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Monetary policy and sentiment-driven fluctuations

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Abstract

Sentiments, or beliefs about aggregate demand, can be self-fulfilling in models departing slightly from the complete information benchmark in the New Keynesian framework. Through its effect on aggregate variables, the policy stance determines the degree of complementarity in firms' production (pricing) decisions and consequently, the precision of endogenous signals that firms receive. As a result, aggregate fluctuations can be driven by both fundamental and non-fundamental shocks. The distribution of non-fundamental shocks is endogenous to policy, introducing a novel trade-off between stabilising output and inflation. Both strong inflation targeting and nominal flexibilities increase the variance of non-fundamental shocks, which are shown to be suboptimal. Moreover, the Taylor principle is no longer sufficient to rule out indeterminacy. Instead, an interest rate rule that places sufficiently low weight on inflation eliminates non-fundamental volatility and thereby the output-inflation trade-off.

Key words: New Keynesian, sunspots, animal spirits, rational expectations, optimal monetary policy, indeterminacy.

JEL classification: E31, E32, E52, E63.

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1 Introduction

Economic forecasts often feature significant uncertainty, suggesting that a range of outcomes may be possible. In part, this uncertainty reflects the interconnectedness of decisions among agents. For instance, investment may be contingent on expected demand. Meanwhile, demand depends on labor market conditions, which in turn rely on supply. These interdependencies imply that uncertainty about how others will behave, and what they believe, may be a source of friction.

However, workhorse models for policy analysis typically abstract from such uncertainty. Instead, by assuming that agents have common knowledge about the state of the economy and its evolution, they limit the potential for fluctuations as a result of dispersed, yet correlated information. Such correlation is plausible, since information is often endogenous, simultaneously reflecting and coordinating agents' actions. How agents use their signals is reflected in their actions, which consequently affects the composition and equilibrium informativeness of such signals. These information frictions leave room for sentiment, or beliefs about aggregate demand, to be a source of fluctuations, orthogonal to those induced by changes in fundamentals such as technology or preferences. Monetary policy, through its effect on the strategic interactions of firms and households, can potentially engender or inhibit such fluctuations.

This paper addresses the role of monetary policy in a model in which sentiment-driven fluctuations can arise. Specifically, I embed decision-making under uncertainty about endogenous outcomes into a New Keynesian model, building upon the framework proposed by Benhabib et al. (2015). A continuum of firms is linked through factor prices and aggregate demand externalities, as in the canonical model. While such linkages provide a motive for coordination, firms lack common knowledge about the current economic state due to dispersed information. They commit to production (pricing) before outcomes are known, basing their decision on a signal that confounds aggregate and idiosyncratic demand. Aggregate demand is both an endogenous outcome and a source of correlation. Dispersed information impedes coordination among firms, while endogenous signals correlate their actions. These features give rise to an equilibrium where sentiments, or beliefs about aggregate demand, can drive fluctuations.

The main contribution of this paper is to use this framework to highlight a new channel for monetary policy, qualifying conventional results about the nature of stabilizing policy. Restricting our focus to rational expectations equilibria, beliefs about aggregate demand are consistent with actual outcomes and vice versa. This refinement disciplines the *distribution* of sentiments with structural parameters, which include the stance of monetary policy.[1] Through its effect on aggregate variables, the stance of policy will affect the degree of strategic complementarity in production and therefore how firms optimally use their signals to make production (pricing) decisions.[2] An individual firm's production (pricing) decision depends not only on expected idiosyncratic and aggregate demand but also how they expect other firms to respond to these shocks. The range of potential outcomes will be shaped by the nature of strategic interaction among firms. Hence, monetary policy affects the distribution of sentiments, which will correspond to the actual distribution of aggregate output in a rational expectations equilibrium.

Although the deviation from the benchmark New Keynesian model is minimal, the policy implications differ due to the endogenous nature of sentiments. Policy itself becomes a source of fluctuations, as the frequency and size of shocks that affect the economy are no longer invariant to its stance. Fluctuations that arise in this model can be non-fundamental in nature, which introduces a new tradeoff for a policymaker whose goal is to stabilize output and inflation. The endogeneity of non-fundamental volatility to the policy stance implies that other predictions of the New Keynesian model no longer hold. Responding strongly to inflation has a destabilizing effect by increasing the likelihood of non-fundamental shocks, and hence output volatility. Adjusting the nominal interest rate too strongly in response to inflation also leads to indeterminacy that arises from expectations of aggregate demand.

I extend these results to the case where beliefs about aggregate demand include both a non-fundamental and fundamental component, such as a productivity shock. As in the baseline model, these beliefs affect production decisions through the firms' signal extraction problem. Information frictions will affect the transmission of fundamental shocks to aggregate output, resulting in aggregate fluctuations with both non-fundamental and fundamental components. However, if the policymaker cannot distinguish between these two sources of fluctuations, monetary policy can no longer implement the constrained efficient allocation.

This paper builds on the extensive literature incorporating information frictions in macroeconomics (Mankiw and Reis, 2002, 2007; Woodford, 2003; Adam, 2007; Lorenzoni, 2009).

[1] Realizations of aggregate demand from this distribution are referred to as non-fundamental shocks, the source of non-fundamental fluctuations. Since the baseline model abstracts from fundamental sources of fluctuations in order to highlight the properties of non-fundamental shocks, *sentiment* and *non-fundamental* will be used interchangeably. Under the assumed information frictions, there also exists an equilibrium where aggregate fluctuations are driven purely by fundamental shocks. Appendix (E) considers the case where sentiment (a belief about aggregate demand) includes both fundamental and non-fundamental shocks.

[2] These results also hold in the case of firms that set prices under incomplete information (Appendix D.2). Multiplicity of equilibria arises from endogenous signals inducing complementarities in actions, not from strategic complementarity. Non-fundamental fluctuations occur with strategic substitutability and even without coordination motives.

While earlier work explored conditions under which non-fundamental volatility can arise in stylized settings (Azariadis, 1981; Cass and Shell, 1983; Cooper and John, 1988; Benhabib and Farmer, 1994; Farmer and Guo, 1994; Wen, 1998), I contribute to a recent strand of literature that obtains non-fundamental fluctuations by introducing incomplete information in otherwise unique-equilibrium macroeconomic models (Angeletos and La'O, 2013; Benhabib et al., 2015). In such models, equilibrium conditions impose more structure on the process by which agents with dispersed information coordinate, facilitating richer policy analysis.

The unique policy implications in this paper depend on the endogenous nature of sentiments, which builds on seminal work by Benhabib et al. (2015). Sentiments, as referred to here and in Acharya et al. (2021) and Chahrour and Gaballo (2021) correspond to an endogenous variable (in this case, aggregate demand) and are captured by dispersed signals that coordinate agents' actions. As a result, the distribution of sentiments is determined by structural parameters and corresponds to the self-fulfilling distribution of aggregate output.[3]

There, and in this application, the endogenous nature of sentiments arises from the presence of endogenous signals. These signals are endogenous because they capture an outcome that results from the optimizing behavior of agents. This characteristic is relevant in many settings. In financial markets, for instance, prices emerge as noisy statistics driven by traders' decisions (Grossman and Stiglitz, 1980), while in goods markets and in the broader economy, prices and macroeconomic indicators convey information about aggregate actions (see e.g. Lucas, 1972; Amador and Weill, 2010, 2012; Vives, 2017; Gaballo, 2018; Acharya et al., 2021; Chahrour and Gaballo, 2021). Endogenous signals can also be viewed as correlated signals, capturing the role of public forecasts, news, or surveys in coordinating actions. A key theme that emerges from this literature is the relationship between how agents use information to make decisions, and the information that is generated from their actions. This feedback yields notable results regarding externalities, the informational efficiency of signals, and the tradeoff between socially optimal dispersion and non-fundamental volatility.

Despite the relevance of endogenous information in macroeconomic contexts, existing studies have not explored the role of monetary policy and its potential to shape outcomes through endogenous signals. Benhabib et al. (2015) focus on a static environment and ab-

[3] Chahrour and Gaballo (2021) demonstrate the presence of equilibria with the same stochastic properties to the sentiment equilibria discussed here, when forecasting errors occur on the household side. Households misinterpret changes in idiosyncratic conditions as indicative of aggregate ones when deriving information from an endogenous signal. This misattribution contributes to fluctuations in aggregate demand driven by correlated forecasts of consumption across islands.

stract from policy implications. Building on this framework, I introduce a different production structure with nominal rigidities. By embedding the static equilibrium from Benhabib et al. (2015) in the New Keynesian model, I formalize a channel through which monetary policy affects strategic interactions among firms and consequently, the distribution of aggregate outcomes. I study optimal monetary policy in this framework and I extend this analysis to the case where both fundamental and non-fundamental shocks drive aggregate fluctuations.

Endogenous signals and the nature of sentiments in this model lead to different policy conclusions compared to Angeletos and La'O (2019). Considering signals that are purely exogenous, they find that policy cannot improve on the decentralized outcome, as the economy responds efficiently to non-fundamental fluctuations that arise due to dispersed information. Instead, the volatility of sentiments featured in this model will be endogenous to policy, allowing the policymaker to shape outcomes through its influence on the degree of coordination in firms' actions, and thereby the precision of the signals they receive. The policymaker should, and can, eliminate non-fundamental fluctuations, as they represent an inefficiency in the use of dispersed information. This paper contributes to this literature by studying optimal monetary policy under uncertainty about endogenous outcomes.

The optimal policy exercise takes as a benchmark the notion of constrained efficiency (Angeletos and Pavan, 2007, 2009) and extends it to the case of endogenous signals and multiple equilibria. In highlighting the informational efficiency role of monetary policy, this paper shares similarities with Paciello and Wiederholt (2014). The authors consider a model with costs of acquiring information about fundamental shocks. A policy aimed at price stability incentivizes price setters to pay less attention to mark-up shocks, thereby eliminating the tradeoff between output volatility and price dispersion. I also show that monetary policy affects the information environment, but through strategic interactions among firms whose actions shape the endogenous signals they receive. In this paper, policy that pursues price stability increases strategic complementarity in firm production, which will increase the weight that firms place on the correlated component of their signal. This amplifies the non-fundamental shocks that can arise in this model, which affects the precision of the signals that firms receive. Angeletos et al. (2020) also study the ability of monetary policy to influence the precision of endogenous signals, but in a different business cycle context. Unlike the results in this paper, monetary policy that leans even more against the wind (relative to the one that implements flexible prices) attains the socially optimal allocation.

The rest of the paper is organized as follows. Section (2) presents a stylized model illustrating how information frictions generate non-fundamental aggregate fluctuations, where the volatility is determined endogenously. I highlight some key results that are important for understanding the main conclusions of this paper. Section (3) introduces the benchmark model, embedding the dynamics of the preceding section in a richer, micro-founded business cycle model with Calvo wage rigidity to analyze the effect of monetary policy on equilibrium outcomes. Optimal monetary policy is considered in Section (4). Section (5) concludes the paper.[4]

2 Stylized Model

The abstract model in this section demonstrates how sentiment-driven fluctuations can arise with two plausible features of a decentralized economy: interconnectedness and endogenous signals. First, economies consist of agents who simultaneously make decisions before knowing aggregate outcomes. Their payoffs are interdependent, as the decisions of any agent depend on the expected decisions of others. In this framework, it is reasonable to assume that agents monitor signals that are informative of others' action (Coibion et al., 2018; Hellwig and Veldkamp, 2009). This motivates the second feature, where agents make decisions based on a signal that is endogenous, as it reflects the aggregate actions of agents. For example, firms may receive advance orders or conduct market research that provides information about aggregate and idiosyncratic demand. These features lead to an equilibrium in which endogenous signals induce correlated actions, yielding aggregate fluctuations even in the absence of fundamental shocks. In this equilibrium, there exists a distribution for sentiments such that for each realization of the sentiment shock, actions confirm beliefs.

This section uses a beauty contest, a class of games featuring weak complementarity and linear best responses taken under incomplete information, to illustrate the role of strategic interactions and information dynamics in shaping economic outcomes. Given the prevalence of these features within economic interactions, there are many applications of beauty contests in macroeconomic models (Morris and Shin, 2002). These include the pricing decisions of monopolistically competitive firms (Woodford, 2003; Hellwig and Veldkamp, 2009) and investment decisions of firms (Angeletos and Pavan, 2007).

Building on Benhabib et al. (2015), this section demonstrates how policy, through its effect on the strategic interactions of agents, can potentially engender or inhibit such fluc-

[4] Appendix (D.2) shows that these results extend to a model with price rigidity. For reference, the flexible wage and flexible price case are presented in Appendices (C) and (D.1). While the baseline model abstracts from fundamental sources of fluctuations to focus on the role of information frictions in generating aggregate volatility, Appendix (E) introduces technology shocks to demonstrate that the results are robust to the presence of fundamental shocks.

tuations. The channel of monetary policy in this model relies on a key mechanism: the informativeness of endogenous signals depends on agents' behavior. Through its effect on aggregate variables, the stance of monetary policy affects how firms use their information. That is, policy will affect the set of plausible (rational expectations equilibrium) outcomes and therefore how much of their signals firms attribute to aggregate demand. In turn, aggregate actions across firms determine the precision of endogenous signals that firms receive.

Beauty contest. A continuum of agents, indexed by $j \in [0,1]$, choose action y_j to maximize expected utility. This action minimizes the expected distance from an idiosyncratic fundamental, $\varepsilon_j \sim N(0, \sigma_\varepsilon^2)$, as well as the expected distance between its action and the actions of others (y).

$$\max_{y_j} \mathbb{E}[-\alpha(y_j - \varepsilon_j)^2 - \beta(y_j - y)^2 | I_j]. \tag{1}$$

Let I_j denote the information set of agent j and let y represent the aggregate action across agents,

$$y = \int_0^1 y_j \,\mathrm{d}j. \tag{2}$$

The parameters α and β capture the importance that agents place on their action being close to the fundamental and their desire to coordinate, respectively. If $\beta < 0$ ($\beta > 0$), agents' actions are characterized by *strategic substitutability* (*strategic complementarity*). It follows that the best response of agent j is a linear combination of the fundamental and the aggregate action, given a unique information set (I_j)

$$y_j = \mathbb{E}[\alpha\varepsilon_j + \beta y | I_j]. \tag{3}$$

Endogenous signal. Suppose $I_j = s_j$, a signal that confounds the idiosyncratic fundamental (ε_j) and the aggregate action taken by agents (y),

$$s_j = \lambda\varepsilon_j + (1 - \lambda)y. \tag{4}$$

The idiosyncratic component of the signal is weighted by $\lambda \in [0,1]$, which is known to agents.

Note that s_j shapes agent j's beliefs about their idiosyncratic fundamental (ε_j). By symmetry, the signal also shapes agent j's beliefs about others' information. For the results that follow, it may be useful to consider two alternative interpretations of this signal: (i) as a noisy signal of the idiosyncratic fundamental (ε_j), whose precision is inversely related to σ_y^2 or (ii) as a correlated signal, where the common component corresponds to the aggregate action. In this setting, the signal has strategic value in the sense that it is informative of what others know.

Equilibrium. To consider an equilibrium in which y may be stochastic, conjecture $y \sim N(0, \sigma_y^2)$. In this case, the signal that agents receive is noisy and they use Bayesian weighting to disentangle its components. The optimal weight for the signal (μ) reflects the volatilities of its components, σ_ε^2 and σ_y^2,[5]

$$y_j = \underbrace{\frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}}_{\mu} \underbrace{[\lambda\varepsilon_j + (1-\lambda)y]}_{s_j}. \tag{5}$$

One implication from (5) is that agent j's best response conditional on their signal will also depend on how others will respond conditional on their signal. The latter is captured by the endogenous outcome, y, which aggregates the equilibrium strategies.

By (2), the aggregate action across agents is then

$$y = \int_0^1 y_j \, \mathrm{d}j = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y. \tag{6}$$

Since y is an endogenous variable and decisions are made before outcomes are known, it is the belief about y's distribution (σ_y^2) that shapes its realization. It is in this sense that y is indeterminate.

Finally, imposing $y = y$ pins down the rational expectations equilibrium. A rational expectations equilibrium consists of an aggregate action (2), an endogenous signal (4), and an individual best response (5) which maximizes expected utility (1) given all available information. This information includes the endogenous signal and σ_y^2, which parameterizes the distribution of aggregate outcomes, y. The rational expectations condition requires realized outcomes for y to be consistent with beliefs about its distribution.

Under this information structure and among Gaussian random variables, the rational expectations equilibrium is pinned down by a particular σ_y^2, which is shaped by parameters

[5] Consistent with rational expectations, Bayesian weighting assumes that agents know the model and the distribution from which shocks are drawn, but they are uncertain about the realization of the shock. The expectation of fluctuations leads agents to take actions that confirm such fluctuations. This logic can be extended to the case of autocorrelated non-fundamental shocks (see Acharya et al. (2021)).

that specify the relative weight agents place on their objectives (α, β),[6]

$$\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\frac{\alpha - \frac{\lambda}{1-\lambda}}{1-\beta}\right)\sigma_\varepsilon^2. \tag{7}$$

This implies that an agent's best response takes into account others' best responses. For agent j, σ_y^2 is a sufficient statistic for others' equilibrium strategies. In conjunction with their signal, σ_y^2 helps to infer the stochastic state y.

Agents face a signal extraction problem and their optimal response to the idiosyncratic fundamental differs from their optimal response to the aggregate outcome. If agents condition their response on a dispersed, endogenous signal, there can be a *non-fundamental, or sentiment equilibrium* in which y is stochastic, but its distribution is endogenously determined. This feature illustrates the endogenous nature of sentiments in this model. Although y is an endogenous variable corresponding to the aggregate action across agents, its realization is indeterminate since any $y \sim N(0, \sigma_y^2)$ satisfies the equilibrium conditions. As the conjecture and its confirmation show, y is stochastic, even in the absence of any aggregate shocks. Instead, the distribution from which y is drawn is determined by structural parameters.

Key properties of the equilibrium To understand the main findings on this paper, I highlight some key features that emerge from this framework.

1. A *fundamental equilibrium* ($\sigma_y^2 = 0$) with $y = 0$ always exists.

2. A *non-fundamental equilibrium* ($\sigma_y^2 > 0$) exists if agents want to respond differently to the two components of their signal, but it is sufficiently difficult to distinguish between them, i.e., if $\beta < 1$, the non-fundamental equilibrium requires $\alpha > \frac{\lambda}{1-\lambda}$. This can also be restated in terms of a restriction on λ: if $\beta < 1$, then $\sigma_y^2 > 0$ if $\lambda \in \left(0, \frac{\alpha}{\alpha+1}\right)$, i.e., equilibrium multiplicity exists if the signal is sufficiently correlated with y. Conversely, if $\beta > 1$, then $\sigma_y^2 > 0$ if $\lambda \in \left(\frac{\alpha}{\alpha+1}, 1\right)$.[7]

3. If $\lambda = 0$, the non-fundamental equilibrium requires $\beta = 1$. The non-fundamental equilibrium is not knife-edge, since it exists for a range of parameterizations of α, β, λ and is stable under constant gain learning and other simpler learning rules. This equilibrium

[6] Agent j chooses action y_j to minimize the loss between their action and the aggregate action (y, a random variable), given a quadratic loss utility function. To simplify our analysis, assume that the aggregate action is normally distributed so that the conditional expectation of y given the signal $s_{j,t}$ is linear.

