desire to save. What Proposition 4 indicates, is that as σ increases, intertemporal substitution becomes less relevant and the impact on asset demand will eventually dominate the valuation effect. This then causes more persistent shocks to have less of an effect on activity than more temporary changes.[20] In fact, the sign of the effect can even flip, as implied by part (c) of the proposition. To visualize this, Figure 2 plots $\Psi^y(\rho_i)$ as ρ_i varies between 0.5 and 1.[21] The figure illustrates that, for rather transitory shocks, life-cycle forces do not affect the monetary transmission mechanism much (i.e., FLANK behaves much like RANK for relatively low values of ρ_i). But as ρ_i increases sufficiently, the two models show stark divergence: whereas RANK implies that very persistent shocks are incredibly potent (with this potency going to infinity in the limit), FLANK suggests the opposite may arise – with $\Psi^y(1) \approx 0$ being a plausible outcome.

In this regard, it is insightful to consider the analytical expression for Ψ^y that results when taking $\rho_i \to 1$ in equation (26). In that case, one obtains:

$$\Psi^y(1) = \sum_{j=0}^{\infty} \psi_j^y = \underbrace{-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1}}_{\textit{intertemporal substitution}} + \underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{\textit{asset demand}} - \underbrace{\frac{1}{1-\beta(1-\mu)}}_{\textit{asset valuation}}. \tag{27}$$

Written this way, the decomposition central to our paper becomes very clear. The first term captures the intertemporal substitution force that is inherent to more standard Euler equation setups. This is always negative and goes to zero as $\frac{1}{\sigma} \to 0$. The second term captures the asset demand effect. This is primarily driven by the duration of household

[19]In addition, it is also possible to show that $\Psi^y(\rho_i)$ is decreasing in δ_2 and increasing in μ, which would be another way to state the message conveyed by part (d) of Proposition 2.

[20]This result is somewhat reminiscent of Lucas and Rapping (1969), who show that the response of labor supply may vary with the persistence of the wage impulse. When the latter is rather transitory, the substitution effect is likely dominant – making labor supply increase with the wage rate. But when the wage changes in a rather persistent manner, the income effect gains importance – potentially causing labor supply to fall with wages.

[21]This figure was generated using a relatively standard calibration at the annual frequency: $\sigma = 4$, $\beta = 0.96$, $\delta_1 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retired life of 20 years), and $\mu = 0.15$ (average bond maturity of 6.7 years).

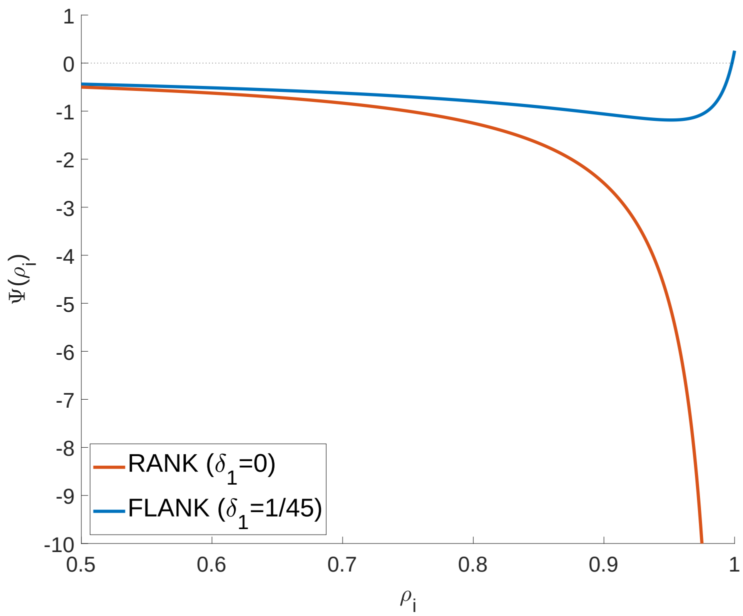

Figure 2: $\Psi^y(\rho_i)$ in RANK and FLANK. Other parameters calibrated as in footnote 21.

liabilities, as governed by the death probability δ_2, which determines the expected length of the retirement state in which the household only enjoys interest income.[22] When $\sigma > 1$ this term is positive and when $\frac{1}{\sigma} \to 0$, this term goes to $\frac{1}{1-\beta}$. Finally, the third term captures the asset valuation effect, which is driven by μ (the duration of the long-term bond). Whenever the household is "short duration", the sum of the last two terms is positive – opening the door to the total effect $\Psi^y(1)$ being close to zero (as the first term in (27) is negative) when the EIS is low. It is interesting to note that in the special case where both $\frac{1}{\sigma} \to 0$ and $\mu \to 0$, we have $\Psi^y(1)$ exactly equal to 0. This arises because consumption becomes driven by the flow value of wealth, rW, while the value of wealth itself is proportional to $\frac{1}{r}$.

For the baseline calibration used in Figure 2, $\Psi^y(1)$ is very close to zero. But since there is considerable uncertainty in the literature regarding the appropriate value for the EIS, Figure 3 goes a step beyond Figure 2 and presents an entire heatmap for $\Psi^y(1)$. This heatmap is associated with the different values taken on by $\Psi(1)$ for different plausible values of the EIS, $1/\sigma$, and bond duration, as governed by μ. For the other two parameters in $\Psi(1)$, we fix $\delta_2 = 1/20$, giving an expected retirement span of 20 years, and we set $\beta = 0.96$.

[22]Note that the role of this effect is maximized for $1/\sigma \to 0$. In that limit case, the household is infinitely risk averse, meaning that it only consumes its interest rate income – never daring to touch the principal itself, for fear of outliving its assets.

When aiming to calibrate a relevant range for $\Psi^y(1)$, our biggest challenge relates to determining the plausible range for the EIS. To this end, we draw from Best et al. (2020) which uses a frontier empirical strategy to identify the EIS. Their preferred estimate for the EIS is 0.1, which we take as a minimal value (as it is comparatively low relative to other available estimates). At the other end, they report values up to 0.3 (see their Table 3B, pooled estimate), so we go up to 0.35 to be inclusive of higher values (which is also consistent with Havránek's (2015) meta-analysis, which reports estimates centered around 0.3-0.4). With respect to average bond maturities, we consider a range between 5 and 20 years (i.e., μ between 0.05 and 0.2), which is aimed at capturing a set of interpretations for assets held. Lower durations (higher μ) are appropriate when only thinking of government bonds, while higher durations (lower μ) is reasonable when thinking of a combination of bonds, equity and real estate.[23] We are aiming to be quite inclusive in the range of parameters explored, as to give a sense of the possible outcomes that can arise in FLANK.

In Figure 3, the red areas represent positive values for $\Psi^y(1)$, while the blue areas represent negative values (with this being the more "conventional" region, as we are considering a permanent rate increase). The white area represents values for $\Psi^y(1)$ that are close to zero, with the black lines representing iso-$\Psi^y(1)$ curves. An iso-$\Psi^y(1)$ curve marked $\pm 1\%$ indicates that a policy which would aim to permanently raise the real rate by 1 percentage point relative to r^*, would cause a 1% deviation of output from its natural level. Note that with a standard formulation of the Euler equation (including the discounted variant), the whole area on the figure would be blue. In particular, the standard RANK model would imply that this entire surface would be valued at $-\infty$. In contrast, Figure 3 shows that positive values for $\Psi^y(1)$ seem almost as plausible as negative values with FLANK. In other words, FLANK gives little reason ex ante to believe that persistently low (high) interest rates are more likely to stimulate (depress) the economy than to depress (stimulate) it. This by itself is an important implication of the FLANK model.[24]

The area in Figure 3 where $\Psi^y(1)$ is *exactly* equal to zero, is by its very nature of measure zero. In this sense, the case where $\Psi^y(1)$ is exactly equal to zero is not very relevant. Nonetheless, the figure shows that there is a considerable area where $\Psi^y(1)$ may be considered to be quite small. In effect, this might represent around a quarter of the

[23] Van Binsbergen (2021) estimates the duration of the S&P 500 at around 20 years. The duration of housing is estimated to be around 8 years (Burgert et al., 2024).

[24] While much of the discussion in this paper focuses on the possibility of having $\Psi^y(1)$ being close to zero, it is worth noting that the possibility of $\Psi^y(1)$ being positive (instead of negative) suggests that low-for-long policies may have contributed to depressing the economy instead of stimulating it.

area. Recall that over the period from 1990 to 2019, the output gap in the US varied by several percentage points without inflation moving much. This suggests that, when inflation expectations are well anchored, variations in activity of a few percentage points away from their natural level may not affect inflation by a lot. Accordingly, Figure 3 hosts a considerable region which could be consistent with a permanent departure of real rates from r^* not creating much inflation, if this interest rate departure is of the order of 1 percentage point (or less).

Figure 3 illustrates that the effect on activity of real rates permanently deviating from r^* is both qualitatively and quantitatively quite different in FLANK, relative to a more standard New Keynesian model. In particular, in FLANK the effect can be positive, negative or close to zero – as opposed to simply always being negative. While we think that the potentially most intriguing take away from this figure is that $\Psi^y(1)$ may be close to zero, Figure 3 only advances this as one possibility. For this reason, it would be of interest to investigate whether modifications to our simple FLANK setup can make the $\Psi^y(1) \approx 0$-region larger or smaller. Although we leave this question to future work, we will return to this issue when discussing bequest motives in Section 7 (the incorporation of which seems able to widen the area).

