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Abstract

Credit market interventions have become a widespread policy tool deployed by governments around the world to support their corporate sectors following shocks like the global financial crisis and the pandemic. Among those policies, forbearance programmes allowed firms to temporarily stop making payments on certain debt obligations or obtain debt forgiveness. However, the impact of these policies is not fully understood. In particular, forbearance lending is generally believed to keep unviable firms alive and contribute to the zombification of the corporate sector. To inform this debate, we examine the effects of Japan's small and medium-sized enterprise (SME) Financing Facilitation Act, which encouraged banks to offer loan forbearance to troubled SMEs. We develop a framework to quantify the aggregate impact of the policy using a difference-in-differences approach combined with back-of-the-envelope counterfactual exercises. Our evaluation indicates that, when coupled with business restructuring plans, forbearance lending can temporarily boost output without contributing to the widespread zombification of the corporate sector. Forbearance is more effective when credit market disruptions impede the reallocation of capital.

Key words: Forbearance lending, zombie firms, credit frictions, misallocation, productivity, search and matching, credit market interventions, policy evaluation.

JEL classification: E22, E43, E44, E65, G21, O40.

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1 Introduction

Interventions in corporate credit markets have featured as prominent tools in the policy response to a series of crisis episodes over the last two decades, including the global financial crisis, the European sovereign debt crisis, and the COVID-19 pandemic. Governments across the globe deployed policies to alleviate the liquidity difficulties encountered by businesses in response to these economic shocks. These policies included loan guarantees, direct lending schemes, and steps by regulators and lenders to implement loan forbearance. Loan forbearance is a practice whereby banks grant temporary relief to struggling borrowers (e.g., extended repayment periods and reduced interest payments) to avoid default. The academic and policy literature is mostly critical of loan forbearance because of its potential to contribute to zombification - a situation where bank lending keeps unviable firms alive, resulting in lower aggregate productivity through credit misallocation and zombie congestion.[1]

An evaluation of Japan's 2009 SME Financing Facilitation Act, however, suggests that forbearance lending might not entirely deserve its bad reputation. In particular, when coupled with business restructuring plans, it can provide temporary relief for struggling firms without contributing to the zombification of the corporate sector, thereby protecting productive capital. In other words, forbearance has the potential to be used as a crisis management tool as part of a carefully designed credit market intervention. Under the SME Financing Facilitation Act, financial institutions were required to make their "best efforts" to ease repayment conditions for qualifying SMEs that asked for support. At the same time, the Japanese Financial Services Agency allowed financial institutions to exclude the restructured SME loans from their reported non-performing loans, under the condition that they came up with business restructuring plans that were expected to make the loans perform again within five years. The law provides a quasi-experiment which enables us to quantify the aggregate consequences of loan forbearance using a difference-in-differences approach combined with back-of-the-envelope counterfactual exercises. The latter are guided by a partial equilibrium search-and-matching model of credit markets that incorporates forbearance incentives.

Our first finding is that the Act worked as an interest rate subsidy. It depressed the average interest rates that firms pay on their debt by about 18.5% on average for treated firms over 2010-2018. While payment deferrals depressed interest rates by around 11% on average, debt forgiveness was associated with a much larger subsidy of 39%. The overall treatment effects are larger in the years closer to the implementation of the Act and fade away over time. Our second finding is that cheap credit boosted the aggregate capital stock at the expense of aggregate productivity. The Act boosted the aggregate capital stock by 1.4% and depressed capital productivity by 0.5% on average over 2010-2018. Our third result is that the extent of credit reallocation determines whether the policy leads to output gains or losses. In the more plausible scenario of impaired reallocation, the Act is estimated

[1] See, e.g., Acharya et al. (2021, 2022, 2023), Andrews and Petroulakis (2019), Barbaro and Tirelli (2021), Faria-e-Castro et al. (2024), Caballero et al. (2008), Adalet McGowan et al. (2018), Banerjee and Hoffmann (2022), Álvarez et al. (2023).

to have boosted output by 2.5% on average. By contrast, if we assume seamless credit reallocation, the Act is estimated to have depressed output by 1.5% on average. Seamless reallocation is unrealistic as there will be frictions impeding the process and capital reallocation is pro-cyclical, i.e. depressed during recessions (e.g., Caballero and Hammour, 2005; Eisfeldt and Rampini, 2006). Our reallocation counterfactuals therefore provide an upper-bound estimate of output gains and losses. The aggregate effects of the law are concentrated in the years close to its implementation and fade away over time. Finally, we find that the Act did not contribute to the creation of zombie firms. On the contrary, we find that a greater exposure to the policy improved firm-level performance. This suggests that the business restructuring plans, which troubled SMEs in receipt of forbearance were requested to submit and adhere to, allowed them to resurrect. To the extent that banks granted forbearance to viable firms, the Act enabled them to weather temporary difficulties while limiting the negative impact on aggregate productivity in the short and long term.