[7] The case of $\beta > 1$ typically generates explosive dynamics in a linear system. Nevertheless, in this equilibrium, a more than proportionate response of y_j to y is moderated by the endogenous signal, if the signal is only weakly related to y.

is robust to a range of parameterizations since the endogeneity of the signal ensures that the best response of agent j has a slope of one.[8,9]

4. The degree of complementarity or substitutability in actions (parameterized by β) affects the distribution of aggregate outcomes. By the rational expectations condition in (7), how agents use their signal affects its precision as an indicator of ε_j,

$$\frac{\partial \sigma_y^2}{\partial \beta} = \frac{\lambda}{1-\lambda}\left(\frac{\alpha - \frac{\lambda}{1-\lambda}}{[1-\beta]^2}\right)\sigma_\varepsilon^2.$$

When agents hold rational expectations, a property of equilibrium strategies is that the variance of aggregate outcomes will depend on the nature of strategic interaction. When β changes, the rational expectations equilibrium condition that pins down σ_y^2 implies that agents internalize how others will respond by adjusting their beliefs about the distribution of aggregate outcomes. In a rational expectations equilibrium, strategies and beliefs (σ_y^2) are therefore consistent with model parameters, including the nature of interaction (β) among players. In other words, agents have expectations that are consistent with the model framework and the equilibrium strategies of others. Strategic complementarity amplifies non-fundamental fluctuations, while strategic substitutability diminishes it.[10]

This simple framework, which nests a form of the New Keynesian model, allows us to highlight a few results that may be helpful for understanding the positive and normative effects of monetary policy in the richer model and its robustness to fundamental shocks. These results arise from the key feature that model parameters pin down the distribution of equilibrium outcomes.

Alternate channel for policy. The features of the non-fundamental equilibrium imply that a policy rule can affect equilibrium outcomes through its influence on the nature of strategic interaction among agents. Consider a policy rule ($\omega = \phi y$) and modified version

[8] In the case of $\lambda = 0$ and payoff irrelevant noise ($s_j = y + \nu_j$), a non-fundamental equilibrium exists when $\beta \geq 1$. See appendix (B.3) for an explanation of why, when firms' actions are strategic substitutes, a sentiment-driven equilibrium exists only if the private signal contains ε_j and z_t in proportions different from the firms' first order condition; i.e. where $\lambda \neq \alpha$ and $(1-\lambda) \neq \beta$.

[9] See Benhabib et al. (2015) for a discussion of off-equilibrium dynamics and equilibrium stability under constant-gain learning. In a similar framework, Chahrour and Gaballo (2021) use adaptive learning to study the stability properties of alternate equilibria.

[10] In the absence of a coordination motive ($\beta = 0$), aggregate fluctuations would still exist. In this case, the equilibrium is pinned down by $\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\alpha - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. This underscores how the reliance of each agents' action on the aggregate action (through the signal) can induce complementarities even if the primitives of the model do not feature any coordination motive.

of the best response function of agent j,

$$y_j = \mathbb{E}[\alpha\varepsilon_j + \tilde{\beta}y - \tau\omega | I_j]. \tag{8}$$

Note that this best response is isomorphic to (3) for $\beta = \tilde{\beta} - \tau\phi$.

Information externality. In the non-fundamental equilibrium, there is a fixed point relationship between how agents react to available information and how information is generated from their actions. In terms of model concepts, the precision of the private signal (s_j) as a measure of the idiosyncratic fundamental (ε_j) depends on the actions of agents. As a result, there is an information externality in which the use of information by agents affects its aggregation. In other words, the best response of each agent, conditioning on their signal, affects the aggregate response. The aggregate response in turn affects the precision of the endogenous signals that agents receive.

Fundamental shocks. Non-fundamental fluctuations in y occur even in the absence of aggregate fundamental shocks. Instead, aggregate fluctuations are the result of agents misattributing aggregate dynamics to idiosyncratic dynamics in their signal extraction problem. In this framework, non-fundamental fluctuations occur as endogenous signals (which are informative about the actions and beliefs of others) correlate agents' actions and beliefs. The size of this correlated component varies with the weight that agents place on various objectives (α and β), which reflects how agents' actions affect the precision of their signals.

While y can be driven entirely non-fundamentally, this does not preclude y from being driven by fundamental sources of fluctuations as well. When agents condition on an endogenous signal, the sentiment equilibrium follows from verifying a conjecture that y is stochastic. These results established for this equilibrium do not depend on whether y is stochastic as a result of fundamental or non-fundamental sources.

3 Extended Model

The abstract model in the preceding section demonstrated how sentiment-driven fluctuations can emerge within a framework where endogenous signals lead to correlated actions. This simple framework nests a form of the New Keynesian model. In this section, I introduce the following deviations from the standard New Keynesian framework to study the effects of monetary policy in the non-fundamental equilibrium. Households form beliefs about consumption and set wages consistent with their beliefs, under Calvo wage rigidity (see Appendix (C) for the flexible wage case). Their beliefs about consumption will be incorporated into a signal that firms receive. Monopolistically competitive firms choose quantity produced, a response that is characterized by strategic complementarity through higher aggregate demand, as well as strategic substitutability through the effect of the real wage on marginal cost. Firms make production decisions (and therefore labor demand decisions) before demand is known. They decide production, conditional on an endogenous signal that confounds idiosyncratic demand ($\varepsilon_{j,t}$) and aggregate demand (y_t). Monetary policy follows a simple Taylor rule, targeting wage inflation and output.[11]

Firms make production decisions before demand is realized, while households make labor supply decisions and consumption plans before production takes place. Under these timing assumptions, firms' decisions are based on expected demand and households decisions are based on expected income. This leaves room for beliefs about aggregate demand to influence aggregate output in equilibrium.

To the extent that monetary policy affects firms' use of information, it will influence the precision of the endogenous signals they receive in equilibrium. Information frictions provide a new channel for monetary policy to affect aggregate outcomes, challenging some standard results of the New Keynesian model regarding stabilization policy. First, both wage flexibility and a strong response of the nominal interest rate to wage inflation introduce non-fundamental fluctuations, thereby increasing the volatility of output. Second, such fluctuations introduce a new tradeoff between stabilizing output and inflation. Third, the Taylor principle is no longer sufficient to rule out indeterminacy from expectations of aggregate demand.

3.1 Households

Following Erceg et al. (2000), there is a continuum of differentiated labor services indexed by $i \in [0, 1]$, all of which are used by each firm. Each households specializes in one type of labor, which it supplies monopolistically. Households face Calvo wage rigidity: in each period, only a constant fraction $(1 - \theta_w)$ of labor types, drawn randomly, are able to adjust their nominal wage.

Optimal Wage Setting. Consider the wage chosen by a household that is able to re-optimize. Household i, supplying labor $N_{i,t}$, chooses wage $W_{i,t}$ to maximize utility (see

[11] In a model with nominal wage rigidity and completely flexible prices, a policymaker attains the Pareto-optimal social welfare level by stabilizing wage inflation (Erceg et al. (2000)). For this reason, the baseline model features wage stickiness and a policy rule that targets wage inflation. See Appendix (D.2) for the case where firms set prices under Calvo price rigidity and the policymaker seeks to stabilize price inflation.

Appendix (B.2) for robustness to alternate preferences on labor supply),

$$\max_{W_{i,t}} \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{{C_{i,t+k|t}}^{1-\gamma}}{1-\gamma} + \Psi(1 - N_{i,t+k|t}) \right) \right]. \tag{9}$$

Let $C_{i,t+k|t}$ and $N_{i,t+k|t}$ represent the consumption and labor supply in period $t+k$ of a household that last reset its wage in period t. Household i's consumption index is given by

$$C_{i,t} = \left[\int_0^1 \epsilon_{i,j,t}^{\frac{1}{\theta}} C_{i,j,t}^{1-\frac{1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}},$$

where $C_{i,j,t}$ represents household i's consumption of good j and $\theta > 1$ the elasticity of substitution between goods. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is intended to simplify expressions.

As Calvo-type wage setting is a constraint on the frequency of wage adjustment, equation (9) can be interpreted as the expected discounted sum of utilities generated over the period during which the wage remains unchanged at the level set in the current period. Optimization of (9) is subject to a sequence of labor demand schedules and flow budget constraints that are effective while $W_{i,t}^*$ is in place. Labor expenditure minimization by firms implies the following demand for labor (see Appendix (B.1) for intermediate steps),

$$N_{i,t+k|t} = \left(\frac{W_{i,t}^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}, \tag{10}$$

where $N_{t+k} = \int_0^1 N_{j,t+k} \, \mathrm{d}j$ denotes aggregate employment in period $t+k$. Households face budget constraint

$$P_{i,t+k} C_{i,t+k|t} + \mathbb{E}_{t+k}(Q_{i,t+k,t+k+1} D_{i,t+k+1|t}) \leq D_{i,t+k|t} + W_{i,t}^* N_{i,t+k|t} + \Pi_{t+k}, \tag{11}$$

where $D_{t+k|t}$ represents the market value of the portfolio of securities held in the beginning of the period by a household that last reoptimized their wage in period t. The corresponding market value in period $t+k$ of the portfolio of securities purchased in that period, $\mathbb{E}_{t+k}(Q_{t+k,t+k+1} D_{t+k+1|t})$, yields a random payoff $D_{t+k+1|t}$. Π_t represents dividends from ownership of firms.

The first order condition associated with this problem is

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k \mathbb{E}_t\left[N_{i,t+k|t}U_c(C_{i,t+k|t},N_{i,t+k|t})\left(\frac{W_{i,t}^*}{P_{t+k}}-\frac{\varepsilon_w}{\varepsilon_w-1}MRS_{i,t+k|t}\right)\right]=0,$$

where $U(C,N)\equiv\frac{C^{1-\gamma}}{1-\gamma}+\Psi(1-N)$, $U_c\equiv\frac{\partial U}{\partial C}$, and $MRS_{i,t+k|t}\equiv-\frac{U_n(C_{i,t+k|t},N_{i,t+k|t})}{U_c(C_{i,t+k|t},N_{i,t+k|t})}$. Log-linearizing this expression, an approximate expression for the optimal wage is given by

$$w_{i,t}^*=\log\left(\frac{\varepsilon_w}{\varepsilon_w-1}\right)+(1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k\mathbb{E}_t(mrs_{i,t+k|t}+p_{t+k}).$$

Assuming full consumption risk sharing across households, facilitated by a complete set of securities markets that equalizes the marginal utility of consumption across households, all households resetting their wage in a given period will choose the same wage, w_t^*, as they face the same problem. An alternative expression for the optimal nominal wage chosen by monopolistically competitive households who can adjust in time t is given by

$$w_t^*=\beta\theta_w\mathbb{E}_t(w_{t+1}^*)+(1-\beta\theta_w)(w_t-[1-\varepsilon_w\varphi]^{-1}\hat{\mu}_t^w), \tag{12}$$

where $\hat{\mu}_t^w\equiv\mu_t^w-\mu^w$ defines the deviations of the economy's log average wage markup ($\mu_t^w\equiv w_t-p_t-mrs_t$) from its steady state level (μ^w).

Defining W_t as the aggregate nominal wage index,

$$W_t\equiv\left[\int_0^1 W_{i,t}^{1-\epsilon_w}di\right]^{\frac{1}{1-\epsilon_w}},$$

the evolution of the aggregate wage index is given by

$$W_t=\left[\theta_w W_{t-1}^{1-\varepsilon_w}+(1-\theta_w)(W_t^*)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}.$$

Log-linearized around a zero wage inflation steady state,

$$w_t=\theta_w w_{t-1}+(1-\theta_w)w_t^*. \tag{13}$$

Combining (12) and (13) yields the New Keynesian wage Philips curve, which describes the resulting dynamics for wage inflation,

$$\pi_t^w=\beta\mathbb{E}_t\pi_{t+1}^w-\lambda_w\hat{\mu}_t^w,$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$ is a measure of wage flexibility and $\hat{\mu}_t^w = \hat{w}_t^r - \gamma\hat{c}_t$.

Intertemporal consumption. Optimizing consumption intertemporally for a household that last reset its wage in $t-k$ yields the following condition,

$$Q_t = \beta\mathbb{E}_t\left[\frac{U_c(C_{t+1}, N_{t+1|t-k})}{U_c(C_t, N_{t|t-k})}\frac{P_t}{P_{t+1}}\right].$$

Letting $i_t \equiv -\ln Q_t$ (the nominal yield on a one-period bond) and the discount rate $\rho \equiv -\ln\beta$, this is log linearized as follows,

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \rho - \mathbb{E}_t\hat{\pi}_{t+1}). \tag{14}$$

At this point, production has not yet taken place, so actual output and consumption are not yet known. Households only form demand schedules for each differentiated good and labor supply schedules, all contingent on shocks to idiosyncratic demand ($\epsilon_{j,t}$) and shocks to aggregate demand (Z_t), to be drawn from their respective distributions.

3.2 Intermediate goods firms

A continuum of monopolistically competitive intermediate goods producers indexed by $j \in [0,1]$ decide production $Y_{j,t}$ before knowing idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Y_t). Instead, they infer these shocks from a signal ($S_{j,t}$) that is endogenous in the sense that it captures aggregate demand, an endogenous variable. This signal, which can be interpreted as early orders, advance sales, or market research, captures both the idiosyncratic preference for good j and the household's belief about consumption (Z_t). Let $\log\epsilon_{j,t} \sim N(0,\sigma_\epsilon^2)$ and if Z_t is stochastic, conjecture $\log Z_t \sim N(\phi_0, \sigma_z^2)$,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{15}$$

Given the household's labor supply schedule and demand schedule for good j, intermediate goods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t}$) subject to production function $Y_{j,t} = AN_{j,t}$,

$$\max_{Y_{j,t}} \mathbb{E}_t\left[P_t Y_{j,t}^{1-\frac{1}{\theta}}(\epsilon_{j,t}Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A}Y_{j,t}|S_{j,t}\right].$$

The first-order condition for the firm's optimal production decision is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A\mathbb{E}_t \left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right) \right]^{\theta}. \tag{16}$$

Log-linearizing this condition around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}_t^r | s_{j,t}]. \tag{17}$$

From (17), higher aggregate demand affects firm j's optimal production decision in two opposing ways: while it leads to an increase in demand for good j (strategic complementarity), the real wage will also be higher (strategic substitutability through marginal cost). For standard calibrations of this model, the first effect (derived from households' optimal consumption across goods) is dominated by the second (which follows from the wage setting decision of household). Although firms' actions are generally strategic substitutes, the next sections will show that the rational expectations equilibrium may not be unique if firms base production decisions on an endogenous signal that confounds aggregate and idiosyncratic demand.

3.3 Central bank

A credible central bank commits to setting the nominal interest rate to target wage inflation and output,

$$i_t = \rho + \phi_\pi^w \pi_t^w + \phi_y \hat{y}_t.$$

3.4 Timing

A key feature of this model is that decisions are made by firms and households before goods are produced and exchanged, and therefore before market clearing prices are realized. Consumption and labor supply decisions are made by households, while among firms, endogenous signals correlate beliefs and therefore production and employment decisions. This feature leaves room for a continuum of beliefs about aggregate demand to satisfy the equilibrium conditions under rational expectations. To see this more clearly, the timeline below delineates the sequence of actions by consumers, firms, and the policymaker. Let Z_t denote households' belief about aggregate demand and $\epsilon_{j,t}$ represent idiosyncratic demand for good j.

1. Households form expectations for aggregate income, based on their beliefs about aggregate demand, Z_t. They form demand schedules for each good, $(C_{j,t}(Z_t, \epsilon_{j,t}))$. A

fraction $(1-\theta_w)$ of households can optimize, setting wages $\frac{W_{i,t}}{P^e_t}(Z_t)$ consistent with their beliefs about aggregate demand and the expected aggregate price level.[12] These schedules are contingent on expected prices $P^e_t(Z_t)$ and $P^e_{j,t}(Z_t, \epsilon_{j,t})$, which are determined when goods markets open.

2. Firms believe that aggregate demand may be stochastic. However, they are unable to observe aggregate demand and idiosyncratic demand, and they must infer the two shocks from a private signal that confounds them ($S_{j,t} = \epsilon^{\lambda}_{j,t} Z^{1-\lambda}_t$).

3. The central bank commits to setting the nominal interest rate on bonds $Q_t(Z_t)$.

4. Firms decide production $Y_{j,t}(S_{j,t})$ and hence labor demand $N_{j,t}(S_{j,t})$, based on their signal. At this point, the goods markets have not yet opened and goods prices have not been realized.

5. The goods market opens and Z_t, $\epsilon_{j,t}$ are observed by all agents. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$) and state-contingent contracts are settled: the real wage $\frac{W_{i,t}}{P_t}(Z_t)$ for the $(1-\theta_w)$ households who have reset wages. $\Pi_t(Z_t)$ and $\Pi^w_t(Z_t)$ are consistent with Z_t.

6. In any rational expectations equilibrium, $Z_t = C_t = Y_t$.

3.5 Rational Expectations Equilibrium

A rational expectations equilibrium satisfies the following system of equations. Wage inflation dynamics follow from households optimizing wages subject to Calvo-type constraints on the adjustment frequency,

$$\pi^w_t = \beta \mathbb{E}_t \pi^w_{t+1} - \lambda_w(\hat{w}^r_t - \gamma \hat{c}_t). \tag{18}$$

Optimal inter-temporal consumption is given by the Euler equation,

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \rho - \mathbb{E}_t \hat{\pi}_{t+1}). \tag{19}$$

Firm production, conditional on signal $s_{j,t}$ is

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}^r_t | s_{j,t}], \tag{20}$$

[12] The degree of wage flexibility (λ_w) and the policymaker's response to wage inflation (ϕ^w_π) determine how much the real wage is expected to change (or how much the price level falls for a given nominal wage). These parameters, known the households, determine an equilibrium relationship between real wages and sentiment.

where

$$s_{j,t} = \lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t. \tag{21}$$

The central bank follows the policy rule

$$i_t = \rho + \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t. \tag{22}$$

Market clearing implies

$$\hat{y}_t = \hat{c}_t.$$

The real wage identity can be used to determine equilibrium price inflation,

$$\hat{w}_{t+1}^r = \hat{w}_t^r + \hat{\pi}_{t+1}^w - \hat{\pi}_{t+1}.$$

Lastly, beliefs about aggregate demand are correct,

$$\hat{z}_t = \hat{y}_t. \tag{23}$$

Definition 1. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t), Q_t = Q(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$*, such that for each realization of* Z_t*, (i) equations (12) and (14) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*,* $W_t = W(Z_t)$*, and* $Q_t = Q(Z_t)$ *(ii) equation (16) maximizes intermediate goods firm's expected profits for all j given the equilibrium prices* $P_t = P(Z_t), W_t = W(Z_t)$*, and the signal (15) (iii) a credible central bank commits to setting the nominal interest rate in response to wage inflation and output (22),* $Q_t = Q(Z_t)$ *(iv) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int_0^1 N_{j,t}\,\mathrm{d}j$*, and (v) expectations are rational: households' beliefs about* W_t, P_t *and* Π_t^w, Π_t *are consistent with their belief about aggregate demand* Z_t*, and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

There exist at least two rational expectations equilibria: (i) a fundamental equilibrium and (ii) a non-fundamental equilibrium.

Fundamental equilibrium. Under the signal given by (15), there is a unique fundamental equilibrium with constant output, $\hat{y}_t = 0$. Beliefs about aggregate demand do not play a role in determining the level of aggregate output. The properties of the fundamental equilibrium are well known; if we had assumed exogenous sources of fundamental variation, such as technology or markups, then these would be the drivers of fluctuations in aggregate output in this equilibrium.