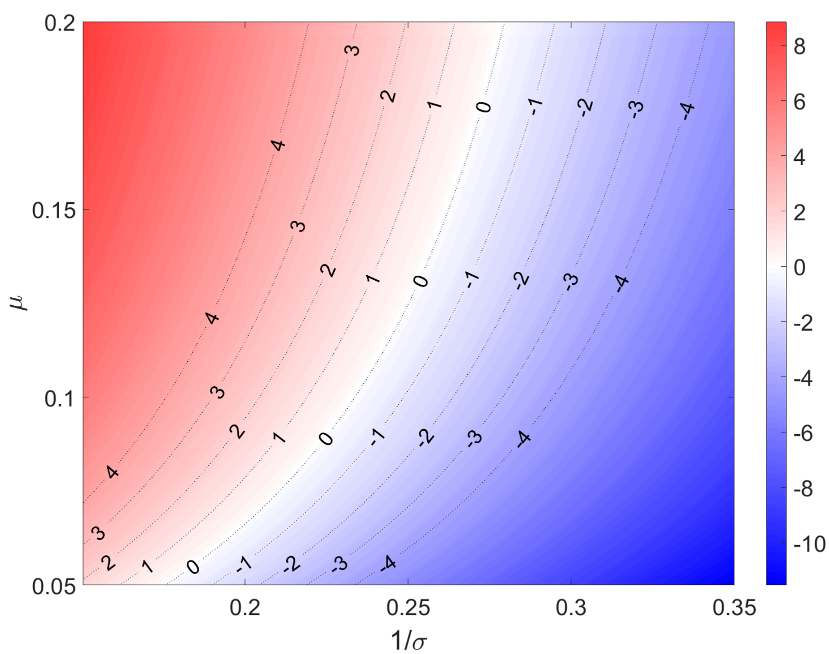

Figure 3: $\Psi^y(1)$ as a function of σ and μ in FLANK. Other parameters calibrated as in footnote 21.

At this stage, one may wish to recall that the above representation of equilibrium outcomes was derived under the assumption that wealth shares across the two sets of agents (retired versus working) was held constant through a tax-transfer scheme. An obvious question is whether this simplification substantially affects the properties of the model – especially regarding $\Psi^y(1)$. For this reason, we also solved the model numerically *without* imposing this restriction. Figure 4 displays the resulting equivalent to Figure 3. As can be seen by comparing the two figures, both the qualitative and quantitative properties of $\Psi^y(1)$ are essentially unchanged when removing this assumption.

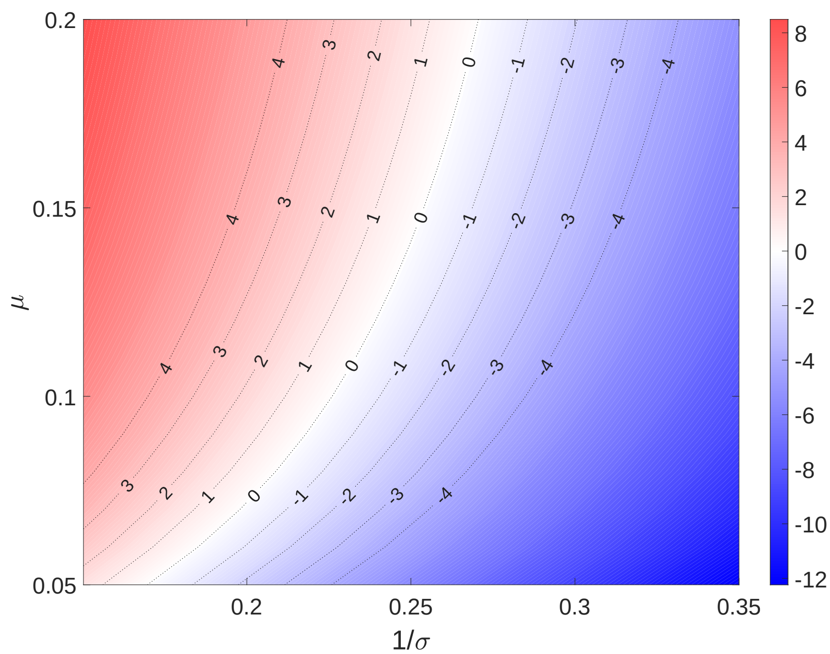

Figure 4: $\Psi^y(1)$ as a function of σ and μ in FLANK, not imposing constant wealth shares across groups. Other parameters calibrated as in footnote 21.

4.4 Why a monetary shock is not equivalent to a demand shock in FLANK

Another interesting feature of the FLANK model is that it breaks down the equivalence (for example present in RANK) between monetary shocks and other types of demand shocks. This section illustrates this point by considering shocks "ε_t^β" to the discount rate (recall equation (14)), but the point is more general.

To see this, observe that there exists an equivalent representation to equations (22)-(23), which were derived setting all $\varepsilon_t^\beta = 0$, when allowing for discount rate shocks. In particular, Appendix B shows that the effects of discount rate shocks "ε_t^β" on output are given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{28}$$

$$\hat{\pi} = \sum_{j=0}^{\infty} \omega_j^\pi \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{29}$$

with $\omega_0^y = -\frac{1}{\sigma}$, $\omega_0^\pi = -\frac{\kappa}{\sigma}$,

$$\begin{aligned}\omega_j^y &= (1-\delta_1)\,\omega_{j-1}^y + \xi_j^\omega,\\ \omega_j^\pi &= \beta\omega_{j-1}^\pi + \kappa\omega_j^y\end{aligned}$$

and

$$\xi_j^\omega \equiv -\frac{1}{\sigma}\left[\delta_1 - \gamma\,(1-\delta_1)\,\frac{1-\beta\,(1-\delta_2)^{\frac{1}{\sigma}}}{\beta\,(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j\,(1-\delta_2)^{\frac{j}{\sigma}}.$$

Crucially, whenever $\delta_1 > 0$, the coefficients on the discount rate shock at each horizon $j > 0$ are *not* proportional to those for the monetary policy shock. For RANK (i.e., when setting $\delta_1 = 0$) the coefficients *are* proportional. In that case, a monetary policy shock induces the exact same dynamics as a discount rate shock – meaning that the former is extremely well-suited to offset the latter. However, in FLANK that is no longer the case. In this world, while discount rate shocks continue to operate through the intertemporal substitution channel, policy-induced shocks to the interest rate are "special" as they come with an offsetting effect (by changes in interest income affecting asset demand) that render more persistent monetary policy shocks less potent. In fact, the time-t impact of a persistent AR(1) discount rate shock is given by:

$$\begin{aligned}\Omega^y(\rho_\beta) &\equiv \sum_{j=0}^{\infty}\omega_j^y\rho_\beta^j\\ &= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_\beta(1-\delta_1)} - \frac{1}{\sigma}\frac{\gamma+\frac{\delta_1(1-\gamma)}{1-\rho_\beta(1-\delta_1)}}{1-\rho_\beta\beta(1-\delta_2)^{\frac{1}{\sigma}}}\end{aligned}$$

From this, it is easy to see that $\Omega^y(\rho_\beta) < 0$ for all $\rho_\beta \in [0,1]$, with $\partial\Omega^y(\rho_\beta)/\partial\rho_\beta < 0$ (meaning that more persistent shocks are more potent in the conventional direction).

These observations suggest that monetary policy may be less well equipped to offset demand-type shocks in a FLANK world, especially when demand shocks are very persistent.

5 Do the effects of monetary policy shocks vary along the yield curve?

An important implication of the FLANK setup is that monetary policy shocks located on the front end of the yield curve should do more to affect real activity in the conventional direction, than shocks to longer-term rates; recall Proposition 2(c). While Section 2 already presented evidence hinting in this direction, we now assess this hypothesis by building on recent advances in monetary policy shock identification.

In particular, we follow Gürkaynak, Sack, and Swanson (2005) and Swanson (2021) by distinguishing between shocks that load primarily on the front end of the yield curve ("target shocks") and shocks further out in the yield curve ("path shocks").[25]

Armed with Swanson's (2021) target- and path shocks (mps^{target} and mps^{path}, respectively) for the US, we proceed by estimating Local Projections, using monthly data, with the LHS featuring the "long difference" in the unemployment rate u:[26]

$$u_{t+h} - u_{t-1} = \alpha_h + \beta_h mps_t^k + \gamma_h X_t + \varepsilon_{t+h},$$

where $k = \{target, path\}$, while X_t is a vector of standard controls, containing twelve lags of: the dependent variable u, the monetary policy shock, the natural log of the CPI, and the central bank's policy rate (data sources are listed in Appendix D). When looking at the impact of target (path) shocks, we also control for path (target) shocks – contemporaneously and twelve lags – in X_t.

As can be seen in Panel (a) of Figure 5, the response of the unemployment rate suggests that target shocks (i.e., those hitting the short-end of the yield curve) contract economic activity, as is conventionally thought. But the figure also shows that path shocks (i.e., those hitting further out into the term structure) tend to have a zero-, or even perverse effect. In Panel (b) and (c), we perform the same analysis, using the exact

[25] Swanson (2021) also identifies a "QE shock" but since our paper is not about the direct impact of QE (we only touch on possible interactions with the potency of interest rate policy; recall Proposition 2(d)) we do not use these in our analysis.

[26] Using the "long difference" $\Delta^h y_t \equiv y_{t+h} - y_{t-1}$ is recommended by Jorda and Taylor (2024), but using a levels-specification yields very similar results.

same regression specification, on data for the Eurozone and the UK. In both cases, we see a similar pattern: target shocks act in the conventional direction, while path shocks either have no significant effect or act in the unconventional direction.[27]

The results presented in Figure 5, while not tightly estimated, do nonetheless support our model's implication that policy-induced changes in longer-term interest rates might do relatively little to affect the level of real activity, or even have perverse effects. In Appendix D we show that looking at the response of industrial production indices (instead of the rate of unemployment) yields similar results, except for the Eurozone where there is no clear difference in the response of industrial production to target versus path shocks.[28]