Non-fundamental equilibrium. There also exists an equilibrium where aggregate output, $\hat{y}_t$, is stochastic and corresponds to self-fulfilling beliefs about aggregate demand, $\hat{z}_t$. This equilibrium is pinned down by a distribution of non-fundamental shock, σ_z^2, such that for every realization of the non-fundamental shock, firms' expected aggregate demand is equal to realized aggregate demand, households' expected aggregate income is equal to realized aggregate output, and expected prices and real wages are equal to realized prices and real wages. The rest of this section will focus on the results of this equilibrium.[13]

Consider an *iid* non-fundamental shock ($\hat{z}_t \sim N(0, \sigma_z^2)$) and conjecture policy functions for $\hat{c}_t$, $\hat{w}_t^r$, $\hat{\pi}_t$, and $\hat{\pi}_t^w$ where the state variables are $\hat{z}_t$, $\hat{w}_{t-1}^r$. The following policy functions verify the conjecture

$$\hat{c}_t = \hat{z}_t, \tag{24}$$

$$\hat{w}_t^r = \frac{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y}{1+\lambda_w\phi_\pi^w}\hat{z}_t, \tag{25}$$

$$\pi_t^w = -\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\hat{z}_t, \tag{26}$$

$$\pi_t = -\left[\frac{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y(1+\lambda_w)}{1+\lambda_w\phi_\pi^w}\right]\hat{z}_t + \hat{w}_{t-1}^r. \tag{27}$$

The policy function for the real wage (25) indicates that it increases in response to a positive sentiment shock. This occurs through a fall in price inflation (27) that exceeds the fall in wage inflation (26). Firm j's optimal production decision (20), incorporating the relationship between the real wage and sentiments (25) is given by

$$\hat{y}_{j,t} = \mathbb{E}_t\left[\hat{\varepsilon}_{j,t} + \left(1-\theta\underbrace{\left[\frac{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y}{1+\lambda_w\phi_\pi^w}\right]}_{a_w}\right)\hat{z}_t|s_{j,t}\right], \tag{28}$$

where $a_w \equiv \frac{\partial\hat{w}_t^r}{\partial\hat{z}_t}$. Note that the best response of firm j is isomorphic to (3) with $\beta = 1-\theta a_w$ and that comparative statics in β imply the comparative statics in (ϕ_π, λ_w). Through its

[13] Appendix (E) will demonstrate the robustness of these results to the case where aggregate fluctuations in the non-fundamental equilibrium have a fundamental and non-fundamental component. In that extension, the fundamental equilibrium will exhibit fluctuations driven by technology shocks.

effect on the real wage, the stance of monetary policy (ϕ_π^w relative to ϕ_y) and the degree of wage flexibility (λ_w) affects strategic interaction among firms. Policy therefore regulates strategic complementarity, as discussed in Section 2. Conditional on its signal (21), firm j's best response is

$$\hat{y}_{j,t} = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)(1-\theta a_w)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t). \tag{29}$$

Summing across firms, aggregate output is

$$\hat{y}_t = \frac{\lambda\sigma_\varepsilon^2 + (1-\lambda)(1-\theta a_w)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda)\hat{z}_t.$$

In a rational expectations equilibrium, there is a fixed-point relation between expectations of aggregate demand and actual aggregate demand (23), which pins down a distribution for aggregate output

$$\sigma_y^2 = \sigma_z^2 = \frac{1}{a_w}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{30}$$

Non-fundamental volatility, and hence output, is determined by structural parameters. In a rational expectations equilibrium, monetary policy affects the optimal response of firm production to aggregate demand, which shapes the distribution of aggregate output.

Proposition 1. Policy stance affects non-fundamental volatility
Let $\lambda \in (0, \frac{1}{2})$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π^w and λ_w, and decreasing in ϕ_y,

$$\sigma_z^2 = \frac{1+\lambda_w\phi_\pi^w}{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{31}$$

Discussion. Proposition 1 states that the stance of monetary policy affects the volatility of self-fulfilling beliefs about aggregate demand. Before discussing the effects of monetary policy on non-fundamental volatility, note that self-fulfilling fluctuations occur even in the absence of changes in the nominal interest rate. Consider the case in which interest rates are constant ($\phi_\pi = 0$, $\phi_y = 0$). Without loss of generality, consider the scenario of a positive sentiment shock, which induces firms to produce more. For demand to accommodate increased supply, the price of consumption must fall and the marginal benefit of labor must increase. This occurs through a fall in the price level that exceeds the fall in nominal wages due to sticky wages. From the perspective of the household, this increase in the real wage is consistent with an increase in aggregate demand, with the same magnitude as the positive sentiment shock.

Next, to understand the effects of monetary policy, consider a policymaker who sets the nominal interest rate according to a Taylor rule where $\phi_\pi \neq 0$, $\phi_y \neq 0$. Consider how a positive sentiment shock transmits in this model using equilibrium conditions (18) - (23). Combining the Euler equation and the Taylor rule, a fall in wage inflation prompts a fall in interest rates, which stimulates aggregate demand.[14] By the New Keynesian Philips Curve for wage inflation, for wage inflation to fall when aggregate demand increases, the real wage must increase. From the household's perspective, an increase in consumption is consistent with a fall in wage inflation if prices are expected to fall by even more. A positive sentiment shock is therefore associated with an increase the real wage.[15] This can be verified by the policy functions (25) - (27). Following a positive sentiment shock and for reasonable parameterizations ($\gamma > 0, \lambda_w > 0, \phi_y \geq 0, \phi_\pi^w \geq 0$), the real wage increases through a fall in price inflation that exceeds the fall in wage inflation ($\frac{\partial \pi_t}{\partial z_t} < \frac{\partial \pi_t^w}{\partial z_t}$),

$$\frac{\partial \pi_t}{\partial z_t} = \frac{\partial \pi_t^w}{\partial z_t} - \underbrace{\left(\gamma + \frac{\phi_y}{1 + \lambda_w \phi_\pi^w}\right)}_{>0}.$$

As a common marginal cost, how the real wage co-varies with sentiment will affect the degree of strategic complementarity in firm production (β). On the *supply side*, consider how these policy functions shape firms' beliefs about possible outcomes. By (23), a rational expectations equilibrium is pinned down by firms' beliefs about the distribution of aggregate outcomes. A positive sentiment shock affects firm j's optimal production through two opposing channels. First, and as previously discussed, the real wage increases ($\frac{\partial w_t^r}{\partial z_t} > 0$) with a positive sentiment shock, raising marginal cost. However, an increase in aggregate demand also increases demand for good j. As the first effect dominates ($\theta a_w > 1$), the optimal response of a firm to a sentiment shock will be to reduce production when sentiment increases (see (28)). In other words, firm production is characterized by strategic substitutability.

[14]The real interest rate, $r_t = i_t - \mathbb{E}_t \pi_{t+1}$, falls in one of two ways: either the nominal interest rate falls and/or expected price inflation increases (current price level falls), since $\mathbb{E}_t \pi_{t+1} \equiv \mathbb{E}_t p_{t+1} - p_t$. In response to an *iid* sentiment shock and a central bank that targets wage inflation, expected price inflation (27) is equal to the real wage. In this model, for expected price inflation to increase, either the real wage increases or the current price level falls.

[15]Conversely, consider the process through which a negative sentiment shock transmits in this model. A fall in demand follows from an increase in interest rates (19), which is prompted by an increase in wage inflation (22). An increase in wage inflation is consistent with a fall in demand if households expect prices to increase by even more than wages, which amounts to a fall in expected real wages (18). A negative sentiment shock is therefore associated with a fall in the real wage. The inverse co-movement of nominal variables and expected demand is consistent with households having a supply-side view of inflation (Candia et al., 2020; Hajdini et al., 2022).

As individual firm j internalizes the possibility that other firms will increase production in response to an increase in sentiment, substitutability in production implies that firm j will attenuate their production in response to an increase in sentiment. Aggregated across all firms, production will be muted in response to an increase in sentiment, compressing the distribution of outcomes. Actual aggregate output shapes beliefs, and vice versa: in equilibrium, beliefs about volatility in aggregate demand determine actual aggregate output. By Proposition 1, there is a rational expectations equilibrium where aggregate demand (Y_t) is stochastic, and any realization from a distribution parameterized by σ_y^2 clears markets. In summary, the stance of monetary policy affects aggregate outcomes through a new channel. Through its influence on the nature of firm coordination, it affects firm production, and hence aggregate output and beliefs thereof.

Note that this proposition requires a restriction on λ, which parameterizes the relative weight of the idiosyncratic component in a firm's signal. Surveys of firms suggest that expectations of marginal costs depend on both firm-specific and aggregate factors (Coibion et al., 2018; Okuda et al., 2021). Conventional wisdom holds that idiosyncratic conditions may be more salient to firms than aggregate conditions (Born et al., 2023). However, Flynn and Sastry (2023) document that firms pay increased attention to macroeconomic conditions when economic conditions are poor. Boneva et al. (2020) further suggest that the importance of idiosyncratic versus aggregate factors varies depending on the macroeconomic variable being forecasted. Firm-specific influences are particularly important for expectations of price and wage growth, new orders, and employment, while aggregate factors play a larger role in wage growth forecasts, although less so for price growth expectations. Finally, Afrouzi (2023) shows that firms with more competitors allocate greater attention to aggregate variables.

Role of ϕ_π^w and λ_w. Next, consider how equilibrium outcomes are affected by the response of monetary policy to wage inflation (ϕ_π^w) or the degree of wage flexibility (λ_w). In an equilibrium where these beliefs can be self-fulfilling, stabilizing wage inflation or introducing wage flexibility increases the volatility of realized output,

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} = \frac{\lambda_w \phi_y}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0,$$
$$\frac{\partial \sigma_z^2}{\partial \lambda_w} = \frac{\phi_\pi^w \phi_y}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0.$$

To illustrate this point, note that these parameters will determine the degree to which a fall in the nominal interest rate substitutes for an increase in the real wage required for a positive non-fundamental shock to be self-fulfilling. Both an increase in wage flexibility and a stronger response to wage inflation have the same effect of mitigating the degree to which the real wage rises when beliefs about aggregate demand increase. This is because a strong response to wage inflation (ϕ_π^w) caps the amount by which wage inflation needs to decrease in order to trigger a fall in the nominal interest rate required for a given sentiment shock. By (18), in order for wage inflation to fall when aggregate demand rises, the real wage must increase. However, if nominal interest rates are very sensitive to changes in wage inflation, or if wages are flexible, this mitigates the extent to which the real wage must increase to reach a given level of wage deflation.[16]

To summarize, in an equilibrium with nominal rigidities, changes in the real wage are a by-product of changes in the interest rate that are required to bring about a given sentiment shock. This implies that ϕ_π^w and λ_w will determine to degree to which the real wage increases (decreases) in response to a positive (negative) sentiment shock. In the terminology of Section 2, firms' production decisions are strategic substitutes, but both an increase in wage flexibility and a stronger response to wage inflation increase the degree of complementarity in firm production. In equilibrium, this increases the volatility of aggregate output.

Tradeoffs for monetary policy. So far, we have seen how conventionally stabilizing monetary policy introduces non-fundamental volatility to aggregate output. However, policies to stabilize output will also introduce volatility to inflation. Therefore, the information frictions we have assumed will introduce a tradeoff that that breaks divine coincidence, but without the cost-push shocks assumed in the New Keynesian model.

Proposition 2. Tradeoff between stabilizing output and inflation
In an equilibrium with non-fundamental fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. As the central bank increases its response to wage inflation (ϕ_π^w), the volatility of wage inflation declines, but this comes at the expense of higher output volatility:

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} = \frac{\lambda_w \phi_y}{[\gamma(1+\lambda_w \phi_\pi^w) + \phi_y]^2} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta} \sigma_\varepsilon^2 > 0.$$

Conversely, the more the policymaker stabilizes output, the more it introduces volatility to wage

[16] Another way to see this is to replace w_t^r in (18) with the real wage identity and rearrange terms to obtain $\pi_t^w = -\frac{\lambda_w}{1+\lambda_w}(\pi_t + c_t - w_{t-1}^r)$. The greater λ_w is, the less price inflation needs to fall to reach a given level of wage inflation. The net effect is that the real wage increases by less when wages are more flexible.

inflation,

$$\frac{\partial \sigma^2_{\pi^w}}{\partial \phi_y} = \frac{\lambda_w^2 \phi_y [\phi_y + 2\gamma(1+\lambda_w \phi_\pi^w)]}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2} \frac{1}{1+\lambda_w\phi_\pi^w} \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0.$$

To arrive at these results, note that equation (26) can be used to derive a relationship between the volatility of inflation and the volatility of output, $\sigma^2_{\pi^w} = \left(\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\right)^2 \sigma_y^2$. Consequently, we can express σ_y^2 and $\sigma^2_{\pi^w}$ solely in terms of model parameters,

$$\sigma_y^2 = \frac{1+\lambda_w\phi_\pi^w}{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2,$$

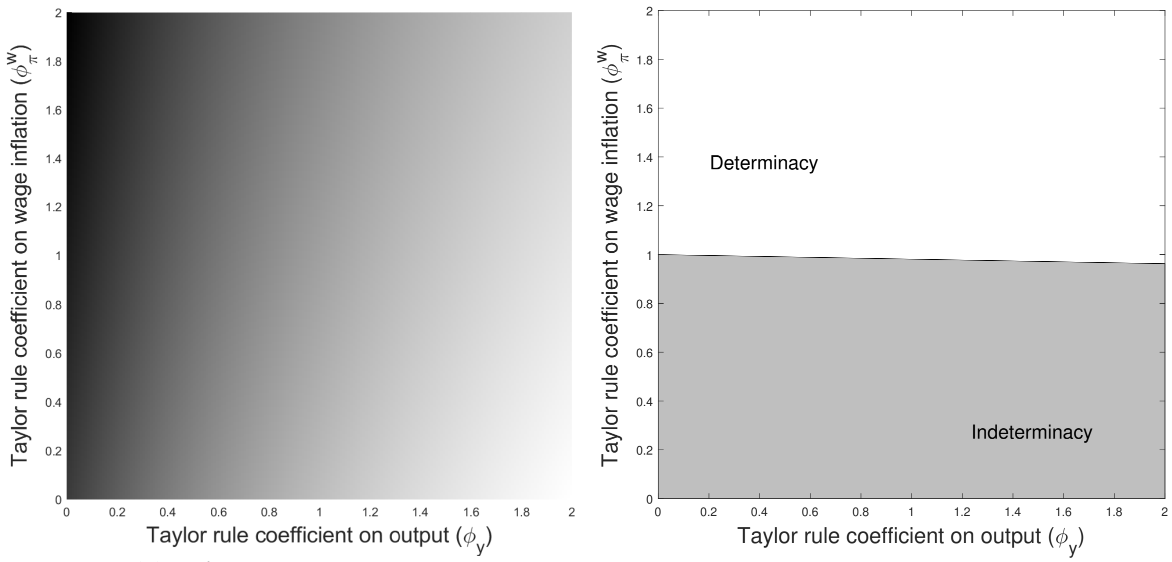
$$\sigma^2_{\pi^w} = \frac{(\lambda_w\phi_y)^2}{(1+\lambda_w\phi_\pi^w)[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2.$$

Proposition 3. The Taylor principle is not sufficient to rule out indeterminacy
There is indeterminacy even when the Taylor principle is satisfied. However, by (31), the policymaker can mitigate non-fundamental fluctuations with an interest rate rule that places low weight on wage inflation.

As we have seen, a strong response of the nominal interest rate to wage inflation introduces non-fundamental volatility to aggregate output. Figure 1a shows the region of indeterminacy in this model. In contrast to the Taylor principle, a nominal interest rate rule that responds more than one-for-one to inflation cannot rule out indeterminacy that arises from expectations of aggregate demand. Instead, such a rule would introduce a multiplicity of rational expectations equilibrium paths for real variables, including equilibria in which fluctuations are unrelated to any variation in fundamentals. This is because a rule that satisfies the Taylor principle does not account for the effect of policy on firms' coordination motives. The stance of policy not only affects how much the real interest rate changes but also how the real wage changes in relation to aggregate demand in equilibrium. In a rational expectations equilibrium, an individual firm's production decision internalizes how the nature of strategic interaction affects other firms' production, and therefore the distribution of aggregate outcomes (σ_y^2). Real indeterminacy is possible in this model because firms make production decisions before shocks are known, based on an endogenous signal of demand.

However, by placing a sufficiently low weight on wage inflation, a policymaker is able to minimize non-fundamental fluctuations. The intuition follows from the previous section, which showed that a positive sentiment shock is self-fulfilling through a fall in the nominal interest rate, which affects how the equilibrium real wage increases. For reasonable

Figure 1: Indeterminacy and Determinacy Regions



(a) Information Frictions

(b) Complete Information

Note: These figures show the indeterminacy region for a model with $\beta = 0.99$ (which implies a steady state real return on bonds of about 4 percent), $\gamma = 1$ (log utility), $\theta_w = 0.66$ (an average wage duration of 1.5 years), and a weight of $\lambda = 0.2$ for the idiosyncratic component of the signal. Under complete information, the condition for indeterminacy is given by $\phi_\pi^w > 1 - \frac{1-\beta}{(1-\nu)\kappa_p+\nu\kappa_w}\phi_y$, where $\nu = \frac{\lambda_p}{\lambda_p+\lambda_w}$. See Blasselle and Poissonnier (2016).

calibrations, the real wage increases through a decrease in price inflation that exceeds the decrease in wage inflation. However, by not responding strongly to wage inflation, the policymaker allows the real wage to co-vary more strongly with sentiment. Thus, the stance of monetary policy affects how firms use their signal, with the result that its equilibrium precision increases, mitigating sentiment-driven fluctuations.

4 Constrained Efficient Allocation

The previous section considered a minor deviation from the full information New Keynesian model: firms made production decisions before shocks were known, conditioning on a signal that confounded idiosyncratic and aggregate demand. The decentralized equilibrium featured aggregate fluctuations with a non-fundamental source. Moreover, conventionally stabilizing monetary policy increased the volatility of such fluctuations. Such policy limits the degree to which the real wage (and therefore marginal cost) rises in equilibrium, thereby affecting how firms want to respond to aggregate demand. This increases the degree of strategic complementarity in firm production. Firms internalize this in their beliefs about the distribution of aggregate outcomes, which is equivalent to the actual distribution in a rational expectations equilibrium. This section considers whether the degree of coordination in the decentralized equilibrium is socially efficient.

An appropriate efficiency benchmark is the solution to the problem of a planner who cannot centralize or transfer information, but instead directs firms' actions based on an endogenous signal that confounds aggregate and idiosyncratic demand. In other words, the social planner takes the dispersion of information as given in the decentralized equilibrium and directs firm j's production based on the firm's signal. In the aggregate, how firms use their signal will affect the volatility of aggregate output, and hence expected household welfare. By characterizing the efficient use of information and its relationship to the socially optimal degree of coordination, this exercise extends the analysis of Angeletos and Pavan (2007) to an endogenous information structure and the case of multiple equilibria.

Comparing the constrained efficient equilibrium to the decentralized equilibrium highlights the source of inefficiency: the use of information by firms affects the precision of the signal, an externality that firms and policymakers do not internalize. While this benchmark abstracts from policy instruments to identify the best allocation that satisfies feasibility constraints, the next section shows that the constrained efficient allocation will have a realistic policy counterpart.

Restricting the set of solutions for output to $Y_t \sim N(\phi_0, \sigma_z^2)$, a planner chooses the mean and variance of output to maximize expected household utility.[17]

$$\max_{\phi_0(B), \sigma_z^2(B)} \mathbb{E}_t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

subject to the following constraints,

$$Y_{j,t} = FS_{j,t}^{B}, \tag{32}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}, \tag{33}$$

$$Y_t = \left(\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right)^{\frac{\theta}{\theta-1}}, \tag{34}$$

$$Y_{j,t} = AN_{j,t}, \tag{35}$$

$$N_t = \int_0^1 N_{j,t} \, \mathrm{d}j, \tag{36}$$

$$Y_{j,t} = C_{j,t}, \tag{37}$$

$$Y_t = C_t, \tag{38}$$

[17] Restricting $Y_t \sim N(\phi_0, \sigma_z^2)$ may rule out other solutions. As the social planner's problem is concave in σ_z^2, the solution is unique.

where $F = e^{-\phi_0}$. By (32) and (33), the planner directs each firm's production decision to depend solely on its own information set. Aggregate output and labor are given by (34) and (36), while production and market clearing are given by (35), (37), and (38), respectively.