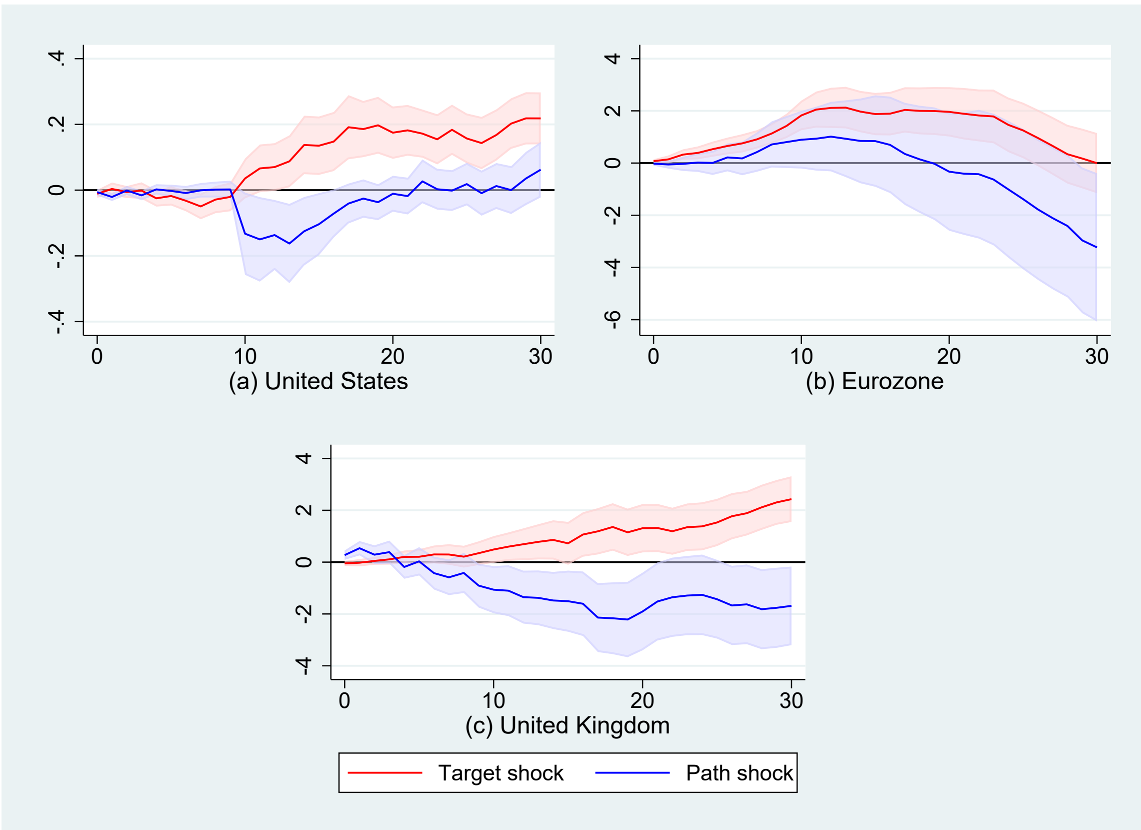

Figure 5: Response of the unemployment rate to "target" and "path" shocks. LPs estimated at the monthly frequency. Shaded areas represent 68% confidence bands.

[27] For the UK, the shocks are taken from Braun, Miranda-Agrippino and Saha (2024); for the Eurozone, we apply the procedure of Braun et al. to the Eurozone data underlying Altavilla et al. (2019). We thank Braun, Miranda-Agrippino and Saha for sharing their code.

[28] Of note: using a very different empirical approach (exploiting cointegrating relationships), Uribe (2022) also finds that output only responds with the conventional sign in response to temporary shocks.

6 Reflections on r*

The FLANK model we have developed implies that the effects of interest rates on economic activity will vary along the yield curve, likely switching sign along the way. In the remainder of this section, we will show that this has important implications for both the relevance of the natural rate of interest "r^*" as a policy anchor, and for the estimation of r^*. In particular, our FLANK setup implies that proper knowledge of r^* may not be very important for inflation-targeting central banks – because the system may be very "forgiving" to the central bank working with a biased value for r^*. This indicates that central banks might still be able to fulfill their mandate in a satisfactory way, even if they happen to be ill-informed about the true value of r^*. In addition, we will show that a common method used to infer r^* may be biased and essentially deliver the central bank's own prior beliefs regarding the location of r^*, as opposed to the actual value of r^*.

6.1 The (ir)relevance of r*

As mentioned before, standard models suggest that the location of r^* is crucial for central banks to be aware of, since keeping rates away from that level for too long is bound to force inflation away from target.[29] In contrast, the FLANK model suggests that central banks may be much less constrained by r^*, potentially making r^* a quasi-irrelevant object and opening the door for monetary policy to influence longer-term real rates. To further clarify the extent to which monetary policy is constrained by r^*, consider the class of models where activity $\hat{y}_t$ can be related to the future path of interest rates in the form:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t (r_{t+1+j} - r^*).$$

This formulation (first shown in equation (22)) hosts the standard RANK model as well as our FLANK setup – where the to models differ only with respect to different implied coefficients for ψ_j^y.

Now consider a situation where the central bank misperceives r^*, where we denote the central bank's perception of r^* by r^L (which can be seen as the central bank's long-run target for r). So, the central bank thinks that activity is governed by:

[29] This notion also appears to be gaining popularity in practice, with the number of central bank speeches referring to the "natural/neutral interest rate" having risen sharply since 2015 (Borio, 2021).

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^L).$$

To what extent would this misperception be problematic? In the presence of such a misperception, the actual determination of output will be given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^L) + \Psi^y(1)(r^L - r^*),$$

where $\Psi(1) = \sum_{j=0}^{\infty} \psi_j^y$. As seen in the above expression, the relevance of r^* for $\hat{y}_t$ depends crucially on the value of $\Psi^y(1)$. When activity is determined by a standard representative agent Euler equation, $\Psi(1) = -\infty$. In this case, making sure that r^L equals r^* is *absolutely crucial* for monetary authorities as deviations of r^L from r^* would have huge implications for activity and consequently inflation.[30]

However, in the FLANK model, $\Psi^y(1)$ may actually be close to *zero*. In this case, deviations of r^L from r^* do not affect activity much. And if the Phillips curve is not very steep, as for example argued by Hazell et al. (2022), an $(r^L - r^*)$-gap would only have a small effect on inflation. Therefore, when $\Psi^y(1)$ is small, a central bank could potentially adopt a policy rule where its long-term anchor for real rates r^L is substantially different from the true r^* without causing any major economic disruption.

In this sense, knowing r^* becomes quasi-irrelevant for the conduct of monetary policy, as the system is very forgiving to the central bank working with a biased r^*-belief. In particular, in the special case where $\Psi^y(1)$ is *exactly* zero, then r^* becomes indeterminate and the central bank can set its long-term goal r^L freely, without any direct implications for output and inflation. Still, the choice for r^L will have implications for asset prices.

6.2 Biased estimation of r*

Our FLANK model also has important implications for estimations of r^*. To see this, note that a very typical formulation (that sits at the core of many popular DSGE models) for the consumption Euler equation reads:

$$\hat{c}_t = \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t(r_{t+1} - r^*_{t+1}) + v_t, \tag{30}$$

[30] This logic captures why central banks are often thought to be heavily constrained by r^*, while it also explains why there is a Forward Guidance Puzzle (Del Negro et al., 2013).

where the parameter $\alpha \leq 1$ reflects a generalization which allows the Euler equation to be "discounted" (in the sense of McKay et al. (2017)) and v_t represents a stationary demand shock. For illustrative purposes we can assume that v_t is an i.i.d. disturbance and assume that r^* follows a random walk: $r^*_{t+1} = r^*_t + w_t$, where w_t is again taken to be i.i.d.

If the data are thought to be driven by such an Euler equation, the work by Laubach and Williams (2003; "LW") suggests a way to estimate r^*_t. In essence, it consists of first creating an "observation" variable z_t as $z_t \equiv (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_t)\sigma$. Given this definition, which gives $z_t = r^*_{t+1} + \sigma v_t$, any long-run variation in z_t will be driven by r^*_{t+1} – implying that one can apply the Kalman filter to the z_t series and successfully recover an estimate of r^*_{t+1}.[31]

We now explore what the above approach would uncover if the data were generated by the FLANK model, but it was misinterpreted as being generated by a more standard Euler equation. In particular, we want to examine the case where one *thinks* the consumption data are generated by (30), but the actual data are generated by FLANK:

$$\hat{c}_t = \sum_{j=0}^{\infty} \psi^y_j \mathbb{E}_t r_{t+1+j} + \Psi^y(1) r^*_{t+1} + v_t, \tag{31}$$

with ψ^y_j as in (22). Combining (31) with an interest rate rule of the form:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}^{CB}_t r^*_{t+1} + \varepsilon^i_t, \tag{32}$$

where $\mathbb{E}^{CB}_t r^*_{t+1}$ represents the central bank's perception of r^*_{t+1} (also following a random walk), we can again create $z_t = (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_{t+1})\sigma$ as suggested by the LW methodology. But in this case z_t will no longer be a noisy reflection of r^*_{t+1} only, as it is now given by:

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}^{CB}_t r^*_{t+1} + \Psi^y(1)(1-\alpha) r^*_{t+1} \right] + (\sigma - 1) v_t. \tag{33}$$

Equation (33) shows that z_t will only succeed in being a noisy reflection of r^*_{t+1}, uncontaminated by the central bank's own belief $E^{CB}_t r^*_{t+1}$, when $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$. But $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$ only arises if the data are actually generated by an Euler equation of the form (30). Whenever $\Psi^y(1) \neq \frac{-1}{(1-\alpha)\sigma}$ (which is the case for FLANK; recall (27)), z_t will in part end up reflecting variations in the central bank's own perceptions $\mathbb{E}^{CB}_t r^*_{t+1}$. If $\Psi(1)$ is close to zero, then z_t will *mainly* reflect $\mathbb{E}^{CB}_t r^*_{t+1}$ instead of the true r^*_{t+1}.

[31] Throughout this section, we give the LW methodology its best chance by assuming that the central bank knows the private expectation $\mathbb{E}_t \hat{c}_{t+1}$. However, similar results arise if we assume that the central bank approximates this expectation with $\hat{c}_{t-1}$.