The social planner has the choice of directing each firm to weight their signal ($S_{j,t}$) by B. If $B \neq 0$, then the planner is subject to an additional constraint: aggregate output is equal to aggregate demand captured by the signal,

$$Y_t = Z_t,$$

which requires $B = \frac{1}{1-\lambda}$. Otherwise, the planner can direct firms to disregard their signal entirely ($B = 0$), in which case $\sigma_z^2 = \sigma_y^2 = 0$. Note that the planner would choose $B = 0$, indicating a coordination failure in the market solution. Nonetheless, I consider the solution of a planner constrained to choose $B > 0$ to highlight other externalities entwined in the signal extraction problem. Here, I impose $B = \frac{1}{1-\lambda}$ to underscore some properties of the inefficiency, beyond those leading the market to converge on $B > 0$. Consequently, the planning concept is adversarial, since it selects the less favorable equilibrium from the planner's perspective (Bergemann and Morris, 2019). Although σ_z^2 is an endogenous variable in the decentralized equilibrium, this is no longer the case when $B = \frac{1}{1-\lambda}$. The only restriction is that aggregate demand captured by the signal is equal to aggregate output. Otherwise, this exercise eliminates private motives for alignment among firms in order to isolate the social value of coordination.

Proposition 4. Constrained efficient allocation
In an equilibrium with endogenous signals and $B \neq 0$, the optimal mean and variance for output is given by

$$\phi_0^* = \frac{1}{2}\frac{[1+(\theta-1)\lambda B]^2}{\theta(\theta-1)}\sigma_\epsilon^2,$$
$$\sigma_z^{2*} = \max\left\{0, -\frac{2}{(1+\varphi)^2-(1-\gamma)^2}\left[\ln\left(\frac{1+\varphi}{1-\gamma}\right)+(1+\varphi)\ln\kappa_2-(1-\gamma)\ln\kappa_1\right]\right\},$$

where

$$\ln\kappa_1 = \phi_0^*,$$
$$\ln\kappa_2 = \frac{1}{2}(\lambda B)^2\sigma_\epsilon^2.$$

See Appendix (A.1). From the expression for σ_z^{2*}, we can see the optimality of fluctuations depends on household risk aversion relative to Frisch elasticity of labor supply. For

$\gamma \geq 1$, optimal volatility is negative, since risk averse households would prefer to avoid fluctuations in aggregate output. For $\gamma < 1$, the optimal volatility of aggregate output reflects household preferences over dispersion and coordination, which in turn depends on the elasticity of substitution between goods. Aggregate volatility reduces the precision in firms' signals about idiosyncratic demand, which is less consequential if goods are highly complementary.[18]

4.1 Sources of inefficiency in the decentralized equilibrium

Constant sources of inefficiency. The steady state of the decentralized equilibrium with information frictions,

$$\phi_0 = \ln\left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\right] + \frac{1}{2(\theta-1)}\sigma_\epsilon^2\left[\frac{1}{\theta}+\frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right]^2 + \frac{\Omega_s}{2},$$

features the following inefficiencies.[19] The first term $\left(\ln\left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\right]\right)$ represents the usual role that market power plays in lowering steady state aggregate output. The less substitutable goods are, the higher markups firms can charge, and it is optimal to lower production to equate marginal cost and price. This term is missing in the social planner's steady state output, since the setup abstracts from prices and downward sloping demand for firm level output. The planner's problem considers the firms' use of productive inputs conditional on information frictions, and its implications for household welfare.

The effect of information frictions on steady state output is captured by the next term, $\frac{1}{2(\theta-1)}\sigma_\epsilon^2\left[\frac{1}{\theta}+\frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right]^2$. When firms are unable to distinguish between idiosyncratic and aggregate demand, some idiosyncratic demand is misattributed to aggregate demand, and there is a degree of utility from output variety that is lost. This term also appears in the planners' steady state output, since the planner is also subject to the decentralization of information and the implementability constraint.

[18]In an equilibrium in which firms condition production on endogenous signals of demand, firms misattribute some idiosyncratic demand to aggregate demand, resulting in a loss of expected household utility from variety of consumption. κ_1 relative to κ_2 measures how much information frictions (captured by λB) decrease $\mathbb{E}(C_t)$ relative to $\mathbb{E}(N_t)$, with implications for the optimality of σ_z^2. This means that the desirability of aggregate fluctuations depends on the elasticity of substitution between goods. When goods are highly complementary, ($\theta \to 1$), and if households derive utility from variety of consumption, then reducing the responsiveness of firms to idiosyncratic demand with information frictions is desirable. Thus, a positive level of σ_z^2 is optimal. For $\theta \in (1,\infty)$, κ_1 exceeds κ_2, and approaches it when $\theta \to \infty$ (perfect substitutability). Although $\theta \in (0,\infty)$, assume $\theta > 1$, as $0 < \theta \leq 1$ is inconsistent with taste for variety and with firms' second order conditions.

[19]Under perfect information, steady state output ($\phi_0 = \ln\left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\right] + \frac{1}{2(\theta-1)}\sigma_\epsilon^2$) is a function of idiosyncratic demand volatility (σ_ϵ^2) and θ, as the CES aggregation of output with idiosyncratic preference shocks implies households derive utility from the intensive margin of consumption.

In summary, there are two sources of steady state distortion in this model. In addition to the steady state distortion that monopolistic competition introduces, there is another that arises from information frictions, particularly the inability of firms to perfectly disentangle idiosyncratic and aggregate demand. This has implications for steady state output when households derive utility from consumption variety.

Time varying sources of inefficiency. Comparing signal responses in the decentralized equilibrium and the social planner's solution allows us to isolate the inefficiency that originates in the way firms process available information. In the decentralized equilibrium with information frictions, firms place the following weight on their signal (29),

$$B = \frac{\lambda\sigma_\epsilon^2 + (1-\theta a_w)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2 + \sigma_\nu^2}.$$

The decentralized equilibrium therefore features an interaction between the use of information and the aggregation of information that is inefficient. As long as there are fluctuations in aggregate output, firms' beliefs about aggregate demand should also be stochastic ($\sigma_z^2 > 0$), since this helps predict marginal cost. In addition, due to the endogeneity of the signal, σ_z^2 affects the precision of the signal with respect to idiosyncratic demand. As a result of correlated signals, correlated actions by firms lead to aggregate fluctuations in output. In the aggregate, the actions of firms conditioning on an endogenous signal affect the precision of the signals that they receive, an externality that the social planner internalizes.

In the standard New Keynesian model, nominal rigidities are a source of allocative inefficiency. Assuming a subsidy to offset the effects of monopolistic competition on the steady state, targeting inflation strongly replicates the flexible wage allocation, allowing relative wages to adjust to shocks so that relative wage distortions do not affect the optimal allocation. However, the policy stance that achieves allocative efficiency in the complete information New Keynesian model creates an informational inefficiency when incomplete information is introduced. For reasonable parameterizations of γ, φ, and θ, the allocation in the decentralized equilibrium is inefficient: there a mapping from signals to actions that improves upon the decentralized equilibrium, which features no sentiment-driven fluctuations (see Appendix A.1).

4.2 Implementation

The previous section abstracted from policy instruments to show that a social planner choosing among allocations that respect resource feasibility and the decentralization of information can improve upon the decentralized equilibrium. The lower welfare in the latter reflects an inefficiency in the use of information, coupled with an inefficiency in the aggregation of information.

As the stance of monetary policy affects aggregate variables, it influences how firms use their signals and the degree of strategic complementarity in firm production, thereby determining the degree to which the business cycle is driven by non-fundamental forces. By the same reasoning, the nominal interest rate can also be used to minimize non-fundamental fluctuations.

In the social planner's problem, there is a continuum of equilibria, each corresponding to a particular volatility of aggregate fluctuations. These equilibria can be ranked by welfare, and a monetary policymaker can implement a particular σ_z^2 through the stance of policy (ϕ_π^w, ϕ_y). Although $\sigma_z^2 > 0$ indicates indeterminacy (i.e., any value of aggregate output drawn from this distribution is a rational expectations equilibrium), these realizations for aggregate output are all equivalent in terms of welfare, as household expected utility depends only on the volatility of outcomes.

Assuming a subsidy for incomplete information and monopolistic competition that aligns the steady state of the decentralized economy with its counterpart in the constrained efficient allocation, a policymaker can implement this allocation using the nominal interest rate. By (31), a simple interest rule that targets inflation sufficiently weakly can approximate the constrained efficient allocation. This finding qualifies the Taylor principle, whereby a strong response to inflation is stabilizing. In the presence of information frictions, a strong response to inflation can be destabilizing since it increases the volatility of output driven by non-fundamental shocks.

This is because a higher weight on inflation stabilization in the Taylor rule caps the degree to which the real wage (and therefore marginal cost) increases in equilibrium. As a result, firm production is characterized by more strategic complementarity. In a rational expectations equilibrium, firms internalize the best responses of other firms. When complementarities in firm production increase, volatility in aggregate output can increase, and firms' beliefs about aggregate outcomes account for this possibility. Instead, a monetary policy stance that allows wage inflation to increase when beliefs about aggregate demand rise (and vice versa) introduces strategic substitutability to firm production. In this case, firms' beliefs internalize the possibility of smaller fluctuations in aggregate output.

In summary, the nature of information frictions matters for policy. These findings are in contrast to Angeletos and La'O (2019), who find no inefficiency in the equilibrium use of information, and hence no room for policy intervention, as long as signals are exogenous. In that case, optimal monetary policy replicates the flexible-price allocation. However, the endogeneity of the signal here and the assumption that agents make decisions before shocks are known allows for non-fundamental sources of fluctuations, altering the positive and normative implications of monetary policy.

5 Conclusion

In this paper, I propose a new channel of transmission for monetary policy. I incorporate decision-making under uncertainty about endogenous outcomes in a New Keynesian model. The stance of monetary policy affects how firms strategically interact, with implications for the distribution of aggregate outcomes. The complete information assumption is not trivial. When production decisions are based on expected demand while labor supply and consumption plans are made before production is realized, aggregate fluctuations can have a non-fundamental component. The volatility of non-fundamental shocks will depend on the policy stance. As a result, several key policy implications of the New Keynesian model no longer hold. Both wage flexibility and targeting wage inflation increase the degree of non-fundamental volatility in aggregate output. However, since non-fundamental shocks introduce a tradeoff between stabilizing output and inflation, stabilizing output also leads to higher inflation volatility. In addition, the Taylor principle does not rule out indeterminacy that arises from expectations of aggregate demand. These results are robust to the introduction of fundamental shocks.

The unconventional effects of monetary policy in this paper depend on the endogenous nature of sentiments and the information externality they introduce. How firms decide production (pricing) based on their signals will depend on the policy stance. Contrary to the standard framework whereby monetary policy responds to mitigate the effects of shocks, policy itself can be a source of extrinsic variation. This implies that optimal monetary policy should consider informational efficiency and how it interacts with allocative efficiency. To internalize how policy affects the strategic interaction among firms and the effect this has on the precision of endogenous signals, I show that policymakers should place less weight on stabilizing inflation.

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A Appendix

A.1 Proof of Proposition 4

Combining (32) and (33), firm level output can be represented as

$$Y_{j,t} = F\epsilon_{j,t}^{\lambda B} Z_t^{(1-\lambda)B}.$$

From (34), aggregate output is

$$Y_t = FZ_t^{(1-\lambda)B} \underbrace{\left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}+\frac{\theta-1}{\theta}\lambda B} \, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}}_{\kappa_1}.$$

The log normal assumption for $\epsilon_{j,t}$ and Z_t and the moment generating function for a normal random variable imply

$$Y_t = FZ_t^{(1-\lambda)B} e^{\frac{1}{2}\frac{(1+\lambda B(\theta-1))^2}{(\theta-1)\theta}\sigma_\epsilon^2}.$$

As the signal is endogenous, implementability ($Y_t = Z_t$) requires $B = \frac{1}{1-\lambda}$, $F = e^{-\frac{1}{2}\frac{(1+\lambda B(\theta-1))^2}{(\theta-1)\theta}\sigma_\epsilon^2}$. Aggregate labor is

$$N_t = \int_0^1 N_{j,t} \, \mathrm{d}j,$$

and for these values of F and B,

$$\begin{aligned} N_t &= A^{-1} F \underbrace{\int_0^1 \epsilon_{j,t}^{\lambda B} \, \mathrm{d}j}_{\kappa_2} Z_t^{(1-\lambda)B}, \\ &= A^{-1} Z_t^{(1-\lambda)B} e^{\frac{1}{2}(\lambda B)^2 \sigma_\epsilon^2}. \end{aligned}$$

Letting $\phi_0 \equiv \log F$, the expected utility of households is given by

$$\begin{aligned} \mathbb{E}[U(C_t, N_t)] &= \frac{1}{1-\gamma}\mathbb{E}(C_t^{1-\gamma}) - \frac{1}{1+\varphi}\mathbb{E}(N_t^{1+\varphi}), \\ &= \frac{1}{1-\gamma} e^{(1-\gamma)\phi_0 + \frac{(1-\gamma)^2}{2}\sigma_z^2} - \frac{1}{1+\varphi} e^{(1+\varphi)(-a+\ln(\frac{\kappa_2}{\kappa_1})+\phi_0)+\frac{(1+\varphi)^2}{2}\sigma_z^2}. \end{aligned}$$

If $\gamma \geq 1$, expected utility is strictly decreasing in σ_z^2 as risk averse households avoid aggregate volatility,

$$\frac{\partial \mathbb{E}(U)}{\partial \sigma_z^2} = \frac{1-\gamma}{2} e^{(1-\gamma)\phi_0 + \frac{(1-\gamma)^2}{2}\sigma_z^2} - \frac{1+\varphi}{2} e^{(1+\varphi)(\log(\frac{\kappa_2}{\kappa_1})+\phi_0)+\frac{(1+\varphi)^2}{2}\sigma_z^2} < 0.$$

Now consider the case of $\gamma < 1$. Although σ_z^2 is an endogenous variable in the decentralized equilibrium, this is no longer the case in the social planner's problem. The only restriction is that aggregate demand captured by the signal is equal to aggregate output. Optimizing household welfare with respect to σ_z^2,

$$\sigma_z^{2*} = \max\left\{0, \frac{2}{(1+\varphi)^2 - (1-\gamma)^2}\left[\log\left(\frac{1-\gamma}{1+\varphi}\right) - (\gamma+\varphi)\phi_0 - (1+\varphi)\log\left(\frac{\kappa_2}{\kappa_1}\right)\right]\right\}.$$

The extent to which risk seeking households would prefer aggregate fluctuations is increasing if steady state output is large relative to steady state labor (i.e., κ_1 is sufficiently large relative to κ_2). In turn, this depends on the degree of substitutability among goods. Aggregate volatility reduces the endogenous signal's precision about idiosyncratic demand, which is inconsequential if goods are highly substitutable.

If $\gamma > 0, \varphi > 0$, then $(1+\varphi) > (1-\gamma)$ and $(1+\varphi)^2 > (1-\gamma)^2$.

$$\sigma_z^{2*} = \underbrace{\frac{2}{(1+\varphi)^2 - (1-\gamma)^2}}_{>0}\left[\underbrace{\ln\left(\frac{1-\gamma}{1+\varphi}\right)}_{<0} \underbrace{-(1+\varphi)\left(-a + \ln\left[\frac{\kappa_2}{\kappa_1}\right]\right)}_{>0} \underbrace{-(\varphi+\gamma)\phi_0}_{<0}\right],$$

where

$$\ln\left(\frac{\kappa_2}{\kappa_1}\right) = \frac{1}{2}\sigma_\epsilon^2\left(\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 \frac{\theta}{\theta-1} - (\lambda B)^2\right).$$

For reasonable calibrations ($\gamma > 0, \varphi > 0$), the optimality of non-fundamental fluctuations depends on θ, the elasticity of substitutability between goods. In the case of perfect substitutability, $\lim_{\theta\to\infty} \ln\kappa_1 = \ln\kappa_2$ and $\sigma_z^{2*} < 0$. In the case of perfect complementarity, $\lim_{\theta\to 0} \ln\kappa_1 > \ln\kappa_2$ and $\sigma_z^{2*} > 0$.

Note, for $\theta \in (0, \infty)$, $\kappa_1 > \kappa_2$ and so $\ln\left(\frac{\kappa_2}{\kappa_1}\right) < 0$ as

$$\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 \frac{\theta}{\theta-1} > (\lambda B)^2,$$
$$\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B\right]^2 > (\lambda B)^2 \frac{\theta-1}{\theta},$$
$$\left[\frac{1}{\theta-1} + \lambda B\right]^2 \left(\frac{\theta-1}{\theta}\right)^2 > (\lambda B)^2 \frac{\theta-1}{\theta}.$$

Also, $\lambda B < 1$ if $B = \frac{1}{1-\lambda}$ and $\lambda \in (0, \frac{1}{2})$.

Steady State (ϕ_0^{SP}). CES aggregation for Y_t and the firm's response in the social planner's problem are given by

$$Y_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$
$$Y_{j,t} = S_{j,t}^B.$$

Combining these expressions,

$$Y_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} S_{j,t}^{B\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}.$$
$$= Z_t^{B(1-\lambda)} \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}+\lambda B\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}.$$

Taking logs,

$$\phi_0 + z_t = z_t + \frac{\theta}{\theta-1}\frac{1}{2}\left(\frac{1}{\theta} + \lambda B\frac{\theta-1}{\theta}\right)^2 \sigma_\epsilon^2, \tag{A.1}$$
$$\phi_0^{SP}\left(B = \frac{1}{1-\lambda}\right) = \frac{\theta}{\theta-1}\frac{1}{2}\left(\frac{1}{\theta} + \lambda B\frac{\theta-1}{\theta}\right)^2 \sigma_\epsilon^2. \tag{A.2}$$

The social planner could also choose $B = 0$, in which case

$$Y_{j,t} = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}},$$
$$\phi_0^{SP}(B = 0) = \frac{1}{2\theta(\theta-1)}\sigma_\epsilon^2.$$

Online Appendix

B General Appendix

B.1 Labor demand

Firm j produces output $Y_{j,t}$ according to the production function

$$Y_{j,t} = AN_{j,t},$$

where $N_{j,t}$ is an index of labor input used by firm j and is defined as

$$N_{j,t} = \left[\int_0^1 N_{i,j,t}^{1-\frac{1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

capturing the use of a continuum of differentiated labor services. $N_{i,j,t}$ is the quantity of type i labor employed by firm j in period t. The parameter ϵ_w represents the elasticity of substitution among labor varieties. From firm minimization of labor expenditure, the following labor demand schedules are obtained,

$$N_{i,j,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\epsilon_w} N_{j,t}.$$

W_t is the aggregate nominal wage index, defined as

$$W_t \equiv \left[\int_0^1 W_{i,t}^{1-\epsilon_w} di \right]^{\frac{1}{1-\epsilon_w}}.$$

Aggregating across firms, the demand for type i labor is

$$N_{i,t} = \int_0^1 N_{i,j,t}\, \mathrm{d}j = \left(\frac{W_{i,t}}{W_t} \right)^{-\epsilon_w} \int_0^1 N_{j,t}\, \mathrm{d}j = \left(\frac{W_{i,t}}{W_t} \right)^{-\epsilon_w} N_t.$$

B.2 Non-linear disutility of labor, firm sets quantity

In the quantity setting case, a non-linear disutility of labor implies that the real wage must increase by more in a sentiment-driven equilibrium (relative to the case of linear disutility of labor).[20] As a result, firm level output is characterized by more substitutability with respect to aggregate output, and sentiments are less volatile.

[20] With a linear disutility of labor, labor supply responds strongly to a change in the real wage.

Consider a more general utility function for households that is non-linear in labor supply. Households choose labor supply (N_t) to maximize utility

$$\max_{N_t} \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

subject to budget constraint

$$P_t C_t \leq W_t N_t + \Pi_t.$$

The resulting first order condition,

$$\frac{-U_n}{U_c} = \frac{W_t}{P_t}$$
$$C_t^{\gamma} N_t^{\varphi} = \frac{W_t}{P_t}$$

implies that the price level is

$$P_t = \frac{W_t}{C_t^{\gamma} N_t^{\varphi}}.$$

Substituting N_t with the production function $Y_t = AN_t$ and applying the market clearing condition, $Y_t = C_t$,

$$P_t = \frac{W_t}{C_t^{\gamma+\varphi}} A^{\varphi}. \tag{B.3}$$

The firms' first order condition is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[(\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} \frac{P_t}{W_t} | s_{j,t} \right] \right]^{\theta}.$$

Substituting P_t with (B.3),

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A^{1+\varphi} \mathbb{E}_t \left[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-\gamma+\varphi} | s_{j,t} \right] \right]^{\theta}.$$

B.3 Private signal correct up to *iid* noise

When agents actions are strategic substitutes, a private signal that perfectly conveys information needed for the agents' first order condition, but with *iid* noise, results in only the fundamental equilibrium. Consider the first order condition of a general beauty contest model, where a continuum of agents indexed by $j \in [0,1]$ take action conditional on a private signal s_j

$$y_j = \mathbb{E}[\underbrace{\alpha\varepsilon_j + \beta y}_{x_j} | s_j],$$

$$s_j = \alpha\varepsilon_j + \beta y + \nu_j.$$

Note that $s_j = x_j + \nu_j$. Agent j's optimal response depends on an idiosyncratic *iid* shock $\varepsilon_j \sim N(0, \sigma^2_{\varepsilon_j})$, as well as on the aggregate response of other agents ($y = \int_0^1 y_j dj$), where $y \sim N(0, \sigma_y^2)$. The parameters α and β capture the elasticity of actions to the idiosyncratic shock and the aggregate variable. If $\beta > 0$, agents face strategic complementarities. If $\beta < 0$, agents face strategic substitutabilities.

Agent j's optimal response is

$$y_j = \frac{\alpha^2\sigma_\varepsilon^2 + \beta^2\sigma_y^2}{\alpha^2\sigma_\varepsilon^2 + \beta^2\sigma_y^2 + \sigma_\nu^2}(\alpha\varepsilon_j + \beta\varepsilon_j y + \nu_j).$$

As $\frac{\alpha^2\sigma_\varepsilon^2+\beta^2\sigma_y^2}{\alpha^2\sigma_\varepsilon^2+\beta^2\sigma_y^2+\sigma_\nu^2} \in (0,1)$, we can only have sentiment-driven equilibrium with this private signal if $\beta > 1$.

However, if the private signal is instead $s_j = \lambda\varepsilon_j + (1-\lambda)y + \nu_j$, where $\lambda \neq \alpha$ and $(1-\lambda) \neq \beta$, then

$$y_j = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(\lambda\varepsilon_j + (1-\lambda)y + \nu_j),$$

$$y = \int_0^1 y_j dj = \frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda)y.$$

In this case, any y is an equilibrium if

$$\frac{\alpha\lambda\sigma_\varepsilon^2 + \beta(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda) = 1.$$

The volatility of y is determined by parameters of the model.

$$\sigma_y^2 = \frac{\alpha\lambda(1-\lambda) - \lambda^2}{(1-\lambda)^2(1-\beta)}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2(1-\beta)}\sigma_\nu^2.$$

The private signal that is correct up to *iid* noise allows firms to respond to the two shocks in the correct proportions. In order for sentiment-driven equilibria to exist when firms' actions are strategic substitutes, information frictions must be such that firms misattribute some of the sentiment component in their signal to idiosyncratic preference for their good.

C Flexible Wages

Consider a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0, 1]$. Households supply labor and form *demand schedules* for differentiated goods conditional on shocks that have not yet been realized. The key friction is that intermediate goods firms commit to labor demand and output, based on an imperfect signal of the aggregate demand and firm level demand, prior to goods being produced and exchanged and before marketing clearing prices are realized.

After production decisions are made, the goods market opens, demand is realized, and prices adjust to clear the market. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

Households. The representative household chooses labor N_t to maximize utility

$$\max_{N_t} \log C_t + \Psi(1 - N_t),$$

subject to budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t},$$

where C_t is aggregate an consumption index, $\frac{W_t}{P_t}$ is the real wage, $\frac{\Pi_t}{P_t}$ is real profit income from all firms, Ψ is disutility of labor. Their first order condition is

$$C_t = \frac{1}{\Psi} \frac{W_t}{P_t}, \tag{C.4}$$

where

$$C_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{C.5}$$

C_t represents an aggregate consumption index, $\theta > 1$ is the elasticity of substitution between goods, $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify expressions. The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t}C_{j,t}\,\mathrm{d}j$, where $P_{j,t}$ is the price of intermediate good j.

Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} C_t \epsilon_{j,t}. \tag{C.6}$$

The resulting aggregate price level is obtained by substituting (C.6) into (C.5),

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t}\,\mathrm{d}j\right)^{\frac{1}{1-\theta}}.$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and aggregate income. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (C.6)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})}\right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{C.7}$$

and decide labor supply, using (C.4) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t,

$$P_t(Z_t) = \frac{W_t}{\Psi\left[\frac{1}{P_t(Z_t)}N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)}\right]}. \tag{C.8}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

Intermediate goods firms. The intermediate goods firms decide production level $Y_{j,t}$ without perfect knowledge of idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Y_t). Instead, they infer these quantities from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda},$$

where $\log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$ and $\log Z_t \sim N(\phi_0, \sigma_z^2)$.

Given the nominal wage, intermediate goods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t}$) subject to production function ($Y_{j,t} = AN_{j,t}$) and demand for its good (C.6). Substituting out labor demand of firm j, ($N_{j,t} = \frac{Y_{j,t}}{A}$) and the price of its good ($P_{j,t}$) using (C.6), firm j's problem is

$$\max_{Y_{j,t}} \mathbb{E}_t \left[P_t Y_{j,t}^{1-\frac{1}{\theta}} (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{j,t} | S_{j,t} \right],$$

The first order condition of intermediate goods firm j is given by,

$$\left(1 - \frac{1}{\theta}\right) Y_{j,t}^{-\frac{1}{\theta}} \mathbb{E}_t \left[P_t (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} | S_{j,t} \right] = \frac{W_t}{A}.$$

Rearranging terms,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[(\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right] \right]^{\theta}, \tag{C.9}$$

Substitute P_t with the household's first order condition, $P_t = \frac{1}{\Psi} \frac{W_t}{Y_t}$, where $Y_t = C_t$ due to the absence of savings in this model. As nominal variables are indeterminate in the flexible wage extension, the nominal wage can be normalized to 1,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}_t [\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta} - 1} | S_{j,t}] \right]^{\theta}.$$

Higher aggregate demand affects firm j's optimal production decision in two ways; while it implies an increase in demand for good j, it also implies that the real wage will be higher. The first effect derives from households' optimal consumption across goods, while the second follows from the labor supply decision of household. Given a nominal wage, the aggregate price level will be lower as aggregate demand increases. This will result in a fall in demand for $C_{j,t}$, which decreases firm j's optimal output level. As $\frac{1}{\theta} - 1 < 0$, the latter effect dominates, with the result that firm j's optimal output decreases with aggregate output. Although firms' actions are strategic substitutes, the rational expectations equilibrium may not be unique if firms condition production on an endogenous signal confounding aggregate and idiosyncratic demand.

Timing. With Z_t as aggregate demand and $\epsilon_{j,t}$ as idiosyncratic preference for good j, the timing of this model is as follows,

1. Households form labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks.

2. Z_t, $\epsilon_{j,t}$ are drawn from their respective distributions.

3. Firms receive a private signal of aggregate demand and idiosyncratic preference for their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$).

4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to production and hence labor demand, based on an imperfect private signal. They produce $Y_{j,t}(S_{j,t})$ and demand labor $N_{j,t}(S_{j,t}) = \frac{Y_{j,t}(S_{j,t})}{A}$.

5. Goods market opens. Z_t, $\epsilon_{j,t}$ observed by everyone. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and $P_t = \frac{1}{\Psi Z_t}$.

Equilibrium. In equilibrium, aggregate output, intermediate goods supply, and the private signal are given by

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}, \tag{C.10}$$

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}] \right]^{\theta}, \tag{C.11}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{C.12}$$

The first equation indicates that in equilibrium, goods markets clear: $Y_t = C_t$, $C_{j,t} = Y_{j,t}$. In the sentiment driven equilibrium, an additional condition stipulates that beliefs about aggregate demand are correct in equilibrium,

$$Z_t = Y_t. \tag{C.13}$$

After the realization of Y_t, and after goods markets clear, the aggregate price index, market clearing prices for each good, aggregate labor, and aggregate profits are given by

$$P_t = \frac{1}{\Psi Y_t}, \tag{C.14}$$

$$P_{j,t} = (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} Y_{j,t}^{-\frac{1}{\theta}} P_t, \tag{C.15}$$

$$N_t = \int_0^1 N_{j,t} \, \mathrm{d}j = \int_0^1 \frac{Y_{j,t}}{A} \, \mathrm{d}j, \tag{C.16}$$

$$\Pi_t = P_t Y_t - N_t = \frac{1}{\Psi} - N_t. \tag{C.17}$$

In the first equation, the actual aggregate price level in equilibrium is determined by realized aggregate output. The second equation indicates that in equilibrium, the market clearing price for good j is determined by realized aggregate output, production of good j, and the realized aggregate price level. In the third equation, labor supply equals aggregate labor demand. In the fourth equation, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition 2. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P(Z_t), P_j(Z_t, \epsilon_{j,t}), W_t = 1\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (C.7) and (C.8) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = 1$ *(ii) equation (C.11) maximizes intermediate goods firm's expected profits for all j given the equilibrium prices* $P(Z_t), W_t = 1$*, and the signal (C.12) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t}\,\mathrm{d}j$*, and (iv) expectations are rational such that the household's beliefs about* P_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition) and* $Y_t = Z_t$*: actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output and (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output. As firms make their production decisions based on the correctly anticipated distribution of aggregate demand and their own idiosyncratic demand shocks, these self-fulfilling stochastic equilibria are consistent with rational expectations.

Fundamental equilibrium. Under perfect information, firms receive signals that reveal their idiosyncratic demand shocks, and we will show that there is a unique rational expectations equilibrium in which output, aggregate demand, and the aggregate price level are constant. Using the equilibrium conditions in (C.11), (C.10), (C.15), and (C.14), $Y_t, P_t, Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows: From (C.11),

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta}. \tag{C.18}$$

Using (C.10), and substituting $Y_{j,t}$ with (C.18),

$$\begin{aligned}
Y_t &= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} \, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} \left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta-1} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t} \, \mathrm{d}j\right]^{\frac{1}{\theta-1}}.
\end{aligned}$$

Let variables with * denote their counterparts in the fundamental equilibrium. As $C_t = Y_t$ in equilibrium,

$$C^* = Y^* = \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t} \, \mathrm{d}j\right]^{\frac{1}{\theta-1}}. \tag{C.19}$$

Using (C.14), the equilibrium aggregate price level is

$$P^* = \frac{1}{\Psi Y^*} = \frac{\theta}{\theta-1}\frac{1}{A}\left[\int_0^1 \epsilon_{j,t} \, \mathrm{d}j\right]^{\frac{1}{1-\theta}}.$$

In the fundamental equilibrium, as Y_t is known, $S_{j,t}$ reveals $\epsilon_{j,t}$ perfectly. Any shift in $\epsilon_{j,t}$ results in a corresponding change in $Y_{j,t}$ without affecting $P_{j,t}$. Substituting the previous expressions for Y_t, P_t, and $Y_{j,t}$ into (C.15),

$$P_{j,t} = \frac{\theta}{\theta-1}\frac{1}{A}.$$

Let $y^* \equiv \log(Y^*)$. Without loss of generality, let $\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi} = 1$. Then (C.19) can also be expressed as follows

$$y^* = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2. \tag{C.20}$$

Sentiment-driven equilibrium. When firms face incomplete information, there exists a sentiment-driven equilibrium, in addition to the fundamental equilibrium. The sentiment-driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but fluctuates following a stochastic variable, (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[21] $\hat{z}_t \sim N(0, \sigma_z^2)$,

[21] See the next section (appendix C) for a calculation of the steady state in this equilibrium.

where σ_z^2 is determined endogenously.

Equation (C.11) gives firm j's optimal output conditional on its signal. As it is derived using equations (C.4) and (C.6), it already incorporates market clearing for labor and consumption,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}. \tag{C.21}$$

Firm j's private signal is

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Log-linearizing around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + (1-\theta)\hat{y}_t | s_{j,t}].$$

Conditional on its signal, firm j's best response is

$$\begin{aligned}
\hat{y}_{j,t} &= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} s_{j,t}, \\
&= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (\lambda\hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t).
\end{aligned}$$

Aggregate supply is then

$$\begin{aligned}
\hat{y}_t &= \int_0^1 \hat{y}_{j,t}\, \mathrm{d}j, \\
&= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (1-\lambda)\hat{z}_t.
\end{aligned}$$

In this equilibrium, household's beliefs about aggregate demand are correct ($\hat{y}_t = \hat{z}_t$). This implies

$$1 = \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (1-\lambda).$$

The volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0, \frac{1}{2})$ and $\sigma_\varepsilon^2 > 0$, then there exists a sentiment driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where[22]

$$\sigma_y^2 = \sigma_z^2 = \underbrace{\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2}_{B}. \tag{C.22}$$

Let B denote the volatility of sentiments under the baseline model. The volatility of the sentiment shock must be commensurate with the degree of complementarity/substitutability in actions across firms (θ), information content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock.

Note that if $\lambda = 1$, the signal contains only the idiosyncratic preference shock, the result is that an equilibrium with constant output is the unique equilibrium. If $\lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate components. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs.

The intuition for why the sentiment-driven equilibrium is a rational expectations equilibrium is as follows: given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that aggregate output will be equal to the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. In particular, when firms' actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal output depends negatively on the level of $\hat{z}_t$ and positively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (C.22). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

Steady state of the sentiment-driven equilibrium. The firm's optimal production, incorporating households' optimal labor supply decision (C.4), and contingent on signal $s_{j,t}$

[22] Alternatively, $\sigma_y^2 = \sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\varepsilon^2$, where the elasticities of firm j's production with respect to $\epsilon_{j,t}$ and y_t are $\beta_0 = 1$ and $1-\beta_1 = \theta$, as in section (2).

is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}_t [\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $z_t \equiv (\log Z_t) - \phi_0 \sim N(0, \sigma_z^2)$, firm j's signal is

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Without loss of generality, normalize $\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi}$ to 1. Firm production is then

$$Y_{j,t} = \left(\mathbb{E}_t [\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | s_{j,t}]\right)^{\theta}.$$

Define $y_t \equiv (\log Y_t) - \phi_0$. Unless specified otherwise, let lower-case letters represent the variable in logs. In this equilibrium, as aggregate demand is sentiment driven, we can replace y_t in the firm's response with z_t,

$$y_{j,t} = (1-\theta)\phi_0 + \theta \log \mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right].$$

To compute the conditional expectation, note that $\mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right]$ is the moment generating function of normal random variable $\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}$. Then

$$\begin{aligned}
&\mathbb{E}_t \left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right] \\
&= \exp\left[\mathbb{E}_t \left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right) + \frac{1}{2}\mathrm{Var}\left(\frac{1}{\theta}, \varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right)\right],
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}_t \left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right) &= \frac{\mathrm{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t, s_{j,t})}{\mathrm{var}(s_{j,t})} s_{j,t}, && \text{(C.23)} \\
&= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} (\lambda\varepsilon_{j,t} + (1-\lambda) z_t). && \text{(C.24)}
\end{aligned}$$

For now, let $\Omega_s \equiv \mathrm{Var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right)$. As $\frac{1}{\theta}\varepsilon_{j,t}, \frac{1-\theta}{\theta} z_t$ are Gaussian, Ω_s does not depend on $s_{j,t}$.

$$y_{j,t} = (1-\theta)\phi_0 + \theta \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t) + \frac{\theta}{2}\Omega_s, \tag{C.25}$$

$$= \varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{C.26}$$

where

$$\mu \equiv \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}, \tag{C.27}$$

$$\varphi_0 \equiv (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s. \tag{C.28}$$

Using equilibrium condition (C.10) which equates aggregate demand and aggregate supply, get an expression for y_t in terms of $y_{j,t}$

$$\left(1-\frac{1}{\theta}\right)\log Y_t = \log\left(\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right),$$

$$\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}}\right),$$

$$= \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}y_{j,t}\right]\right).$$

Replacing $y_{j,t}$ with (C.26) and using the properties of a moment generating function for normal random variable $\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]$,

$$\left(1-\frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]\right), \tag{C.29}$$

$$= \left(1-\frac{1}{\theta}\right)\varphi_0 + \left[\frac{\theta-1}{\theta}\theta\mu(1-\lambda)\right]z_t + \frac{1}{2}\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right]^2\sigma_\varepsilon^2, \tag{C.30}$$

$$\left(\frac{\theta-1}{\theta}\right)(\phi_0 + z_t) = \frac{\theta-1}{\theta}\varphi_0 + \frac{\theta-1}{\theta}\theta\mu(1-\lambda)z_t + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{C.31}$$

Matching the coefficients in (C.31) to get two constraints for the parameters to be determined (ϕ_0, σ_z^2)

$$\theta\mu = \frac{1}{1-\lambda}, \tag{C.32}$$

$$\frac{\theta-1}{\theta}\phi_0 = \frac{\theta-1}{\theta}\varphi_0 + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{C.33}$$

Next, σ_z^2 can be solved for in terms of the structural parameters using (C.32) and (C.27)

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{C.34}$$

Rearranging terms for a more intuitive expression,

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\epsilon^2.$$

Next, solve for the steady state (ϕ_0), using (C.31),

$$\phi_0 = \varphi_0 + \frac{1}{2}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1}+\frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

Substituting for φ_0 and simplifying,

$$\phi_0 = \frac{\Omega_s}{2} - \log\psi + \frac{1}{2\theta}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1}+\frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

As $\Omega_s \equiv \text{var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t | s_{j,t}\right)$,

$$\begin{aligned}
\Omega_s &= \text{var}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t) - \frac{[\text{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t, s_{j,t})]^2}{\text{var}(s_{j,t})}, \\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \mu\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right], \\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right], \\
&= \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2 + \frac{1-\theta}{\theta^2}(-\theta\sigma_z^2),
\end{aligned}$$

where the third equality uses (C.23) and (C.27). Incorporating (C.34),

$$\Omega_s = \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\left(1+(1-\theta)\left(-\frac{\lambda}{1-\lambda}\right)\right)\sigma_\epsilon^2.$$

Simplifying,

$$\Omega_s = \frac{(1-\lambda)(1-2\lambda)+(\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2.$$

Then by (C.28) and (C.33),

$$\phi_0 = \frac{(1-\lambda)(\theta-1)\lambda}{\theta(1-\lambda)} \underbrace{\frac{1}{2(\theta-1)}\sigma_\varepsilon^2}_{\phi_0^*},$$

where ϕ_0^* denotes the steady state of the fundamental equilibrium (C.20).

D Price Setting Firms

D.1 Flexible Prices

As in the baseline model, there is a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$. Households supply labor and form demand schedules for differentiated goods conditional on shocks that have not yet been realized. However, the key friction is that intermediate goods firms must set prices first and commit to meeting demand at the announced price, based on an imperfect signal of the aggregate demand and firm level demand.