Matters only get worse if one were to specify a more general interest rate rule. In particular, consider replacing (32) by:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}_t^{CB} r_{t+1}^* + \theta v_t + \varepsilon_t^i,$$

where $\theta > 0$ allows the central bank to respond to demand shocks v_t. We then get:

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}_t^{CB} r_{t+1}^* + \Psi^y(1)(1-\alpha) r_t^* \right] + \left[(\sigma - 1) + \theta \right] v_t.$$

Now, "θv_t" shows up in z_t, implying that the central bank's perception of r_t^* starts to co-move with its own short-term actions in response to demand shocks v_t. While standard logic suggests that any co-movement between a central bank's policy rate and r^*-estimates is due to the central bank successfully tracking the latter, our results suggest that the causality may run the other way: an initial negative, purely transitory demand shock, which induces the central bank to cut its policy rate, might ignite a dynamic that leads the central bank to lower its estimate of r^* – which then has the unintended consequence of giving the initial rate cut more persistence through an unanticipated downward revision in the intercept of the policy rule (32). If $\Psi^y(1) \approx 0$, persistent rate changes don't affect activity and inflation much, meaning that there is no strong feedback from the system and hence no strong force pulling the central bank back towards the true r^* (recall Section 6.1).[32] In this case, $\mathbb{E}_t^{CB} r_{t+1}^*$ obtains a self-fulfilling aspect and it then becomes rational for markets to pay attention to the central bank's belief on r^*, even if markets do not think that the central bank has private information regarding r^*.

7 Discussion: assumptions and extensions

7.1 Assumptions

In this section we want to briefly discuss three assumptions underlying our model. The first is related to the absence of hand-to-mouth agents, the second is related to the absence of equity and the third relates to the absence of bequest motives. In all three cases, we will point out why our current assumptions could be easily relaxed and why we think

[32]John H. Williams (1931) famously argued that "The natural rate is an abstraction; like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate must have been brought in line with the natural rate, but if it does not, it must not have been." Our FLANK model suggests that these "works" might be rather weak, implying that there is not much to be learned from outcomes.

they would not likely change our key insights. We also discuss why our results should be considered as “local”, placing implicit bounds on how far interest rates could deviate persistently from the true r^*.

Hand-to-mouth agents. Our model treats all households as intertemporal optimizers. This may appear inappropriate given the evidence supporting the presence of hand-to-mouth consumers (Kaplan et al., 2014). Accordingly, the mechanisms in the model may appear relevant only for the more financially well off. We concur with this assessment but do not view it as a drawback of our approach. One of the main insights from the hand-to-mouth literature is that the dynamics of aggregate activity will primarily be driven by the behaviour of optimizing households – even if the later are only a fraction of the total population (Werning, 2015). With hand-to-mouth households, the decisions of optimizing agents are transmitted to wider economy through the non-optimizing households – potentially yielding amplification (Bilbiie, 2020; 2024). But as long as the fraction of total income going to hand-to-mouth households is not changing very much, treating the economy as if driven only by the optimizing households becomes a good approximation. This is the interpretation we favor, with the recognition that the actual modelled behavior may only reflect a subset of the population. While our model's structure is flexible enough to easily allow for the incorporation of hand-to-mouth households, we choose not to follow this route as this would complicate the setup without adding anything new.

Equity. The only asset that agents can hold in our model are government bonds. This may seem restrictive, as it neglects equity. Introducing an equity market in the model is quite straightforward. In our current setup, working households own all firms. An alternative would be to allow firm equity to be traded in a market featuring both workers and retirees, and where the equity price would respond to interest rates as implied by standard arbitrage conditions. We have explored this modification and have not found it to affect our main results – motivating our choice for the simpler setup. The reason that allowing for equity does not materially affect the mechanisms central to our paper, is that interest rates affect equity and long-term bond prices in the same direction. So, while allowing for equity makes the model's asset valuation channel slightly more involved, it does not change its nature. There are nonetheless two aspects that would change with the inclusion of equity. The first relates to the strength of the valuation channel. With only long-term bonds, the strength of this channel is governed by bond duration. In contrast, with equity, the strength of this channel would also be governed by the equity risk premium.[33] This does not change the main mechanism, but it influences how to calibrate the model (as discussed in footnote 23). The second aspect that would change with equity, is that it would open the door to exploring changes in risk premiums (see Caramp and Silva (2021) for an analysis along these lines), which is also related to the literature on safe asset demand (Caballero et al., 2016; 2017). We believe the latter would be interesting to explore, but leave this to future work.

Housing. Along very similar lines, the logic of the model would continue to hold if households were also allowed to save in a housing asset. So, while our model contains a long-term bond as the asset through which saving takes place, the exact nature of the asset is of secondary importance. The more important issue is that this asset has positive duration, i.e., that its price "q" is inversely related to the interest rate.

Bequests. While our FLANK model does not include a bequest motive, we believe that its main insights should carry through and may even be strengthened with such an extension. In particular, it is likely that the presence of bequest motives would accentuate the asset demand force present in FLANK. One of the difficulties with respect to modelling bequests relates to how best to capture the objective of the savers involved. A simple way to capture a bequest motive within our framework, would be to think of bequests as consumption past death. In that case, a bequest motive would be similar to having longer longevity, that is, a lower δ_2. To explore how bequests could affect our results, especially with respect to $\Psi^y(1)$ (i.e., the effect that a permanent increase in real rates has on consumption demand), we explored implications of reducing δ_2. As can be seen in Figure 6, where δ_2 is reduced from $\frac{1}{20}$ to $\frac{1}{30}$, the range of parameter values where $\Psi^y(1)$ is close to zero expands – just for slightly higher values of the EIS. For example, with an EIS just below 0.3, there is now a large range for μ (governing asset duration) that produces values for $\Psi^y(1)$ that are small.

Local analysis versus a global analysis. While we offer only a local analysis of our model in this paper, it is relevant to briefly mention how results would likely change with a global analysis. In our local analysis, real rates may be able to deviate from r^* for long periods of time without doing much to activity or inflation. However, if the deviation became very large, then many of the local properties could change. As shown in Beaudry

[33]The steady-state value of equity would equal $\frac{d}{r+rp}$, where d is the dividend payment, r is the real rate, and rp is an equity risk premium. Recall that the steady-state bond price in the model is given by $\frac{1}{r+\mu}$, where $1/\mu$ governs bond duration. This illustrates that a lower equity premium implies that asset prices are more sensitive to real rate changes, which parallels the role played by bond duration.

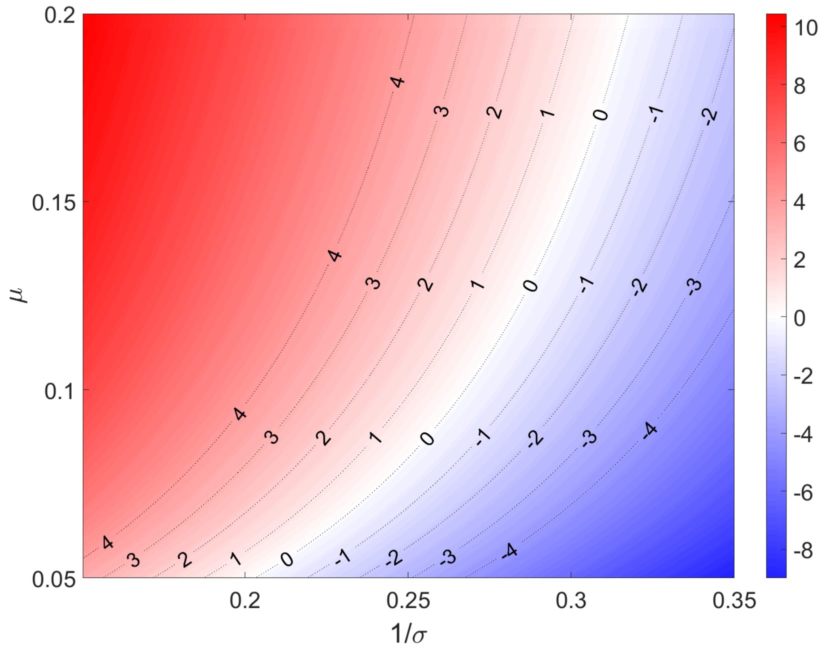

Figure 6: $\Psi^y(1)$ as a function of σ and μ in FLANK when proxying a bequest motive by setting $\delta_2 = 1/30$. Other parameters calibrated as in footnote 21.

et al. (2024) using a similar framework, the underlying asset demand function is C-shaped. This is to say that, at very high real rates, asset demand will eventually always become increasing in returns (even for $EIS << 1$). This implies that large deviations in interest rates away from r^* would not be possible without creating a large economic boom or contraction. Hence, from a global perspective, r^* should be viewed as remaining relevant, but knowing it with great precision is not necessarily very important.

7.2 Possible extensions for future work

By offering a tractable framework combining life-cycle forces and monetary policy, our work opens several avenues for future work. Our finding that conventional monetary policy may be less potent when retirement preoccupations are more prevalent (or when household assets are of shorter duration) suggests that central banks may need to move the interest rate *by more* to achieve a given effect on output and prices in an aging society (or a "post-QE world" where central banks hold significant long-term bond portfolios). This may have adverse consequences for financial stability. We do not model these interactions in the present paper, but such an extension could be warranted.

Second, while the FLANK model is already heterogenous-agent in nature (distinguishing between workers and retirees), it could be interesting to incorporate other dimensions of heterogeneity. A natural candidate would involve allowing for heterogeneity in the marginal propensity to consume (MPC) out of wealth. Empirical studies document that this object varies across the wealth distribution, with richer households having lower MPCs (Di Maggio et al., 2020; Chodorow-Reich et al., 2021). In that case, our model's logic suggests that greater inequality (a smaller fraction of households owning a bigger share of the asset supply) can weaken the monetary transmission mechanism – as the "asset valuation effect" is normally an important force working in the conventional direction. But when consumption demand of asset holders is not very sensitive to valuation effects, as would be the case when most assets are held by low-MPC households, this channel loses potency. To analyze such questions, the model developed by Bardoczy and Velasquez-Giraldo (2024), which combines MPC-heterogeneity with life-cycle dynamics, seems to hold great potential.

Third, to keep the analysis clean, our model intentionally abstracts from various other transmission mechanisms, such as cash flow-related channels (e.g. operating via mortgage debt) or mechanisms running through capital investment. Augmenting our model with such channels could be a natural next step, but we expect that the core lesson from our present analysis will survive: allowing life-cycle forces to influence consumption decisions opens the door to having persistent rate changes affecting aggregate demand by little.