After prices are set, the goods market opens, demand is realized, and production adjust to meet demand at the announced price. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

As in the case of firms who choose production, sentiment-driven equilibria requires firms' optimal response to the idiosyncratic fundamental to differ from their optimal response to the aggregate outcome. In the case of price-setting firms, the optimal price for intermediate goods firm j does not depend on the idiosyncratic preference shock for good j, unless we assume that its marginal cost is positively correlated with its demand. One approach is to assume that idiosyncratic demand $\varepsilon_{j,t}$ affects production technology. For example, if higher idiosyncratic demand leads to lower marketing costs, then the two components of the signal, $\varepsilon_{j,t}$ and Z_t will affect marginal cost differently. In this case, marginal cost is falling in idiosyncratic demand and increasing in aggregate demand (through higher wages). With this modification, if firms condition prices on a dispersed, endogenous signal, sentiment-driven fluctuations are possible when agents misattribute idiosyncratic demand to aggregate demand.

Households. The representative household's problem is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + \Pi_t.$$

where C_t is an aggregate consumption index and $C_{j,t}$ denotes the quantity of good j consumed by the household in period t.[23] The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. Ψ is disutility of labor, while $\theta > 1$ is the elasticity of substitution between goods. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify calculations. Π_t is profit income from all firms, while W_t is the wage.

The household allocates consumption among j goods to maximize C_t for any given level of expenditures. Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} C_t \epsilon_{j,t}. \tag{D.35}$$

The resulting aggregate price level is obtained by substituting (D.35) into the aggregate consumption index,

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj \right)^{\frac{1}{1-\theta}},$$

and implies $\int P_{j,t} C_{j,t} dj = P_t C_t$.

Choosing labor (N_t) optimally, the households' labor supply condition is

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}, \tag{D.36}$$

$$\Psi C_t^{\gamma} = \frac{W_t}{P_t}, \tag{D.37}$$

where $\frac{W_t}{P_t}$ is the real wage. Taking the log of this expression,

$$w_t - p_t = \gamma c_t + \log \Psi.$$

[23]For non-linear disutility of labor, see Appendix (B.2). Specifying the utility function in this way ($\gamma \neq 1$) will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments.

Intertemporal consumption is

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right).$$

In logs,

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption to be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (D.35)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})} \right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{D.38}$$

and decide labor supply, using (D.37) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t

$$P_t(Z_t) = \frac{W_t}{\Psi \left[\frac{W_t}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)} \right]^{\gamma}}. \tag{D.39}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

Intermediate goods firms. Sentiment-driven equilibria requires a signal extraction problem with two shocks, to each of which the optimal response of the firm's price setting decision is different. The model implies that the optimal price for intermediate goods firm j under perfect information does not depend on the idiosyncratic preference shock for good j. To circumvent this, assume that a firm's marginal cost is positively correlated with its demand.

The intermediate goods firms decide price $P_{j,t}$ without perfect knowledge of idiosyncratic demand or aggregate demand. Instead, they infer $\epsilon_{j,t}$ and $Y_{j,t}$ from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Y_t^{1-\lambda}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $y_t \equiv (\log Y_t) - \phi_0 \sim N(0, \sigma_y^2)$.

Given an aggregate price index (P_t), intermediate goods producers choose $P_{j,t}$ to maximize nominal profits

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t} Y_{j,t} - W_t N_{j,t} \right],$$

subject to production function

$$Y_{j,t} = \epsilon_{j,t}^{\tau} N_{j,t}.$$

Note that idiosyncratic demand $\epsilon_{j,t}$ will also need to affect production technology for the sentiment equilibrium to exist (for example, if demand affects marketing costs). Under this assumption, the two components of the signal, $\epsilon_{j,t}$ and Z_t will affect marginal cost differently, and fluctuations are possible when agents misattribute the latter to the former.

Demand schedule for good j (imposing the market clearing condition, $C_t = Y_t$ and $C_{j,t} = Y_{j,t}$),

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} \epsilon_{j,t} Y_t.$$

Substituting $N_{j,t}$ using firm j's production function and $Y_{j,t}$ from its demand schedule, the firms' problem is

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t}^{1-\theta} P_t^{\theta} \epsilon_{j,t} Y_t - W_t P_t^{\theta} P_{j,t}^{-\theta} \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t} \right]. \tag{D.40}$$

The first order condition is given by

$$(1-\theta) P_{j,t}^{-\theta} P_t^{\theta} \mathbb{E}_t(\epsilon_{j,t} Y_t | S_{j,t}) + \theta P_t^{\theta} P_{j,t}^{-\theta-1} \mathbb{E}_t(W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}) = 0.$$

As nominal variables are indeterminate in the flexible price case, the nominal aggregate consumption price index (P_t) can be normalized to 1. Rearranging terms,

$$P_{j,t} = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

Replacing W_t with the household's labor supply decision, firm j's optimal price is

$$P_{j,t} = \left(\frac{\theta}{\theta - 1}\right) \Psi \frac{\mathbb{E}[\epsilon_{j,t}^{1-\tau} Y_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

Timing. Letting Z_t denote aggregate demand and $\epsilon_{j,t}$ represent idiosyncratic preference for good j, the timing of this model is as follows:

1. Households form a labor supply schedule, $N_t(Z_t)$, and demand schedules for each good j, $(C_{j,t}(Z_t, \epsilon_{j,t}))$, contingent on shocks to be realized.

2. Z_t, $\epsilon_{j,t}$ realized.

3. Firms receive a private signal of aggregate demand and idiosyncratic preference for their good, $S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$.

4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to a price, $P_{j,t}(s_{j,t})$, based on an imperfect private signal.

5. The goods market opens. Z_t, $\epsilon_{j,t}$ is observed by everyone. Firms meet supply at posted price $Y_{j,t}(P_{j,t})$, so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and $W_t = \Psi Z_t^{\gamma}$.[24]

Equilibrium. In equilibrium, the aggregate price index, intermediate goods price, and the private signal are given by

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \tag{D.41}$$

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \tag{D.42}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{D.43}$$

Note that the firm's price setting decision already incorporates the household's optimal labor supply decision, $\frac{W_t}{P_t} = \Psi Y_t^{\gamma}$. In the sentiment driven equilibrium, one additional condition applies: that beliefs about aggregate demand are correct in equilibrium.

$$Z_t = Y_t. \tag{D.44}$$

[24]Thus, wages are realized at the end of the period.

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{D.45}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{D.46}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{D.47}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{D.48}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{D.49}$$

The first equality, which follows from the household's demand equation, indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition 3. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (D.38) and (D.39) maximize household utility given the equilibrium prices* $P_t = 1, P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = W(Z_t)$ *(ii) equation (D.42) maximizes intermediate goods firm's expected profits for all* j *given the equilibrium prices* $P_t = 1, W_t = W(Z_t)$*, and the signal (D.43) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t} dj$*, and (iv) expectations are rational such that the household's beliefs about* W_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition), and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$*.*

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output.

Fundamental equilibrium. Under perfect information, there is a unique rational expectations equilibrium in which the price of good j, aggregate price level, and aggregate demand are constant. aggregate output is constant and known. Then, the private signal that firms receive reveals their idiosyncratic demand shocks. Using the equilibrium conditions in (D.42), (D.46), (D.45), and (D.48), Y_t, P_t, $Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows.

Under perfect information, the price of good j (D.42) is

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{W_t \epsilon_{j,t}^{1-\tau} Y_t}{\epsilon_{j,t} Y_t}.$$

Replacing W_t with (D.48),

$$P_{j,t} = \frac{\theta}{\theta - 1} \Psi P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}.$$

Without loss of generality, normalizing $\frac{\theta}{\theta-1}\Psi$ to 1,

$$P_{j,t} = P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}. \tag{D.50}$$

Substituting (D.50) into (D.41), the aggregate price index with flexible prices is indeterminate:

$$\begin{aligned} P_t &= \left[\int \epsilon_{j,t} [P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}]^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \\ &= \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{1-\theta}} P_t Y_t^{\gamma}. \end{aligned}$$

Without loss of generality, normalize P_t to 1. The normalization of $P_t = 1$ can be used to find Y_t,

$$Y_t = \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{\gamma(\theta-1)}}. \tag{D.51}$$

Taking the log of this expression (let $y_t \equiv (\log Y_t) - \phi_0$),

$$y_t + \phi_0 = \frac{1}{\gamma(\theta - 1)} \log \mathbb{E}_t \left[\epsilon_{j,t}^{1-\tau(1-\theta)} \right].$$

As $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$, by the properties of a moment generating function for a normally distributed random variable,

$$y_t + \phi_0 = \frac{1}{\theta - 1}\frac{1}{2}\mathrm{Var}_t([1 - \tau(1-\theta)]\varepsilon_{j,t}), \tag{D.52}$$

$$= \frac{1}{\gamma(\theta-1)}\frac{[1-\tau(1-\theta)]^2}{2}\sigma_\varepsilon^2. \tag{D.53}$$

Equating coefficients implies $y_t = 0$ and

$$\phi_0^* = \frac{1}{2(\theta-1)}\frac{(1+\tau[\theta-1])^2}{\gamma}\sigma_\varepsilon^2 \tag{D.54}$$

As expected, output in the fundamental equilibrium when firms choose quantity (C.20), ($\gamma = 1, \tau = 0$) is equivalent to its counterpart when firms choose prices.

Finally, an expression for $Y_{j,t}$ can be found by using the demand curve (D.45), and substituting $P_{j,t}$ with (D.50)

$$\begin{aligned} Y_{j,t} &= \left(\frac{P_t}{P_{j,t}}\right)^\theta \epsilon_{j,t} Y_t, \\ &= [Y_t^\gamma \epsilon_{j,t}^{-\tau}]^{-\theta}\epsilon_{j,t} Y_t, \\ &= \epsilon_{j,t}^{1+\tau\theta} Y_t^{1-\gamma\theta}. \end{aligned}$$

Replacing Y_t with (D.51),

$$Y_{j,t} = \epsilon_{j,t}^{1+\tau\theta}\left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj\right]^{\frac{1-\gamma\theta}{\gamma(\theta-1)}}.$$

Sentiment-driven equilibrium. When firms set prices conditional on an endogenous signal of aggregate demand, there exists a sentiment driven equilibrium in addition to the fundamental equilibrium. The sentiment driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but equal to a sentiment (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[25] To solve for this equilibrium, conjecture $\hat{z}_t \sim N(0, \sigma_z^2)$, where σ_z^2 is a constant to be determined below.

Consider the case of a positive sentiment shock in the flexible wage and flexible price model. A self-fulfilling equilibrium is possible when σ_z^2 is sufficiently low such that firms attribute just enough of z_t to $\epsilon_{j,t}$ and so that the increase in sentiment leads firms to lower

[25] See appendix (C) for a calculation of the steady state in this equilibrium.

$p_{j,t}$. When goods markets open, the quantity of firm j's product, $(y_{j,t}(p_{j,t}))$, demanded at price $p_{j,t}$ is higher than that under perfect information. Thus, there is a σ_z^2 such that aggregate supply across firms exactly fulfills the positive sentiment formed by households.

Proposition 5. *Let $\lambda \in (0,1)$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic with variance*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma} \sigma_\epsilon^2, \tag{D.55}$$

where $B = \frac{\partial p_t}{\partial z_t}$.

Proof. Equation (D.42) gives firm j's optimal price conditional on its signal. As it is derived using equations (D.48) and (D.45), it already incorporates market clearing for labor and consumption.

$$\begin{aligned} P_{j,t} &= \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \\ &= \frac{\theta}{\theta - 1} \Psi \frac{\mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Z_t | S_{j,t}]}, \end{aligned}$$

where the second equality results from substituting W_t with the household's optimal labor supply (D.48). Taking logs,

$$p_{j,t} = \log\left(\frac{\theta}{\theta - 1}\Psi\right) + \log \mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | s_{j,t}] - \log \mathbb{E}_t[\epsilon_{j,t} Z_t | s_{j,t}].$$

Conjecture a solution of the form $p_{j,t} = D + B s_{j,t}$. According to this guess, $p_t = A + B(1-\lambda) z_t$ where A incorporates $\mathbb{E}(\epsilon_{j,t})$, which affects the steady state. Substituting our guess for p_t,

$$p_{j,t} = \log\left(\frac{\theta}{\theta-1}\Psi\right) + \log \mathbb{E}_t[\exp(p_t + (1-\tau)\varepsilon_{j,t} + (\gamma+1)(z_t+\phi_0))|s_{j,t}] \tag{D.56}$$

$$- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t + \phi_0)|s_{j,t}] \tag{D.57}$$

$$= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A \tag{D.58}$$

$$+ \log \mathbb{E}[\exp(B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}] \tag{D.59}$$

$$- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t)] \tag{D.60}$$

$$= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1 - \Omega_2}{2} + (\mu_1 - \mu_2)s_{j,t} \tag{D.61}$$

$$= \varphi_0 + \bar{\mu}s_{j,t} \tag{D.62}$$

where

$$\varphi_0 \equiv \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1-\Omega_2}{2}, \tag{D.63}$$

$$\bar{\mu} \equiv \mu_1 - \mu_2, \tag{D.64}$$

$$\mu_1 \equiv \mathbb{E}_t[B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], \tag{D.65}$$

$$\Omega_1 \equiv \frac{1}{2}\text{Var}_t[B(1-\lambda)+\gamma+1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], \tag{D.66}$$

$$\mu_2 \equiv \mathbb{E}_t[\varepsilon_{j,t} + z_t|s_{j,t}], \tag{D.67}$$

$$\Omega_2 \equiv \frac{1}{2}\text{Var}[\varepsilon_{j,t} + z_t|s_{j,t}]. \tag{D.68}$$

Variables in lowercase denote the log of their counterparts, with the exception of $z_t = \log Z_t - \phi_0$. Note that the firm's price is a constant projection of $s_{j,t}$. Hence, in a sentiment-driven equilibrium, all firms set prices in the same proportion to their signal.

Taking the log of the aggregate price index (D.41) and substituting for $p_{j,t}$ with (D.62),

$$\begin{aligned}
(1-\theta)p_t &= \log \mathbb{E}_t[P_{j,t}^{1-\theta}\epsilon_{j,t}], \\
&= \log \mathbb{E}_t[\exp([1-\theta]p_{j,t} + \varepsilon_{j,t})], \\
&= (1-\theta)\varphi_0 + (1-\theta)\bar{\mu}(1-\lambda)z_t + \log \mathbb{E}_t[e^{([1-\theta]\bar{\mu}\lambda+1)\varepsilon_{j,t}}], \\
A + Bz_t &= \varphi_0 + \bar{\mu}(1-\lambda)z_t + \frac{[(1-\theta)\bar{\mu}\lambda+1]^2}{2(1-\theta)}\sigma_\epsilon^2.
\end{aligned}$$

Equating coefficients on z_t,

$$B = \bar{\mu}(1-\lambda). \tag{D.69}$$

Evaluating (D.65) and (D.67), we have

$$B = \frac{(\gamma + B)(1-\lambda)\sigma_z^2 - \tau\lambda(1-\lambda)\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda),$$

which implies[26]

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma}\sigma_\epsilon^2. \tag{D.70}$$

From equating the constant terms, we have

$$A = \varphi_0 + \frac{[(1-\theta)\bar{\mu}\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2.$$

Applying (D.69) and (D.63),

$$\phi_0 = \frac{1}{\gamma}\left(\frac{[(1-\theta)\frac{\lambda}{1-\lambda}B + 1]^2}{2(\theta-1)}\sigma_\epsilon^2 - \log\left(\frac{\theta}{\theta-1}\Psi\right) - \frac{\Omega_1 - \Omega_2}{2}\right).$$

Note that A is the steady state for the price level, which is indeterminate, while ϕ_0 is the steady state for aggregate output. The conditional variances are constants, and functions of σ_ϵ^2, σ_z^2, and other parameters of the model,

$$\Omega_1 - \Omega_2 = [(\gamma + B)^2 + (2-\mu_1)(\gamma + B) - B]\sigma_z^2 + \left[\tau^2 + (\mu_1 - 2)\tau - B\frac{\lambda}{1-\lambda}\right]\sigma_\epsilon^2.$$

$\square$

Thus, the volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0,1)$, $\tau > 0$, and $\sigma_\varepsilon^2 > 0$, then there exists a sentiment-driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where

$$\sigma_y^2 = \sigma_z^2. \tag{D.71}$$

Expression D.70 implies that sentiment volatility is determined by structural parame-

[26]The relationship between the price level and sentiments is indeterminate in the flexible price case.

ters, such as the degree of complementarity/substitutability in actions across firms (τ, γ), information content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock. Note that if $\tau = 0, \lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate demand or price depends only on aggregate demand. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs. Sentiment volatility is decreasing in $1 - \lambda$; as the private signal becomes more informative about aggregate demand ($1 - \lambda$ increases), we approach the certainty equilibrium of the previous section. Sentiment volatility is increasing in $\sigma_\varepsilon^2 > 0$, which implies that a sentiment-driven equilibrium needs sufficient coordination. All firms set the same price regardless of their individual signal, but depending on the (known) distribution of signals. The more volatile the idiosyncratic component of the signal, the more difficult it is to attain coordination. In this case, sentiment volatility must be commensurately larger.

The sentiment-driven equilibrium is a rational expectations equilibrium: given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that price-setting decisions lead to aggregate output equaling the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. Firms increase their price in response to aggregate demand, and decrease their price in response to idiosyncratic demand. Through prices, firms' output decision are strategic substitutes. When firms actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal price depends negatively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$ and positively on the level of aggregate demand, $\hat{z}_t$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (D.70). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

D.2 Sticky Prices

Under Calvo price setting, a fraction θ_p of firms can not adjust their price in period t. Instead, $(1 - \theta_p)$ of firms choose their optimal price taking into account the probability of not being able to adjust for $\frac{1}{\theta_p}$ periods. The representative households sets wages flexibly.

As multiple equilibria arises from coordinated actions when signals are correlated, sticky prices will reduce the set of equilibria by impeding coordination. As a result, sentiment-driven fluctuations are less volatile. Due to the endogeneity of sentiment volatility, when the central bank targets inflation strongly or prices are more flexible, this leads to higher volatility of output. Note that although sentiment shocks are *iid*, the Calvo parameter affects inflation through the proportion of firms who can reset prices.

The following sections will introduce the micro-foundations of this model: the optimization problems of households and firms, timing to clarify what is known when decisions are undertaken, and equilibrium conditions. The quantity of output in the fundamental equilibrium is derived, followed by the mean level of output in the sentiment-driven equilibrium. In addition, the mechanism behind a self-fulfilling equilibrium with sentiments will be described.

Households. The representative household's problem is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + Tr_t.$$

From the household's problem, we obtain optimal conditions for demand ($C_{j,t}$),

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} C_t \epsilon_{j,t},$$

where the resulting aggregate price index

$$P_t \equiv \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

implies $\int P_{j,t}C_{j,t}dj = P_tC_t$. The household's labor supply schedule,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t},$$
$$\Psi C_t^{\gamma} = \frac{W_t}{P_t},$$
$$w_t - p_t = \gamma c_t + \log \Psi.$$

Finally, intertemporal consumption is given by

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right),$$
$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].$$

The representative household chooses labor N_t to maximize utility[27]

$$\max_{N_t} \frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1 - N_t), s$$

subject to budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t},$$

where C_t is aggregate an consumption index, $\frac{W_t}{P_t}$ is the real wage, $\frac{\Pi_t}{P_t}$ is real profit income from all firms, Ψ is disutility of labor. Their first order condition is

$$C_t^{\gamma} = \frac{1}{\Psi} \frac{W_t}{P_t}, \tag{D.72}$$

where

$$C_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{D.73}$$

$\theta > 1$ is the elasticity of substitution between goods, $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to

[27] Specifying the utility function in this way will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments. In the previous setup, $\gamma = 1$.

simplify calculations. The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t}C_{j,t}dj$, where $P_{j,t}$ is the price of intermediate good j.

From optimizing its consumption allocation, household demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} C_t \epsilon_{j,t}. \tag{D.74}$$

The resulting aggregate price level is obtained by substituting (D.74) into (D.73):

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj\right)^{\frac{1}{1-\theta}}.$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and aggregate demand, to be realized. Let Z_t represent the household's beliefs about aggregate demand at the beginning of period t. Households form consumption *plans* using (D.74)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})}\right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{D.75}$$

and decide labor supply, using (D.72) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t,

$$P_t(Z_t) = \frac{W_t}{\Psi\left[\frac{W_t}{P_t(Z_t)}N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)}\right]^{\gamma}}. \tag{D.76}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_tN_t$.

Firms. The firms' marginal cost is derived from the following minimization problem,

$$\min_{N_{j,t}} W_t N_{j,t},$$

subject to

$$Y_{j,t} \leq \epsilon_{j,t}^{\tau} N_{j,t}.$$

The Lagrangian is

$$L = W_t N_{j,t} - \Phi_t(\epsilon_{j,t}^{\tau} N_{j,t} - Y_{j,t}).$$

Substituting for W_t using (D.72), nominal marginal cost is

$$\begin{aligned} \Phi_t &= \Psi \epsilon_{j,t}^{-\tau} Z_t^{\gamma} P_t, \\ \phi_t &= \log(\Psi) - \tau \epsilon_{j,t} + \gamma z_t + p_t. \end{aligned}$$

Under Calvo price setting, the aggregate price index is as follows:

$$P_t^{1-\theta} = \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj + \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj,$$

where $\times_t^c$ denotes the set of firms who can not re-adjust prices in period t and $\times_t$ as the complement of this set. Let

$$P_{t-1}^{1-\theta} \equiv \frac{1}{\theta_p} \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj, \tag{D.77}$$

$$P_t^{*(1-\theta)} \equiv \frac{1}{1-\theta_p} \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj. \tag{D.78}$$

Using these definitions, the aggregate price index is given by

$$P_t^{1-\theta} = \theta_p P_{t-1}^{1-\theta} + (1-\theta_p) P_t^{*(1-\theta)}, \tag{D.79}$$

$$\Pi_t^{1-\theta} = \theta_p + (1-\theta_p) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\theta}. \tag{D.80}$$

A first order approximation to (D.80) around a zero inflation steady state yields

$$\pi_t = (1-\theta_p)(p_t^* - p_{t-1}). \tag{D.81}$$

The firm's profit-maximizing price is

$$p_{j,t}^* - p_{t-1} = (1-\beta\theta_p) \mathbb{E}_t[\gamma z_t - \tau \varepsilon_{j,t} | s_{j,t}] + \mathbb{E}_t[\pi_t | s_{j,t}].$$

Substituting π_t with (D.81),

$$p_{j,t}^* = (1-\beta\theta_p) \mathbb{E}_t[\gamma z_t - \tau \varepsilon_{j,t} | s_{j,t}] + (1-\theta_p) \mathbb{E}_t[p_t^* | s_{j,t}] + \theta_p p_{t-1}. \tag{D.82}$$

To find an expression relating the aggregate price level and sentiment ($p_t^*(z_t)$), conjecture $p_t^* = \tilde{D} + \mu(1-\lambda)z_t$. Use the conjecture and (D.82) to find $p_{j,t}^*$

$$\begin{aligned}
p_{j,t}^* &= (1-\beta\theta_p)\mathbb{E}[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}] + (1-\theta_p)\mathbb{E}_t[\tilde{D} + \mu(1-\lambda)z_t|s_{j,t}] + \theta_p p_{t-1} \\
&= (1-\theta_p)\tilde{D} + \theta_p p_{t-1} + \mathbb{E}_t([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}|s_{j,t})
\end{aligned}$$

Let $p_{j,t}^* = D + \mu s_{j,t}$ where

$$\begin{aligned}
D &\equiv (1-\theta_p)\tilde{D} + \theta_p p_{t-1}, \\
\mu &\equiv \frac{\text{cov}([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}, s_{j,t})}{\text{var}(s_{j,t})}.
\end{aligned}$$

Substitute $p_{j,t}^*$ into (D.78) and equate coefficients to find the steady state for $p_{j,t}^*$ and p_t^*, as well as their responses to z_t. Taking the log of (D.78) and defining $\mathbb{E}_{\times_t}$ as $\frac{1}{1-\theta_p}\int_{\times_t}$,

$$\begin{aligned}
(1-\theta)p_t^* &= \ln \mathbb{E}_{\times_t} e^{(1-\theta_p)p_{j,t}^* + \varepsilon_{j,t}}, \\
p_t^* &= D + \mu(1-\lambda)z_t + \frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2.
\end{aligned}$$

Equating coefficients,

$$\begin{aligned}
\tilde{D} &= p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 \\
D &= p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 \\
\mu &= (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}
\end{aligned}$$

Note that μ is close to $\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}]$ if $\theta_p \to 1$. The more flexible prices are ($\theta_p \to 0$), the larger is μ, and the more pass through of z_t to $p_{j,t}^*$ and thus to p_t^*. When prices are sticky, coordination is more difficult to achieve. The θ_p in the denominator is from the effect of z_t on p_t^*. The implied processes are

$$p_{j,t}^* = p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}s_{j,t}, \tag{D.83}$$

$$p_t^* = p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}(1-\lambda)z_t. \tag{D.84}$$

Substituting for p_t^* in (D.81) with (D.84), we get a form of the New-Keynesian Philips

Curve, which results from the price-setting behavior of firms with imperfect information,

$$\pi_t = \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda+1]^2}{2(1-\theta)}\sigma_\epsilon^2 + (1-\theta_p)(1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}(1-\lambda)z_t. \quad \text{(D.85)}$$

Note that the degree of pass through of z_t to π_t is increasing in the degree of price flexibility ($\theta_p \downarrow$).

Central bank. The central bank sets the nominal interest rate as a function of price inflation and output

$$Q_t^{-1} = \beta^{-1}\Pi_t^{\phi_\pi} + Y_t^{\phi_y}.$$

In logs,

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t.$$

Equilibrium. In equilibrium, the aggregate price index, intermediate goods price, and the private signal are given by

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \quad \text{(D.86)}$$

$$0 = \sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t[Q_{t,t+k} Y_{t+k|t}(P_{j,t}^* - M\psi_{t+k|t})], \quad \text{(D.87)}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \quad \text{(D.88)}$$

With *iid* sentiments, (D.87) simplies to

$$P_{j,t}^* = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | s_{j,t}]}$$

In the sentiment-driven equilibrium, an additional condition requires beliefs about aggregate demand to be correct in equilibrium,

$$Z_t = Y_t. \quad \text{(D.89)}$$

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{D.90}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{D.91}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{D.92}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{D.93}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{D.94}$$

The first equality follows from the household's demand equation and indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

Effect of an *iid* shock to sentiment. The Euler equation and Taylor rule imply the following relationship between inflation and sentiments in partial equilibrium

$$\pi_t = -\frac{\gamma + \phi_y}{\phi_\pi} z_t, \tag{D.95}$$

while the New-Keynsian Philips curve (D.85) describes another relation. In a sentiment-driven equilibrium, the σ_z^2 that satisfies both relationships is

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda} \frac{1}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\theta_p}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2. \tag{D.96}$$

Proposition 6. *Let $\lambda \in (0, 1)$. Under Calvo price setting, there exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π and* *decreasing in ϕ_y,*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2, \tag{D.97}$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\gamma$.

Under sticky prices, the self-fulfilling equilibrium has a different mechanism than in the case where firms set prices and households set wages flexibly. Here, a positive sentiment shock is realized when the nominal interest rate falls, which follows from a decrease in price inflation. For price inflation to fall when sentiment increases, σ_z^2 must be sufficiently low such that firms must misattribute enough of the increase in z_t to $\epsilon_{j,t}$ instead, leading them to lower prices. When goods markets open, households demand $y_{j,t}(p_{j,t})$, which is higher than the quantity that would have been demanded if firms had set prices under perfect information. There is a σ_z^2 such that aggregate supply is equal to the sentiment that households have formed.

Note that as price flexibility facilitates the pass through of z_t, sentiment volatility is increasing in the degree to which firms are able to adjust prices. As $\phi_\pi \to \infty$ or $\lambda_p \to \infty$, σ_z^2 approaches its value under flexible prices (D.55).

By (D.97), a policymaker can suppress non-fundamental fluctuations with a simple interest rate rule that places sufficiently low weight on price inflation,

$$\phi_\pi < \frac{\lambda}{1-\lambda}\frac{1}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\tau}. \tag{D.98}$$

In the complete information case, the condition for indeterminacy is given by (Bullard (2002)),

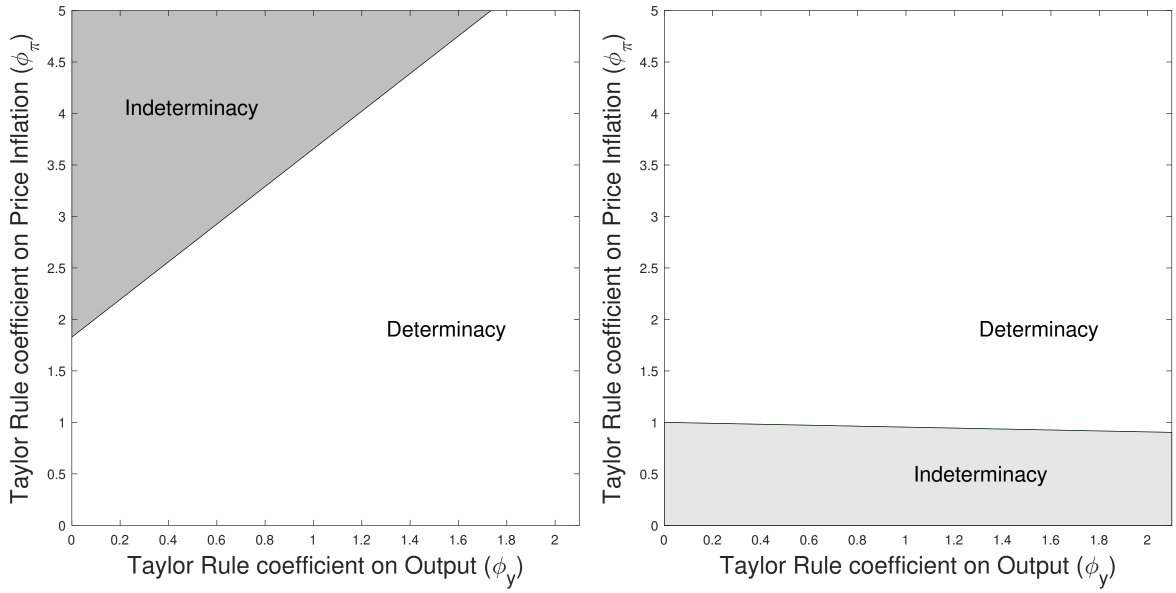
$$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y,$$

where $\kappa = \lambda_p \gamma$.

Proposition 7. *In an equilibrium with sentiment-driven fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. Equation (D.95) can be used to derive a relationship between the volatility of inflation and the volatility of output,*

$$\sigma_\pi^2 = \left(\frac{\gamma+\phi_y}{\phi_\pi}\right)^2 \sigma_y^2.$$

Figure 2: Indeterminacy and Determinacy Regions



(a) Information Frictions (b) Complete Information

Note: These figures show the indeterminacy region for a model with $\beta = 0.99$ (which implies a steady state real return on bonds of about 4 percent), $\gamma = 1$ (log utility), and $\theta_p = 0.66$ (an average wage duration of 1.5 years), and a weight of $\lambda = 0.2$ for the idiosyncratic component of the signal.

Expressing σ_y^2 and $\sigma_{\pi^w}^2$ in terms of model parameters,

$$\sigma_y^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2,$$

$$\sigma_\pi^2 = \left(\frac{\gamma+\phi_y}{\phi_\pi}\right)^2 \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p \lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\gamma}{\lambda_p}\frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2.$$

As the central bank increases its response to price inflation (ϕ_π), the volatility of price inflation declines, but this comes at the expense of higher volatility of output. Assuming $\phi_\pi > \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\tau}$, i.e., we are in an equilibrium with non-fundamental fluctuations ($\sigma_y^2 > 0$),

$$\frac{\partial \sigma_y^2}{\partial \phi_\pi} > 0.$$

Conversely, the more the central bank responds to output, the more volatile price inflation is in *equilibrium.*

$$\frac{\partial \sigma_\pi^2}{\partial \phi_y} > 0.$$

As in D.95, let $\frac{\partial \pi_t}{\partial z_t} = -\frac{\gamma+\phi_y}{\phi_\pi}$. Assuming $\phi_\pi > \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\tau}$, so that we are in an equilibrium with non-fundamental fluctuations ($\sigma_y^2 > 0$),

$$\frac{\partial \sigma_y^2}{\partial \phi_\pi} = \frac{\lambda}{1-\lambda}\sigma_\epsilon^2 \left(\frac{\partial[\frac{\partial \pi_t}{\partial z_t}]}{\partial \phi_\pi}\right)\left[\frac{\tau + \frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}\frac{\partial \pi_t}{\partial z_t}}{\gamma - \frac{\gamma}{\lambda_p}\frac{\partial \pi_t}{\partial z_t}} + \frac{\frac{\lambda}{1-\lambda}\frac{\gamma}{\theta_p\lambda_p}}{\gamma - \frac{\gamma}{\lambda_p}\frac{\partial \pi_t}{\partial z_t}}\right] > 0$$

The same is true for price flexibility, $\frac{\partial \sigma_z^2}{\partial \lambda_p} > 0$.

E Productivity shock

The baseline model has shown how monetary policy that targets inflation strongly can increase the volatility of non-fundamental fluctuations, which arise under a minor deviation from the complete information benchmark of a standard New Keynesian model. We abstracted from fundamental sources of fluctuations in order to isolate the effects of non-fundamental shocks. This section will demonstrate the robustness of these results to the case where aggregate output also consists of a fundamental component, an unobservable technology shock (A_t).

Recall that the results of the previous section were derived from two key conditions, which are maintained in this extension: (1) firms are unable to distinguish between idiosyncratic and aggregate demand and (2) endogenous signals that capture aggregate demand.[28] Therefore, whether aggregate demand is comprised of non-fundamental or fundamental components does not affect the conclusions. Not only do non-fundamental fluctuations introduce a tradeoff between stabilizing output and inflation, they are also not efficient. The stance of policy will also affect how technology shocks are transmitted to aggregate output. However, as long as the policymaker is unable to distinguish fundamental from non-fundamental shocks, it is unable to eliminate the latter.

As before, let Z_t denote households' beliefs about aggregate demand, but let it be com-

[28] Chahrour and Gaballo (2021) note that the non-fundamental equilibria in Benhabib et al. (2015) are not robust to exogenous aggregate variation in the signal. However, equilibria with both non-fundamental and fundamental aggregate components can still emerge when agents have separate exogenous signals about other aggregate exogenous components Acharya et al. (2021).

prised of both a fundamental shock (A_t) and a non-fundamental shock (ζ_t),

$$Z_t = f(\zeta_t, A_t).$$

Let $a_t \equiv \log A_t \sim N(\bar{a}, \sigma_a^2)$ be an AR(1) process,

$$A_t = A_{t-1}^{\rho} \epsilon_{A,t}.$$

As in the previous section, let households' labor supply schedule be a function of their beliefs about aggregate demand

$$\frac{W_t}{P_t} = \frac{1}{\Psi} Z_t^{\gamma}. \tag{E.99}$$

Household demand for good j is given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} \epsilon_{j,t} Y_t. \tag{E.100}$$

In this extension, firm j's production function also depends on an aggregate productivity shock,

$$Y_{j,t} = A_t N_{j,t}. \tag{E.101}$$

Firm j's first order condition, incorporating (E.99), (E.100), and (E.101)

$$Y_{j,t} = \left(\mathbb{E} \left[\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Z_t^{\frac{1}{\theta} - \gamma} A_t | S_{j,t} \right] \right)^{\theta}. \tag{E.102}$$

As before, firms base their production decision on a signal that confounds aggregate and idiosyncratic demand,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Aggregate output is given by

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} \epsilon_{j,t}^{\frac{1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}. \tag{E.103}$$

Finally, in equilibrium, households beliefs about aggregate demand are self-fulfilling

$$Z_t = Y_t.$$

E.1 Flexible wages

Certainty equilibrium. Under complete information, optimal production of firm j is

$$Y_{j,t} = \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta} - \gamma} A_t \right)^{\theta}.$$

Conjecture that aggregate demand Y_t is driven by both technology and a non-fundamental component,

$$Y_t = e^{\phi_0^A} A_t^{\psi_{ya}} \zeta_t,$$

where ϕ_0^A (the steady state of $\log Y_t$), ψ_{ya} (which parameterizes the transmission of the technology shock to aggregate ouput), and σ_ζ^2 (the volatility of the non-fundamental shock) are parameters to be identified. Substituting firm j's optimal production into (E.103), fluctuations in aggregate output depend only on exogenous changes in technology when information is complete,

$$Y_t = \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} A_t \left[\int \epsilon_{j,t} \, \mathrm{d}j \right]^{\frac{1}{\theta - 1}} \right)^{\frac{1}{\gamma}}.$$

Proposition 8. *When firms perfectly observe shocks $\epsilon_{j,t}$ and A_t, there is a certainty equilibrium in which Y_t responds only to fluctuations in technology. $y_t \equiv \log Y_t$ has mean and variance*

$$\begin{aligned} \phi_0^{A*} &= \frac{1}{\gamma} \left[\log \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \right) + \bar{a} + \frac{1}{2(\theta - 1)} \sigma_\epsilon^2 \right], \\ \sigma_y^2 &= \frac{1}{\gamma^2} \sigma_a^2. \end{aligned}$$

The relationship between output and aggregate technology is $\psi_{ya} = \frac{1}{\gamma}$ and output is not driven by any non-fundamental sources ($\sigma_\zeta^2 = 0$).

Non-fundamental equilibrium. Information frictions are essential for an equilibrium in which fluctuations in aggregate output contain a non-fundamental component. To demonstrate this, consider the case where firm production is conditioned on a signal that confounds aggregate and idiosyncratic demand, $S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \mathbb{E}_t \left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{P_t}{W_t} A_t | S_{j,t} \right) \right]^{\theta}.$$

As before, conjecture aggregate demand to be driven by both technology and a non-fundamental component, where ϕ_0^A, ψ_{ya}, and σ_ζ^2 are to be identified,

$$Y_t = e^{\phi_0^A} A_t^{\psi_{ya}} \zeta_t.$$

Proposition 9. *Let $\lambda \in (0, \frac{1}{2})$. When firms condition output on an endogenous signal, Y_t features fluctuations from both fundamental and non-fundamental sources, A_t and ζ_t. Aggregate output, $y_t \equiv \log Y_t \sim N(\phi_0^A, \sigma_y^2)$, is stochastic, with mean ϕ_0^A and variance σ_y^2*

$$\begin{aligned}
\phi_0^A &= \frac{1}{\gamma} \left[\log \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \right) + \bar{a} + \frac{\Omega_s}{2} + \frac{1}{2(\theta - 1)} \sigma_\epsilon^2 \left(\frac{1}{\theta} + \frac{\theta - 1}{\theta} \frac{\lambda}{1 - \lambda} \right)^2 \right] \\
\sigma_y^2 &= \sigma_\zeta^2 + \frac{1}{\gamma^2} \sigma_a^2,
\end{aligned}$$

The volatility of non-fundamental fluctuations is

$$\sigma_\zeta^2 = \frac{1}{\gamma \theta} \tilde{\sigma}_z^2.$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda} \left(1 - \frac{\lambda}{1-\lambda}\right) \sigma_\epsilon^2$. Aggregate technology affects aggregate output by

$$\psi_{ya} = \frac{1}{\gamma}.$$

As long as endogenous signals capture aggregate demand and firms are unable to distinguish between idiosyncratic and aggregate demand, their signal extraction problem will entail misattributing one to the other, leading to fluctuations which have both fundamental and non-fundamental components.