When it comes to adding realism, countries typically do not exclusively rely on fully-funded pension arrangements – also providing retirees with some basic retirement income via a pay-as-you-go (PAYG) system, financed by taxing working individuals. The generosity of such schemes however tends to be limited,[34] leaving an important role for the saving dynamics central to our paper – a role that would only increase in importance if one were to explicitly model bequest motives (in contrast to savings, a PAYG pension cannot be bequeathed to one's offspring). What our model also makes clear, is that the importance of retirement preoccupations to the monetary transmission mechanism is greater in countries where PAYG pensions are less important. As demographic forces (increasing old age dependence ratios) are currently putting PAYG systems under pressure (OECD, 2021), our paper suggests that the importance of retirement preoccupations to monetary policy makers may increase further over time.

Our theoretical model can also serve as a guide to empirical researchers in formulating

[34]For example: 2023 US Social Security payments were about $1,782 per month (see https://www.cbpp.org/sites/default/files/atoms/files/8-8-16socsec.pdf). Most young, working Americans are moreover pessimistic about their future Social Security benefits (Turner and Rajnes, 2021), increasing the importance of their own saving efforts.

the correct econometric specification when trying to estimate the MPC out of wealth. In particular, our model suggests that it is important to control for the accompanying *level* of interest rates. If wealth levels are high because of low discount rates, the MPC to consume out of this wealth is likely to be relatively low, as households would want to hold on to their stock of assets to compensate for the lower flow return. This suggests that the MPC to consume out of wealth not only varies with wealth holdings (with richer households having a lower MPC) but also with the prevailing level of long-term interest rates (with the propensity to consume out of wealth being lower when rates are lower). Recent empirical findings in Di Maggio et al. (2020) and Fagereng et al. (2021) are indeed hinting in this direction, pointing towards a higher MPC out of dividend payouts relative to capital gains stemming from lower rates of interest.

It would also be interesting to characterize optimal policy in FLANK. Since the model suggests that very persistent interest rate changes might not affect demand by much, this implies that interest rate policy may be ill-equipped to offset persistent demand shocks. The latter may be better left for fiscal policy to deal with, with monetary policy instead focusing on stabilization in response to disturbances that are deemed more transient in nature.

Finally, to us, the region of the model's parameter space where $\Psi^y(1) \approx 0$ carries considerable appeal: not only can it explain why central banks appear to have significant control over longer-term real rates, but also why central banks have been quite successful in fulfilling their mandate despite being very imperfectly informed about the location of r^*. In this light, it could be interesting to explore what can widen the range where $\Psi^y(1)$ is small. Our initial explorations suggest that a bequest motive can do so (remember the discussion around Figure 6) but there may be other avenues, including on the modelling front, that can establish the same effect. One possibility is to explore the model when using Epstein-Zin (1989) preferences, which allow the EIS and coefficient of relative risk aversion to be calibrated separately (rather than imposing that they are each other's inverse).

8 Conclusion

As noted in the Introduction, there is considerable evidence suggesting central banks' policy rate decisions have a significant effect on long-term real rates. A common interpretation of this link is that it reflects reverse causality. According to this view, central banks have significant private information about the value of r^* – with this information being transmitted to markets around the time of its policy decisions.

In this paper we instead argue that this link may actually have a causal element to it, albeit not deliberate. In particular, we developed a New Keynesian-type model with life-cycle features ("FLANK") to highlight the potential effects of very persistent policy-induced changes in interest rates. In this setup, we show that persistent rate changes involve different effects (rooted in intertemporal substitution, asset valuation, and asset demand) that act on aggregate demand in opposing directions and that together imply an ambiguous effect on economic activity. A standard calibration suggests that the net effect of very persistent policy-induced rate changes may actually be close to zero.

While we do not claim to know with certainty that the net effects are in fact approximately zero – even though it is consistent with various empirical observations and calibrations offered in this paper – we do argue that such a possibility opens the door to a fundamentally different view regarding the powers of central banks. Especially, it offers an interpretation on the observed link between policy rates and long-term real rates that does not rely on central banks having private information. According to our perspective, central banks may have much less power than commonly thought to affect economic activity over the long run, if they wanted to do so.[35] Instead, our FLANK model implies that if a central bank chooses to keep real interest rates low for a prolonged period, as many central banks did post-GFC, this may not boost the economy much; it might even cause a slight contraction. The main effect of such a low-for-long policy would be to boost asset valuations, but that might not stimulate consumption demand as households may choose to hold on to this expanded wealth given it is now expected to generate less flow income going forward, implying that the household does not feel any richer on balance.

As a result, if central banks misperceive r^*, and used their misperceived r^* to guide policy, they would have very few signals suggesting they are mistaken. In this sense, the economy is rather forgiving to a central bank community that misperceives r^*. Accordingly, central bank decisions may actually drive real rates over long periods of time, without them realizing this to be the case. In particular, it can lead to cases where a rate cut that the central bank initially intends to be purely temporary, acquires additional persistence as it subsequently induces the central bank to erroneously lower its estimate of r^* (and vice versa for a rate hike). In this type of environment, it becomes rational for markets to view central bank decisions and statements as relevant for long-term rates, even if they do not think central banks have private information about r^*.

[35] In the standard New Keynesian model, central banks are able to create long-lasting inflation or deflation via persistent monetary policy shocks – boosting the economy by pushing the policy rate away from r^* for prolonged periods of time.

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Appendix

A Equilibrium and steady state

The equilibrium of the model is described by the following equations:

$$
\begin{aligned}
y_t &= \frac{\vartheta c_t^w + (1-\vartheta) c_t^r}{1 - \frac{\theta}{2} (\pi_t - \bar{\pi})^2} \\
c_t^r &= a_t^r \left[(\Gamma_t)^{\frac{1}{\sigma}} - 1 \right]^{-1} \\
(c_t^w)^{-\sigma} &= \beta_t \left\{ (1-\delta_1) \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} \Gamma_{t+1} r_{t+1} \right] \right\} \\
\left[(\Gamma_t)^{\frac{1}{\sigma}} - 1 \right]^{\sigma} &= (1-\delta_2) \beta_t \mathbb{E}_t \left[r_{t+1} \Gamma_{t+1} \left(r_{t+1}^r \right)^{-\sigma} \right] \\
(\pi_t - \bar{\pi}) \pi_t &= \lambda \left[\chi \left(\frac{y_t}{\vartheta A} \right)^{1+\varphi} - 1 \right] + \mathbb{E}_t \left[\Lambda_{t,t+1}^w (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \\
\Lambda_{t,t+1}^w &= \beta_t \frac{(1-\delta_1) \left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} \Gamma_{t+1}}{(c_t^w)^{-\sigma}} \\
\Lambda_{t,t+1}^r &= (1-\delta_2) \beta \frac{\Gamma_{t+1} \left(r_{t+1}^r \right)^{-\sigma}}{\left(\Gamma_t^{\frac{1}{\sigma}} - 1 \right)^{\sigma}} \\
q_t b^g &= \vartheta a_t^w + (1-\vartheta) a_t^r \\
0 &= \vartheta (1-\alpha_t^w) a_t^w + (1-\vartheta)(1-\alpha_t^r) a_t^r \\
r_{t+1}^r &= r_{t+1} + \left[\frac{1 + (1-\mu) q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^r \\
r_{t+1}^w &= r_{t+1} + \left[\frac{1 + (1-\mu) q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^w \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^r \frac{1 + (1-\mu) q_{t+1}}{q_t} \right] \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^w \frac{1 + (1-\mu) q_{t+1}}{q_t} \right] \\
a_{t+1}^r &= \left[(1-\delta_2) a_t^r r_{t+1}^r + \delta_2 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right) \right] \left[1 - (\Gamma_{t+1})^{-\frac{1}{\sigma}} \right] \\
i_t &= r \bar{\pi} \left(\frac{\mathbb{E}_t [\pi_{t+1}]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t^i} \\
r_{t+1} &= \frac{i_t}{\mathbb{E}_t [\pi_{t+1}]}
\end{aligned}
$$

Assuming that the inflation target is zero ($\bar{\pi} = 1$) and $\tau^r = 0$, the steady state real interest rate r solves:

$$\frac{y}{r-\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}\frac{1+\delta_1\frac{\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}+\frac{\delta_1}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}=\frac{b^g}{r-1+\mu}$$

The left-hand side of this equation represents the steady-state demand for savings, while the right-hand side captures the steady-state value of the assets supplied to the economy. Steady states for the other variables are given by:

$$\begin{aligned}
\Gamma &= \left\{1-\left[\left(1-\delta_2\right)\beta r^{1-\sigma}\right]^{\frac{1}{\sigma}}\right\}^{-\sigma}\\
y &= A\frac{\delta_2}{\delta_1+\delta_2}\left(\frac{1}{\chi}\right)^{\frac{1}{1+\varphi}}\\
\Lambda^r &= \Lambda^w = \frac{1}{r}\\
r^r &= r^w = r\\
q &= \frac{1}{r-1+\mu}\\
a^w &= \frac{qb^g}{\vartheta}\frac{1-\left(1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}{1-\left(1-\delta_1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}\\
a^r &= \varsigma a^w\\
c^w &= \frac{1-\gamma}{\vartheta}y\\
c^r &= \frac{\gamma}{1-\vartheta}y
\end{aligned}$$

with

$$\varsigma\equiv\frac{\delta_2\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}{1-\left(1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}$$

$$\gamma\equiv\frac{\delta_1}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}\left\{1-\left(1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}\right\}+\delta_1}$$

Now assume that $a_t^r=\varsigma a_t^w$, $r=\beta^{-1}$ and τ_{t+1}^r is unexpected. The log-linearized equilibrium equations are then given by:

$$
\begin{aligned}
\hat{y}_t &= (1-\gamma)\,\hat{c}^w_t + \gamma \hat{c}^r_t \\
\hat{c}^r_t &= \hat{q}_t - \frac{1}{\sigma}\frac{1}{\beta\,(1-\delta_2)^{\frac{1}{\sigma}}}\hat{\Gamma}_t \\
\hat{c}^w_t &= (1-\delta_1)\left(\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{\Gamma}_{t+1}\right) - \frac{1}{\sigma}\varepsilon^\beta_t \\
\hat{\Gamma}_t &= \beta\,(1-\delta_2)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} - (\sigma-1)\,\mathbb{E}_t\hat{r}_{t+1} + \varepsilon^\beta_t\right] \\
\hat{\pi}_t &= \lambda\,(1+\varphi)\,\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \\
\hat{q}_t &= \beta\,(1-\mu)\,\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \\
\hat{r}_{t+1} &= \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho \\
\hat{i}_t &= \varrho + (1+\phi)\,\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon^i_t
\end{aligned}
$$

with $a^w_t = a^r_t = q_t$, $r^r_{t+1} = r^w_{t+1} = r_{t+1}$, and $\varrho \equiv \log r$.