E.2 Calvo Wage Rigidity

The equilibrium conditions in sections (3.1) - (3.5) are maintained in this extension, with the exception that $A = A_t$ and $Z_t = f(\zeta_t, A_t)$.

Proposition 10. *Let $\lambda \in (0, \frac{1}{2})$. When firms condition output on an endogenous signal, there exists a rational expectations equilibrium where aggregate output Y_t features fluctuations from* *both fundamental and non-fundamental sources, A_t and ζ_t. Aggregate output, $y_t \equiv \log Y_t \sim N(\phi_0^A, \sigma_y^2)$, is stochastic, with its variance increasing in ϕ_π^w and λ_w,*

$$\sigma_y^2 = \sigma_\zeta^2 + \left(\frac{1+\phi_\pi^w \lambda_w}{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}\right)^2 \sigma_a^2.$$

The volatility of non-fundamental fluctuations is

$$\sigma_\zeta^2 = \frac{1+\phi_\pi^w \lambda_w}{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}\frac{1}{\theta}\tilde{\sigma}_z^2,$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. Aggregate technology affects aggregate output by

$$\psi_{ya} = \frac{\lambda_w(\phi_\pi^w - \rho) + (1-\beta\rho)(1-\rho)}{[\gamma(1-\rho)+\phi_y](1-\beta\rho) + \gamma\lambda_w(\phi_\pi^w-\rho)}.$$

A proof of this proposition is provided below. As $\phi_\pi^w \to \infty$, σ_y^2 approaches its value under flexible wages,

$$\lim_{\phi_\pi^w \to \infty} \sigma_y^2 = \frac{1}{\gamma\theta}\tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2.$$

A nominal interest rate rule that responds strongly to wage inflation will increase volatility in beliefs about aggregate output. In an equilibrium where these beliefs can be self-fulfilling, stabilizing wage inflation increases the volatility of aggregate output. Letting $a_w \equiv \frac{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}{1+\phi_\pi^w\lambda_w}$,

$$\frac{\partial\sigma_y^2}{\partial\phi_\pi^w} = -\left(2\sigma_a^2 a_w^{-3} + \frac{1}{\theta}\tilde{\sigma}_z^2 a_w^{-2}\right)\frac{\partial a_w}{\partial\phi_\pi^w} > 0, \tag{E.104}$$

$$\frac{\partial\sigma_y^2}{\partial\lambda_w} = -\left(2\sigma_a^2 a_w^{-3} + \frac{1}{\theta}\tilde{\sigma}_z^2 a_w^{-2}\right)\frac{\partial a_w}{\partial\lambda_w} > 0, \tag{E.105}$$

since $\frac{\partial a_w}{\partial \phi_\pi^w} = -\frac{\lambda_w\phi_y}{(1+\phi_\pi^w\lambda_w)^2} < 0$. Wage flexibility will also increase non-fundamental volatility, since $\frac{\partial a_w}{\partial\lambda_w} = -\frac{\phi_\pi^w\phi_y}{1+\phi_\pi^w\lambda_w} < 0$.

Stabilizing output increases the volatility of wage inflation,

$$\frac{\partial\sigma_{\pi^w}^2}{\partial\phi_y} = \left(\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\right)^2 \left[\frac{1}{\theta a_w}\tilde{\sigma}_z^2\left(\frac{2}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial\phi_y}\right) + \frac{2\sigma_a^2}{a_w^2}\left(\frac{1}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial\phi_y}\right)\right].$$

Note that $\frac{\partial \sigma^2_{\pi^w}}{\partial \phi_y} > 0$, since

$$\frac{1}{\phi_y} - \frac{1}{a_w}\frac{\partial a_w}{\partial \phi_y} = \frac{1}{\phi_y} - \frac{1}{\gamma(1+\phi^w_\pi \lambda_w) + \phi_y} > 0.$$

As in the baseline model, the presence of non-fundamental shocks creates a tradeoff between stabilizing output and inflation. Equation (E.125) can be used to derive a relationship between the volatility of inflation and the volatility of output,

$$\sigma^2_{\pi^w} = \left(\frac{\lambda_w \phi_y}{1+\lambda_w \phi^w_\pi}\right)^2 \sigma^2_y.$$

Expressing σ^2_y and $\sigma^2_{\pi^w}$ in terms of model parameters,

$$\begin{aligned}\sigma^2_y &= \frac{1}{\theta a_w}\tilde{\sigma}^2_z + \frac{1}{a^2_w}\sigma^2_a,\\ \sigma^2_{\pi^w} &= \left(\frac{\lambda_w \phi_y}{1+\lambda_w \phi^w_\pi}\right)^2 \left(\frac{1}{\theta a_w}\tilde{\sigma}^2_z + \frac{1}{a^2_w}\sigma^2_a\right).\end{aligned}$$

The following proposition summarizes these findings.

Proposition 11. *In an equilibrium with non-fundamental fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. As the central bank increases its response to wage inflation (ϕ^w_π), the volatility of wage inflation declines, but this comes at the expense of higher output volatility (E.104), $\frac{\partial \sigma^2_y}{\partial \phi^w_\pi} > 0$. Conversely, the more the central bank responds to output, output volatility decreases at the expense of more volatile wage inflation, $\frac{\partial \sigma^2_{\pi^w}}{\partial \phi_y} > 0$.*

The dynamics of this extension follow those in the baseline case: as the policymaker tries to stabilize wage inflation, the real wage becomes less responsive to beliefs about aggregate demand. As a result, firm production is characterized by more strategic complementarity. An individual firm's best response will internalize others' best responses in forming a belief about the distribution of aggregate production. In the aggregate, this increases the responsiveness of output to A_t and ζ_t, amplifying both non-fundamental and fundamental shocks. The tradeoff between inflation and output remains; a policymaker that tries to stabilize output will amplify the responsiveness of inflation to these shocks.

Proof - Sentiment Equilibrium with Flexible Wages and Technology Shocks To solve for equilibrium output, conjecture $Y_t = MA_t^{\psi_{ya}}\zeta_t$ and $y_t \equiv \log Y_t \sim N(\phi_0^A, \sigma_y^2)$. In expectation,

$$e^{\phi_0^A + \frac{\sigma_y^2}{2}} = e^{m + \psi_{ya}\bar{a} + \frac{\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2}{2}}. \tag{E.106}$$

This implies

$$\begin{aligned} \phi_0^A &= m + \psi_{ya}\bar{a}, \\ \sigma_y^2 &= \psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2. \end{aligned}$$

Firm level production, in logs,

$$y_{j,t} = \theta \log\left(\frac{\theta-1}{\theta}\frac{1}{\psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \theta\,\underbrace{\mathbb{E}\left[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta} - \gamma)\bar{y}_t + \bar{a}_t | \tilde{s}_{j,t}\right]}_{\mu} + \frac{\theta}{2}\Omega_s,$$

where $\tilde{s}_{j,t} = \lambda\epsilon_{j,t} + (1-\lambda)(\psi_{ya}\bar{a}_t + \bar{\zeta}_t)$, $\bar{a}_t \equiv \log \bar{A}_t \sim N(0, \sigma_a^2)$, $\bar{\zeta}_t \equiv \zeta_t \sim N(0, \sigma_\zeta^2)$, $\bar{y}_t \equiv \log \bar{Y}_t \equiv \log[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t] \sim N(0, \sigma_y^2)$ and $\Omega_s \equiv \mathrm{Var}[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta} - \gamma)\bar{y}_t + \bar{a}_t|\tilde{s}_{j,t}]$ Let firm production be represented by

$$Y_{j,t} = e^{\varphi_0}\tilde{S}_{j,t}^B,$$

where $\tilde{S}_{j,t} = \epsilon_{j,t}^\lambda[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{1-\lambda}$, $\varphi_0 \equiv \theta\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \frac{\theta}{2}\Omega_s$, $\log \bar{Y}_t \sim N(0, \sigma_y^2)$, and $B \equiv \theta\mu$. By (E.103), aggregate output is

$$Y_t = e^{\varphi_0}[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{B(1-\lambda)} \underbrace{\left[\int \epsilon_{j,t}^{\frac{1}{\theta} + \frac{\theta-1}{\theta}\lambda B}\, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}}_{\kappa_1}.$$

In logs,

$$y_t = \varphi_0 + B(1-\lambda)[\psi_{ya}\bar{a}_t + \bar{\zeta}_t] + \log\kappa_1.$$

In expectation, this expression implies

$$e^{\phi_0^A + \frac{\sigma_y^2}{2}} = e^{\varphi_0 + \log\kappa_1 + \frac{1}{2}[B(1-\lambda)]^2[\psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2]}.$$

Equating with the conjecture (E.106),

$$
\begin{aligned}
B &= \frac{1}{1-\lambda}, && \text{(E.107)}\\
\phi_0^A &= \varphi_0 + \log \kappa_1, && \text{(E.108)}\\
&= \theta \log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0^A + \theta\bar{a} + \frac{\theta}{2}\Omega_s + \log\kappa_1, && \text{(E.109)}\\
&= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \bar{a} + \frac{\Omega_s}{2} + \frac{\log\kappa_1}{\theta}\right], && \text{(E.110)}\\
&= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \bar{a} + \frac{\Omega_s}{2} + \frac{1}{2(\theta-1)}\sigma_\epsilon^2\left(\frac{1}{\theta}+\frac{\theta-1}{\theta}\frac{\lambda}{1-\lambda}\right)^2\right], && \text{(E.111)}\\
\psi_{ya} &= \frac{1}{\gamma}, && \text{(E.112)}\\
m &= \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \frac{\Omega_s}{2}\right] + \frac{\log\kappa_1}{\theta}. && \text{(E.113)}
\end{aligned}
$$

In equilibrium, (E.107) implies

$$\sigma_y^2 = \tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2 + (1-\gamma\theta)\sigma_\zeta^2,$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. Equating with the results from our conjecture,

$$
\begin{aligned}
\sigma_y^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2,\\
\sigma_\zeta^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2.
\end{aligned}
$$

When firms condition production on an endogenous signal of aggregate demand, there is an extrinsic component to aggregate output ($\sigma_\zeta^2 > 0$).

Proof - Sentiment Equilibrium with Sticky Wages and Technology Shocks Incorporating the household's labor supply condition and its own production function, firm j conditions production ($Y_{j,t}$) on its signal $S_{j,t}$,

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)\mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{1}{W_t/P_t} A_t | S_{j,t}\right)\right]^\theta.$$

In logs, and letting $\Omega_s \equiv Var\left[\frac{1}{\theta}(\varepsilon_{j,t} + y_t) - \theta w_t^r + \tau a_t | s_{j,t}\right]$,

$$y_{j,t} = \theta \ln\left(1 - \frac{1}{\theta}\right) + \mathbb{E}[\varepsilon_{j,t} + y_t - \theta w_t^r + \theta a_t | s_{j,t}] + \frac{\theta}{2}\Omega_s. \tag{E.114}$$

The other equilibrium conditions include the Euler equation, Taylor rule, New Keynesian Phillips curve for wage inflation, the signal firms receive, labor supply of households, market clearing, and technology process,

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}), \tag{E.115}$$

$$\hat{i}_t = \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t, \tag{E.116}$$

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w - \lambda_w \hat{\mu}_t^w, \tag{E.117}$$

$$s_{j,t} = \lambda \varepsilon_{j,t} + (1-\lambda) y_t, \tag{E.118}$$

$$\hat{\mu}_t^w = \hat{w}_t^r - \gamma \hat{c}_t, \tag{E.119}$$

$$\hat{y}_t = \hat{c}_t, \tag{E.120}$$

$$\hat{y}_t = \int_0^1 \hat{y}_{j,t} dj, \tag{E.121}$$

$$\hat{a}_{t+1} = \rho \hat{a}_t + \hat{\varepsilon}_{t+1}^a. \tag{E.122}$$

Conjecture the following policy functions for output, price inflation, wage inflation, and the real wage,

$$\begin{aligned}
\hat{c}_t &= \hat{\zeta}_t + b_c \hat{w}_{t-1}^r + \psi_{ya} \hat{a}_t, \\
\hat{\pi}_t &= a_\pi \hat{\zeta}_t + b_\pi \hat{w}_{t-1}^r + c_\pi \hat{a}_t, \\
\hat{\pi}_t^w &= a_{\pi^w} \hat{\zeta}_t + b_{\pi^w} \hat{w}_{t-1}^r + c_{\pi^w} \hat{a}_t, \\
\hat{w}_t^r &= a_w \hat{\zeta}_t + b_w \hat{w}_{t-1}^r + c_w \hat{a}_t.
\end{aligned}$$

The following coefficients verify the conjecture

$$\begin{aligned}
a_w &= \frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y}{1 + \phi_\pi^w \lambda_w}, \\
b_\pi &= 1, \\
a_\pi^w &= -\frac{\lambda_w \phi_y}{1 + \lambda_w \phi_\pi^w}, \\
a_\pi &= -\frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y(1 + \lambda_w)}{1 + \lambda_w}.
\end{aligned}$$

Assuming technology shocks are *iid* ($\rho = 0$),

$$\begin{aligned} c_w &= \frac{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}{1+\phi_\pi^w\lambda_w}\psi_{ya}, \\ c_\pi &= -\frac{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y(1+\lambda_w)}{1+\lambda_w\phi_\pi^w}\psi_{ya}, \\ c_\pi^w &= -\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\psi_{ya}. \end{aligned}$$

From the wage inflation equation, $b_\pi^w(1-\beta c_w) = \lambda_w\gamma b_c$, which implies $b_\pi^w = b_c = 0$.

Note that the coefficients imply the same responses to the state variables as the baseline case where z_t was entirely non-fundamental. Now, when z_t is composed of both fundamental and non-fundamental components ($z_t = \zeta_t + \psi_{ya}a_t$), the policy functions can be written as

$$w_t^r = \frac{\gamma(1+\phi_\pi^w\lambda_w)+\phi_y}{1+\phi_\pi^w\lambda_w}(\zeta_t+\psi_{ya}a_t), \tag{E.123}$$

$$\pi_t^w = -\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}(\zeta_t+\psi_{ya}a_t), \tag{E.124}$$

$$\pi_t = -\frac{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y(1+\lambda_w)}{1+\lambda_w\phi_\pi^w}(\zeta_t+\psi_{ya}a_t)+\hat{w}_{t-1}^r, \tag{E.125}$$

$$c_t = \zeta_t + \psi_{ya}a_t. \tag{E.126}$$

Next identify ψ_{ya} from the equilibrium condition (E.121). Let $\hat{y}_{j,t} = y_{j,t} - \varphi_0$, where $\varphi_0 \equiv \theta\left[\ln\left(1-\frac{1}{\theta}\right)+\frac{\Omega_s}{2}\right]$. By (E.114) firm j's first order condition is given by

$$\begin{aligned} \hat{y}_{j,t} &= \mathbb{E}[\varepsilon_{j,t}+y_t-\theta w_t^r+\theta a_t|s_{j,t}] \\ &= \mathbb{E}[\varepsilon_{j,t}+(\hat{\zeta}_t+\psi_{ya}\hat{a}_t)-\theta(a_w\zeta_t+c_w a_t)+\theta a_t|s_{j,t}] \\ &= \mathbb{E}[\varepsilon_{j,t}+(\psi_{ya}-\theta c_w+\theta)\hat{a}_t+(1-\theta a_w)\zeta_t|s_{j,t}] \\ &= \frac{\lambda\sigma_\epsilon^2+(\psi_{ya}+\theta(1-c_w))\psi_{ya}(1-\lambda)\sigma_a^2+(1-\theta a_w)(1-\lambda)\sigma_\zeta^2}{\lambda^2\sigma_\epsilon^2+(1-\lambda)^2(\psi_{ya}^2\sigma_a^2+\sigma_\zeta^2)}\underbrace{[\lambda\varepsilon_{j,t}+(1-\lambda)y_t]}_{s_{j,t}}. \end{aligned}$$

Equilibrium condition (E.121) implies

$$\frac{\lambda\sigma_\epsilon^2+(\psi_{ya}+\theta(1-c_w))\psi_{ya}(1-\lambda)\sigma_a^2+(1-\theta a_w)(1-\lambda)\sigma_\zeta^2}{\lambda^2\sigma_\epsilon^2+(1-\lambda)^2(\psi_{ya}^2\sigma_a^2+\sigma_\zeta^2)} = \frac{1}{1-\lambda}. \tag{E.127}$$

Solving for ψ_{ya},

$$\psi_{ya}^2 = (\psi_{ya} + \theta(1 - c_w))\psi_{ya}.$$

For $\psi_{ya} \neq 0$, $c_w = 1$, which implies

$$\psi_{ya} = \frac{1 + \phi_\pi^w \lambda_w}{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y}.$$

Solving for σ_ζ^2 using E.127,

$$\sigma_\zeta^2 = (1 - \theta a_w)\sigma_\zeta^2 + \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2.$$

Letting $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$, which is equivalent to sentiment volatility in the model without technology shocks,

$$\sigma_\zeta^2 = \frac{1}{\theta a_w}\tilde{\sigma}_z^2$$

Note that as $\phi_\pi^w \to \infty$, we approach the flexible wage case, where $a_w \to \gamma$. Finally, using $\psi_{ya} = \frac{1+\phi_\pi^w \lambda_w}{\gamma(1+\phi_\pi^w \lambda_w)+\phi_y}$, we can express the coefficients (c_π, c_{π^w}) for the technology shock as follows,

$$\begin{aligned}
c_\pi &= -\frac{\gamma(1 + \phi_\pi^w \lambda_w) + \phi_y(1 + \lambda_w)}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}, \\
c_\pi^w &= -\frac{\lambda_w \phi_y}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}.
\end{aligned}$$

Persistent technology. Under the assumption that technology shocks are persistent ($\rho > 0$), a_π, a_π^w, and a_w remain the same, while the coefficients for a_t in our policy functions are as follows,

$$\begin{aligned}
c_w &= \frac{[\gamma(1-\rho) + \phi_y](1 - \beta\rho) + \gamma\lambda_w(\phi_\pi^w - \rho)}{\lambda_w(\phi_\pi^w - \rho) + (1 - \beta\rho)(1 - \rho)}\psi_{ya}, \\
c_{\pi^w} &= \frac{1}{1 - \beta\rho}[-\lambda_w(c_w - \gamma\psi_{ya})], \\
c_\pi &= c_{\pi^w} - c_w.
\end{aligned}$$

Under persistent technology shocks, E.127 still holds. Solving for ψ_{ya}, and assuming $\psi_{ya} \neq$

0, $c_w = 1$, this implies

$$\psi_{ya} = \frac{\lambda_w(\phi_\pi^w - \rho) + (1-\beta\rho)(1-\rho)}{[\gamma(1-\rho)+\phi_y](1-\beta\rho) + \gamma\lambda_w(\phi_\pi^w - \rho)},$$

$$c_{\pi^w} = -\frac{\lambda_w\phi_y}{\gamma\left([(1-\rho)+\frac{\phi_y}{\gamma}](1-\beta\rho) + \lambda_w(\phi_\pi^w-\rho)\right)},$$

$$c_\pi = -\frac{\lambda_w\phi_y}{\gamma\left([(1-\rho)+\frac{\phi_y}{\gamma}](1-\beta\rho) + \lambda_w(\phi_\pi^w-\rho)\right)} - 1.$$