B Proofs of Propositions

B.1 Proof of Proposition 1

When $\phi = 0$, the equilibrium dynamics are captured by:

$$
\begin{bmatrix} \hat{c}^w_t \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & -\frac{\delta_1}{\sigma} & 0 & \beta\delta_1\,(1-\mu) \\ 0 & \beta\,(1-\delta_2)^{\frac{1}{\sigma}} & 0 & 0 \\ \kappa\,(1-\gamma)\,(1-\delta_1) & -\kappa\frac{(1-\gamma)\delta_1+\gamma}{\sigma} & \beta & \beta\kappa\,(1-\mu)\,[(1-\gamma)\,\delta_1+\gamma] \\ 0 & 0 & 0 & \beta\,(1-\mu) \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{y}_{t+1} \\ \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix}
$$

The four eigenvalues of this system are $\left\{\beta, \beta\,(1-\mu)\,, 1-\delta_1, \beta\,(1-\delta_2)^{\,1/\sigma}\right\}$. Since $\beta, \mu, \delta_1, \delta_2 \in (0,1)$ and $\sigma > 0$ then all four eigenvalues are less than 1 in modulus and the system has a unique stable solution.

B.2 Proof of Proposition 2

We start by deriving the "yield curve representation" of $\hat{y}_t$ and $\hat{\pi}_t$, equations (22) and (23). Assume $\phi = 0$, such that $\hat{r}_{t+1} = \varepsilon^i_t$. Solving q and Γ forward yields

$$\hat{q}_t = -\mathbb{E}_t\hat{r}_{t+1} - \sum_{j=1}^{\infty} \beta^j (1-\mu)^j \mathbb{E}_t\hat{r}_{t+1+j}$$

$$\hat{\Gamma}_t = -(\sigma - 1)\sum_{j=0}^{\infty} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1} \mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1} \varepsilon^{\beta}_{t+j}$$

and thus

$$\mathbb{E}_t\hat{\Gamma}_{t+1} = -(\sigma - 1)\sum_{j=1}^{\infty} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{j} \mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=1}^{\infty} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}}\right]^{j} \mathbb{E}_t\varepsilon^{\beta}_{t+j}$$

Plug these into the workers' Euler equation to obtain

$$\hat{c}^w_t = (1-\delta_1)\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1 \sum_{j=1}^{\infty} \beta^j \left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \mathbb{E}_t r_{t+1+j}$$
$$-\frac{1}{\sigma}\left[\delta_1 \sum_{j=1}^{\infty} \beta^j (1-\delta_2)^{\frac{j}{\sigma}} \mathbb{E}_t\varepsilon^{\beta}_{t+j} + \varepsilon^{\beta}_t\right]$$

Let's iterate forward and collect coefficients to obtain

$$\hat{c}^w_t = -\frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} + \left\{-\frac{1}{\sigma}(1-\delta_1) + \delta_1\beta\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{1}{\sigma}} - (1-\mu)\right]\right\}\mathbb{E}_t\hat{r}_{t+2}$$
$$+\left\{\left\{-\frac{1}{\sigma}(1-\delta_1) + \delta_1\beta\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{1}{\sigma}} - (1-\mu)\right]\right\}(1-\delta_1) + \delta_1\beta^2\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{2}{\sigma}} - (1-\mu)^2\right]\right\}\mathbb{E}_t\hat{r}_{t+3} + ...$$
$$-\frac{1}{\sigma}\varepsilon^{\beta}_t - \frac{1}{\sigma}\left[1-\delta_1 + \delta_1\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]\mathbb{E}_t\varepsilon^{\beta}_{t+1} - \frac{1}{\sigma}\left\{(1-\delta_1)\left[1-\delta_1+\delta_1\beta(1-\delta_2)^{\frac{1}{\sigma}}\right] + \delta_1\beta^2(1-\delta_2)^{\frac{2}{\sigma}}\right\}\mathbb{E}_t\varepsilon^{\beta}_{t+2} + ...$$

Therefore, we can write

$$\hat{c}^w_t = \sum_{j=0}^{\infty}\tilde{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\tilde{\omega}_j\mathbb{E}_t\varepsilon^{\beta}_{t+j}$$

where $\tilde{\psi}_0 = \tilde{\omega}_0 = -\frac{1}{\sigma}$ and

$$\tilde{\psi}_j = \tilde{\psi}_{j-1}(1-\delta_1) + \delta_1\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]$$
$$= (1-\delta_1)^j\tilde{\psi}_0 + \delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{i}{\sigma}} - (1-\mu)^i\right]$$

$$
\begin{aligned}
\tilde{\omega}_j &= \tilde{\omega}_{j-1}\left(1-\delta_1\right) - \frac{\delta_1}{\sigma}\beta^j \left(1-\delta_2\right)^{\frac{j}{\sigma}} \\
&= \left(1-\delta_1\right)^j \tilde{\omega}_0 - \frac{\delta_1}{\sigma}\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left(1-\delta_2\right)^{\frac{i}{\sigma}}
\end{aligned}
$$

Plug the equations derived above for q and Γ into the retirees' consumption function to obtain

$$
\begin{aligned}
\hat{c}_t^r = &-\frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} + \beta\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{1}{\sigma}} - \left(1-\mu\right)\right]\mathbb{E}_t\hat{r}_{t+2} \\
&+ \beta^2\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{2}{\sigma}} - \left(1-\mu\right)^2\right]\mathbb{E}_t\hat{r}_{t+3} + ... \\
&- \frac{1}{\sigma}\varepsilon_t^\beta - \frac{1}{\sigma}\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\varepsilon_{t+1}^\beta - \frac{1}{\sigma}\beta^2\left(1-\delta_2\right)^{\frac{2}{\sigma}}\varepsilon_{t+2}^\beta + ...
\end{aligned}
$$

Therefore, we can write

$$
\hat{c}_t^r = \sum_{j=0}^{\infty}\bar{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\bar{\omega}_j\mathbb{E}_t\varepsilon_{t+j}^\beta
$$

where $\bar{\psi}_0 = \bar{\omega}_0 = -\frac{1}{\sigma}$ and

$$
\begin{aligned}
\bar{\psi}_j &= \beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right] \\
\bar{\omega}_j &= -\frac{1}{\sigma}\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}
\end{aligned}
$$

Finally, we can use these representations for $\hat{c}_t^w$ and $\hat{c}_t^r$ to rewrite $\hat{y}_t$ as

$$
\hat{y}_t = \sum_{j=0}^{\infty}\psi_j^y\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\omega_j^y\mathbb{E}_t\varepsilon_{t+j}^\beta
$$

where $\psi_j^y \equiv \left(1-\gamma\right)\tilde{\psi}_j + \gamma\bar{\psi}_j$ and $\omega_j^y \equiv \left(1-\gamma\right)\tilde{\omega}_j + \gamma\bar{\omega}_j$, which imply $\psi_0^y = \omega_0^y = -\frac{1}{\sigma}$ and

$$
\begin{aligned}
\psi_j^y = &-\frac{1}{\sigma}\left(1-\gamma\right)\left(1-\delta_1\right)^j + \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}} - \left(1-\mu\right)^i\right] \\
&+ \gamma\beta^j\left[\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}} - \left(1-\mu\right)^j\right] \\
&= \left(1-\delta_1\right)\psi_{j-1}^y + \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma\left(1-\delta_1\right)\frac{1-\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}}{\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}}\right]\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}} \\
&- \left[\delta_1 - \gamma\left(1-\delta_1\right)\frac{1-\beta\left(1-\mu\right)}{\beta\left(1-\mu\right)}\right]\beta^j\left(1-\mu\right)^j
\end{aligned}
$$

$$\omega_j^y = -\frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j + \frac{1}{\sigma}(1-\gamma)\delta_1 \sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i(1-\delta_2)^{\frac{i}{\sigma}} - \gamma\frac{1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}}$$
$$= (1-\delta_1)\omega_{j-1}^y - \frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}$$

Now, solve $\hat{\pi}_t$ forward to obtain

$$\begin{aligned}
\hat{\pi}_t &= \kappa \sum_{j=0}^{\infty}\beta^j \mathbb{E}_t \hat{y}_{t+j} \\
&= \kappa\left\{\sum_{j=0}^{\infty}\psi_j^y \mathbb{E}_t \hat{r}_{t+1+j} + \beta\sum_{j=0}^{\infty}\psi_j^y \mathbb{E}_t \hat{r}_{t+2+j} + \beta^2\sum_{j=0}^{\infty}\psi_j^y \mathbb{E}_t \hat{r}_{t+3+j} + ...\right\} \\
&+ \kappa\left\{\sum_{j=0}^{\infty}\omega_j^y \mathbb{E}_t \varepsilon_{t+j}^{\beta} + \beta\sum_{j=0}^{\infty}\omega_j^y \mathbb{E}_t \varepsilon_{t+1+j}^{\beta} + \beta^2\sum_{j=0}^{\infty}\omega_j^y \mathbb{E}_t \varepsilon_{t+2+j}^{\beta} + ...\right\} \\
&= \kappa\left\{\psi_0^y \mathbb{E}_t\hat{r}_{t+1} + \psi_1^y \mathbb{E}_t\hat{r}_{t+2} + \psi_2^y \mathbb{E}_t\hat{r}_{t+3} + \psi_3^y \mathbb{E}_t\hat{r}_{t+4} + ... + \beta\left[\psi_0^y \mathbb{E}_t\hat{r}_{t+2} + \psi_1^y \mathbb{E}_t\hat{r}_{t+3} + \psi_2^y \mathbb{E}_t\hat{r}_{t+4} + ...\right] + ...\right\} \\
&+ \kappa\left\{\omega_0^y \varepsilon_t^{\beta} + \omega_1^y \mathbb{E}_t\varepsilon_{t+1}^{\beta} + \omega_2^y \mathbb{E}_t\varepsilon_{t+2}^{\beta} + \omega_3^y \mathbb{E}_t\varepsilon_{t+3}^{\beta} + ... + \beta\left[\omega_0^y \mathbb{E}_t\varepsilon_{t+1}^{\beta} + \omega_1^y \mathbb{E}_t\varepsilon_{t+2}^{\beta} + \omega_2^y \mathbb{E}_t\varepsilon_{t+3}^{\beta} + ...\right] + ...\right\} \\
&= \kappa\left\{\psi_0^y \mathbb{E}_t\hat{r}_{t+1} + \left(\psi_1^y + \beta\psi_0^y\right)\mathbb{E}_t\hat{r}_{t+2} + \left(\psi_2^y + \beta\psi_1^y + \beta^2\psi_0^y\right)\mathbb{E}_t\hat{r}_{t+3} + \left(\psi_3^y + \beta\psi_2^y + \beta^2\psi_1^y + \beta^3\psi_0^y\right)\mathbb{E}_t\hat{r}_{t+4} + ...\right\} \\
&+ \kappa\left\{\omega_0^y \varepsilon_t^{\beta} + \left(\omega_1^y + \beta\omega_0^y\right)\mathbb{E}_t\varepsilon_{t+1}^{\beta} + \left(\omega_2^y + \beta\omega_1^y + \beta^2\omega_0^y\right)\mathbb{E}_t\varepsilon_{t+2}^{\beta} + \left(\omega_3^y + \beta\omega_2^y + \beta^2\omega_1^y + \beta^3\omega_0^y\right)\mathbb{E}_t\varepsilon_{t+3}^{\beta} + ...\right\} \\
&= \sum_{j=0}^{\infty}\psi_j^{\pi}\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\omega_j^{\pi}\mathbb{E}_t\varepsilon_{t+j}^{\beta}
\end{aligned}$$

Therefore, we can write

$$\hat{\pi}_t = \sum_{j=0}^{\infty}\psi_j^{\pi}\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\omega_j^{\pi}\mathbb{E}_t\varepsilon_{t+j}^{\beta}$$

where $\psi_0^{\pi} = \kappa\psi_0^y$, $\omega_0^{\pi} = \kappa\omega_0^y$, and

$$\begin{aligned}
\psi_j^{\pi} &= \beta\psi_{j-1}^{\pi} + \kappa\psi_j^y \\
\omega_j^{\pi} &= \beta\omega_{j-1}^{\pi} + \kappa\omega_j^y
\end{aligned}$$

Proof of 2.a If $\delta_1 = 0$, then $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^{\pi} = -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 0$. If $\delta_1 > 0$, then

$$\psi_1^y = -\frac{1}{\sigma} + \frac{1}{\sigma}\left[\delta_1 + \gamma(1-\delta_1)\right]\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma}}\right] + \left[\delta_1 + \gamma(1-\delta_1)\right]\left[\beta(1-\delta_2)^{\frac{1}{\sigma}} - \beta(1-\mu)\right]$$

$$\psi_2^y = -\frac{1}{\sigma} + \frac{1}{\sigma}\left\{\left[1-\delta_1+\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]\left[\delta_1+\gamma\left(1-\delta_1\right)\right]+\delta_1\right\}\left[1-\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]$$
$$+\left\{\left[\delta_1+\gamma\left(1-\delta_1\right)\right]\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}+\beta\left(1-\mu\right)\right]+\delta_1\left(1-\gamma\right)\left(1-\delta_1\right)\right\}\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}-\beta\left(1-\mu\right)\right]$$
$$\psi_3^y = \ldots$$

If $\delta_2 < \mu$, then they are all strictly grater than $-\frac{1}{\sigma}$. Since $\psi_j^y > -\frac{1}{\sigma}$ for all $j \geq 1$ and $\psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i}\psi_i^y$, then also $\psi_j^\pi > -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$ for all $j \geq 1$.

Proof of 2.b Let $\xi_j^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1-\gamma\left(1-\delta_1\right)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j\left(1-\delta_2\right)^{\frac{j}{\sigma}}-\left[\delta_1-\gamma\left(1-\delta_1\right)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j\left(1-\mu\right)^j$ and solve ψ_j^y forward to obtain, then we can write

$$\psi_j^y = \left(1-\delta_1\right)^j\psi_0^y + \sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\xi_i^\psi$$

Now, since $\lim_{j\to\infty}\xi_j^\psi = 0$ then also $\lim_{j\to\infty}\psi_j^y = 0$, provided that $\delta_1 > 0$. Since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$, then also $\lim_{j\to\infty}\psi_j^\pi = 0$.

Proof of 2.c The derivative of ψ_j^y with respect to σ is

$$\frac{\partial\psi_j^y}{\partial\sigma} = \frac{1}{\sigma^2}\left(1-\gamma\right)\left(1-\delta_1\right)^j + \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{i}{\sigma}}+\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{i}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{i}{\sigma^2}\right]$$
$$+\gamma\beta^j\left[\frac{1}{\sigma^2}\left(1-\delta_2\right)^{\frac{j}{\sigma}}+\frac{\sigma-1}{\sigma}\left(1-\delta_2\right)^{\frac{j}{\sigma}}\left[-\ln\left(1-\delta_2\right)\right]\frac{j}{\sigma^2}\right]$$

Since all of its elements are positive (recall that $\delta_2 \in [0,1]$, therefore $-\ln(1-\delta_2) > 0$), then $\frac{\partial\psi_j^y}{\partial\sigma} > 0$. The derivative of ψ_j^π with respect to σ is

$$\frac{\partial\psi_j^\pi}{\partial\sigma} = \kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial\psi_i^y}{\partial\sigma}$$

which is therefore also positive.

Then, notice that

$$\lim_{\sigma\to+\infty}\psi_j^y = \left(1-\gamma\right)\delta_1\sum_{i=1}^{j}\left(1-\delta_1\right)^{j-i}\beta^i\left[1-\left(1-\mu\right)^i\right]+\gamma\beta^j\left[1-\left(1-\mu\right)^j\right] > 0$$

which is weakly positive, as $\mu \in [0,1]$. Since ψ_j^y is continuous in σ and its limit for $\sigma\to+\infty$ is positive, then $\exists\sigma < +\infty$ such that $\psi_j^y > 0$. Similarly, since $\psi_j^\pi = \kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$ is continuous in σ and

$$\lim_{\sigma \to +\infty} \psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i} \lim_{\sigma \to +\infty} \psi_i^y > 0$$

then $\exists \sigma < +\infty$ such that $\psi_j^\pi > 0$.

Proof of 2.d The derivatives of ψ_j^y with respect to δ_2 and μ are

$$\begin{aligned}
\frac{\partial \psi_j^y}{\partial \delta_2} &= -\frac{1}{\sigma} \frac{\sigma - 1}{\sigma} \frac{(1-\gamma)\,\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i} \beta^i i \,(1-\delta_2)^{\frac{i}{\sigma}} + \gamma \beta^j j \,(1-\delta_2)^{\frac{j}{\sigma}}}{1-\delta_2} < 0, \\
\frac{\partial \psi_j^y}{\partial \mu} &= \frac{(1-\gamma)\,\delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i} \beta^i i \,(1-\mu)^i + \gamma \beta^j j \,(1-\mu)^j}{1-\mu} > 0.
\end{aligned}$$

Since $\psi_j^\pi = \kappa \sum_{i=0}^{j} \beta^{j-i} \psi_i^y$, the derivatives of ψ_j^π with respect to δ_2 and μ are

$$\begin{aligned}
\frac{\partial \psi_j^\pi}{\partial \delta_2} &= \kappa \sum_{i=0}^{j} \beta^{j-i} \frac{\partial \psi_i^y}{\partial \delta_2} < 0, \\
\frac{\partial \psi_j^\pi}{\partial \mu} &= \kappa \sum_{i=0}^{j} \beta^{j-i} \frac{\partial \psi_i^y}{\partial \mu} > 0.
\end{aligned}$$

B.3 Proof of Proposition 3

We start by deriving equation (26). Assume $\phi = 0$ and $\mathbb{E}_t \varepsilon_{t+1}^i = \rho_i \varepsilon_t^i$. Then $\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t \hat{r}_{t+1+j} = \sum_{j=0}^{\infty} \psi_j^y (\rho_i)^j \varepsilon_t^i = \Psi^y(\rho_i)\, \varepsilon_t^i$, where

$$\begin{aligned}
\Psi^y(\rho_i) &= -\frac{1}{\sigma} + \sum_{j=1}^{\infty} \left\{ (1-\delta_1)\psi_{j-1}^r \rho_i^j + \delta_1 \left\{ \frac{\sigma-1}{\sigma} \left[1 - \gamma \frac{1-\delta_1}{\delta_1} \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}} \right] (\beta\rho_i)^j (1-\delta_2)^{\frac{j}{\sigma}} - \left[1 - \gamma \frac{1-\delta_1}{\delta_1} \frac{1-\beta(1-\mu)}{\beta(1-\mu)} \right] (\beta\rho_i)^j (1-\mu)^j \right\} \right\} \\
&= -\frac{1}{\sigma} + (1-\delta_1)\rho_i \Psi(\rho_i) + \delta_1 \left\{ \frac{\sigma-1}{\sigma} \left[1 - \gamma \frac{1-\delta_1}{\delta_1} \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}} \right] \frac{\beta\rho_i (1-\delta_2)^{\frac{1}{\sigma}}}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} - \left[1 - \gamma \frac{1-\delta_1}{\delta_1} \frac{1-\beta(1-\mu)}{\beta(1-\mu)} \right] \frac{\beta\rho_i(1-\mu)}{1-\beta\rho_i(1-\mu)} \right\} \\
&= -\frac{1}{\sigma} \frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} + \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)} \right] \left[\frac{\sigma-1}{\sigma} \frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta\rho_i(1-\mu)} \right]
\end{aligned}$$

Now, if $\delta_1 = 0$, then

$$\Psi^y(\rho_i) = -\frac{1}{\sigma} \frac{1}{1-\rho_i}$$

which is strictly negative, for all $\rho_i \in [0,1]$ and diverges to $-\infty$ as $\rho_i \uparrow 1$.

B.4 Proof of Proposition 4

Notice that

$$\lim_{\rho_i \to 1} \Psi^y(\rho_i = -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1} + \frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}$$

which is finite, since $\delta_1 > 0$, $\beta(1-\delta_2)^{\frac{1}{\sigma}} < 1$ and $\beta(1-\mu) < 1$

The derivative of Ψ^y with respect to ρ_i is

$$\frac{\partial \Psi^y}{\partial \rho_i} = -\frac{1}{\sigma}(1-\gamma)\left[\frac{1-\delta_1}{1-\rho_i(1-\delta_1)}\right]^2 + \frac{\delta_1(1-\gamma)(1-\delta_1)}{[1-\rho_i(1-\delta_1)]^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta\rho_i(1-\mu)}\right]$$
$$+\left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\sigma-1}{\sigma}\frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} - \frac{\beta(1-\mu)}{[1-\beta\rho_i(1-\mu)]^2}\right]$$

At $\rho_i = 1$, this derivative becomes

$$\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = -\frac{1-\gamma}{\sigma}\left(\frac{1-\delta_1}{\delta_1}\right)^2 + (1-\gamma)\frac{(1-\delta_1)}{(\delta_1)^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}\right]$$
$$+\frac{\sigma-1}{\sigma}\frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} - \frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}$$

By setting, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = 0$ we obtain an implicit expression for σ^*:

$$\sigma^* = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^*}} - \beta(1-\mu)}\frac{(1-\gamma)\frac{1-\delta_1}{\delta_1}\left[1-\delta_1+\frac{1}{1-\beta(1-\mu)}\right] + \delta_1\frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}}{(1-\gamma)\frac{1-\delta_1}{\delta_1} + \delta_1\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}}$$

Therefore, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} < 0$ iff $\sigma < \sigma^*$ and $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} > 0$ iff $\sigma > \sigma^*$.

This proves 4.b and the second part of 4.a. To prove 4.c, and the first part of 4.a, we set $\Psi^y(1) = 0$ and solve for σ to obtain and implicit expression for σ^{**} :

$$\sigma^{**} = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}} - \beta(1-\mu)}\left[(1-\gamma)\frac{1-\delta_1}{\delta_1} + \frac{1}{1-\beta(1-\mu)}\right]$$

Therefore, $\Psi^y(1) < 0$ iff $\sigma < \sigma^{**}$ and $\Psi^y(1) > 0$ iff $\sigma > \sigma^{**}$. Finally, we need to show that

$\sigma^{**} > \sigma^{*}$:

$$\sigma^{**} - \sigma^{*} = \frac{\left[1 - \beta\left(1-\delta_2\right)^{\frac{1}{\sigma^{**}}}\right]\left[1-\beta\left(1-\mu\right)\right]}{\beta\left(1-\delta_2\right)^{\frac{1}{\sigma^{**}}} - \beta\left(1-\mu\right)} \frac{\left(1-\gamma-\delta_1\right)\left(1-\gamma\right)\left(\frac{1-\delta_1}{\delta_1}\right)^2 + \frac{\delta_1 + (1-\gamma)(1-\delta_1)\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\beta(1-\mu)\right]}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}}{\left(1-\gamma\right)\frac{1-\delta_1}{\delta_1} + \delta_1 \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}}$$

$$> 0$$

Since Ψ^y is continuous in ρ_i, then $\exists \epsilon > 0$ such that all statements proved for $\Psi^y(1)$ are also valid for $\rho_i \in (1-\epsilon, 1]$.

C Region of determinacy

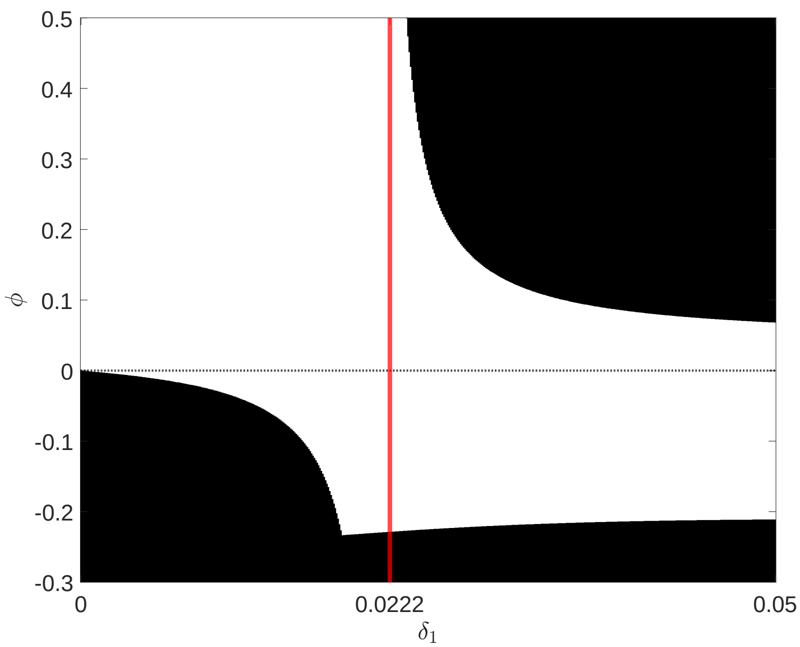

Figure C1: Visual representation of our model's region of determinacy (in white) as a function of ϕ and δ_1; red line represents our baseline choice for $\delta_1 = 1/45$. Other parameters calibrated as in footnote 21.

D Local Projections

D.1 Results for industrial production

As shown in Section 5, the effects of monetary policy shocks on unemployment appear to be more in line with conventional wisdom when occurring at the short end of the yield curve (i.e., for the so-called "target shocks"). Figure D1 below shows that this result is robust to looking at industrial production. The underlying regression specification is the exact same as the one used in Section 5, just with the natural logarithm of industrial production swapped for the unemployment rate. As with the unemployment rate, we continue to find that the target shocks tend to produce more "conventionally looking" responses (with contractionary shocks lowering industrial production) compared to path shocks. The only exception is the Eurozone, where there does not seem to be a material difference.

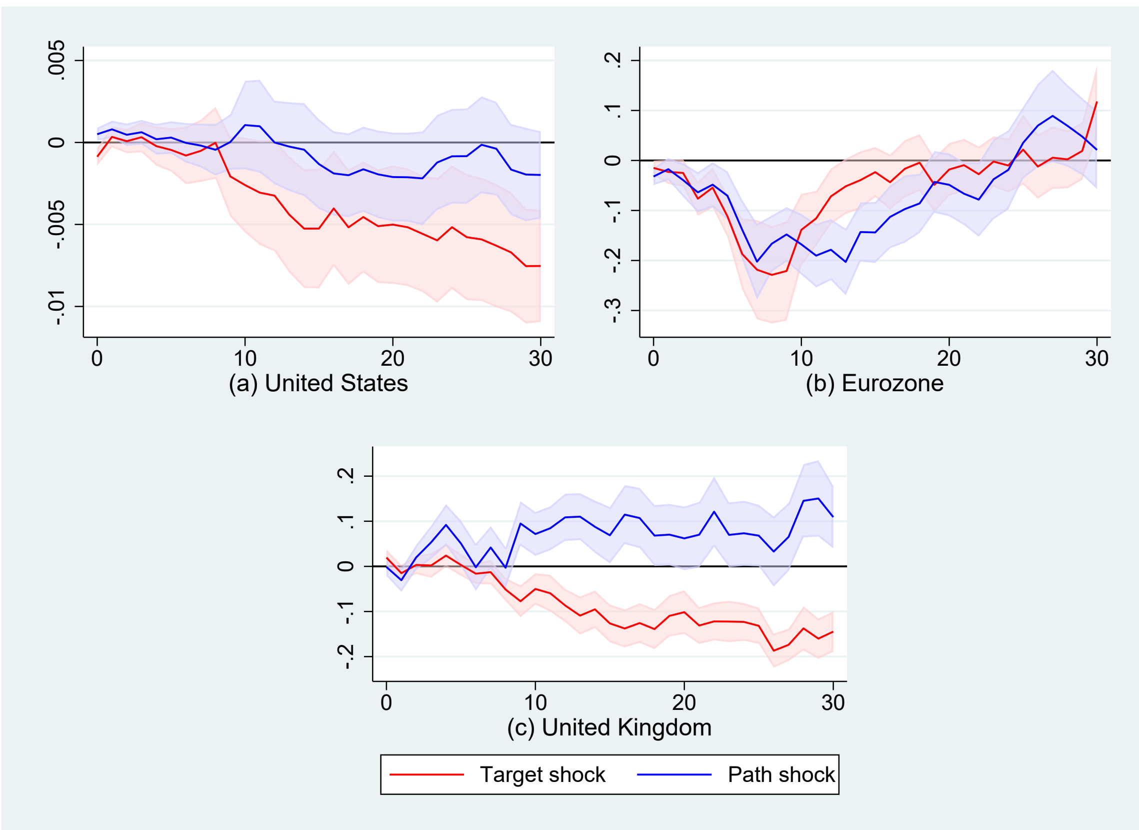

Figure D2: Response of industrial production to "target" and "path" shocks. LPs estimated at the monthly frequency. Shaded areas represent 68% confidence bands.

D.2 Data sources

The unemployment rate is taken from the BLS (for the US), Eurostat (for the Eurozone), and ONS (for the UK).

The industrial production index is taken from the Fed Board (for the US), Eurostat (for the Eurozone), and ONS (for the UK).

The CPI is taken from the BLS (for the US), Eurostat (for the Eurozone), and ONS (for the UK).

The policy rate is taken from the central bank websites. For the US, we use the Federal Funds Rate; for the Eurozone, we use the deposit rate; for the UK, we use Bank Rate.