Our results challenge the view that loan forbearance necessarily contributes to the zombification of the corporate sector - a notion grounded in Japan's experience during the Lost Decade. We argue that "this time is different" (see Section 2). Importantly, when coupled with business restructuring plans, forbearance can provide relief for struggling firms that are otherwise solvent and will recover from a temporary shock. In other words, a carefully designed credit market intervention based on forbearance has the potential to be used as part of the policy toolkit to respond to severe stress in the corporate sector. In addition, our results indicate that forbearance lending is a more effective credit market intervention when credit reallocation is subdued, for example during recessions, especially those that are accompanied by credit market disruptions.

We make several contributions to the literature. First, we provide evidence on the impact of loan forbearance using a plausibly exogenous policy shock from the point of view of the lenders. Such evidence is rare as loan forbearance policies are typically selective and only applied to a subset of bank-firm relationships with specific characteristics. The SME Financing Facilitation Act is exceptional in that it mandated forbearance lending to all banks. In other words, forbearance was not motivated by bank-specific motives. The exogenous nature of the policy shock is reflected in the following statement by Financial Services Minister Shizuka Kamei:

> "As long as I'm financial services minister, I'm not going to leave small companies in the lurch, unable to get loans. If a bank takes that approach, I'll hit them with a business improvement order." *The Japan Times*, 7 October 2009.

This sets forbearance lending post-GFC apart from that practised during the Japanese Lost Decade (1991-2001), during which low-capitalised banks were more likely to forbear (e.g. Peek and Rosengren 2005, Caballero et al. 2008). Although there was no penalty imposed on banks that did not follow the "best efforts" requirement, almost all requests for loan restructurings were accepted (see Yamori, 2019). The law therefore provides a quasi-experiment which enables us to develop an approach to estimate the plausibly causal impact of forbearance lending. The Act is discussed in detail in Section 2.2. To our knowledge, we are the first to holistically evaluate the consequences of this specific large-scale policy intervention.

Second, we provide evidence on the *aggregate* consequences of loan forbearance. The vast literature on forbearance lending during the Lost Decade in Japan and the sovereign debt crisis in Europe consists mostly of firm-level, bank-level, or industry-level studies[2]. Studies that provide an estimate of its impact on aggregate economic performance include Kwon, Narita and Narita (2015) on Japan's Lost Decade, Tracey (2021) on the sovereign debt crisis in Europe, and Faria-e-Castro, Paul, and Sánchez (2024) on the US. An overall quantitative assessment matters for policy. An important question is indeed whether loan forbearance can be used as a crisis management tool when the corporate sector is under severe financial stress, without contributing to zombification. To answer this question, we have to weigh the costs and benefits of forbearance in aggregate counterfactual exercises.

Third, our methodological contribution is to design a theory-driven empirical strategy that enables us to estimate the effect of the SME Financing Facilitation Act in the absence of data on loan restructurings. The approach proceeds as follows. First, we develop a simple search and matching model of the credit market where banks have incentives to forbear. This model serves to guide our difference-in-differences (DiD) estimation and back-of-the-envelope counterfactual exercises. Second, we use accounting data from Tokyo Shoko Research (TSR) and survey data from the Research Institute of Economy, Trade and Industry (RIETI) to estimate the impact of the policy on the average interest rates paid by firms using DiD methods. Since we do not have access to data on the universe of loan restructurings, we combine the eligibility criteria with survey data from RIETI to measure treatment exposure at the firm level. Third, the estimated annual treatment effects on interest rates are used to underpin back-of-the-envelope counterfactual exercises guided by the model. These consist in removing the estimated annual treatment effects and comparing the counterfactual population of firms to the observed population in terms of the capital stock, capital productivity, and output.

The rest of the paper is structured as follows. Section 2 briefly discusses the literature on zombie lending in Japan and presents the SME Financing Facilitation Act in more detail. Section 3 presents our search-and-matching model. Section 4 describes our difference-in-differences methodology and explains how we perform the counterfactual exercises. Section 5 discusses our data sources and measurement issues. Section 6 presents the results of the DiD analysis on interest rates, which forms the starting point for the counterfactual exercises in Section 7. Section 8 explores whether the SME Financing Facilitation Act encouraged zombification. Section 9 examines the robustness of the counterfactuals to alternative assumptions. Section 10 concludes.

[2]See Sekine, Kobayashi and Saita (2003), Ahearne and Shinada (2005), Hoshi (2006), Caballero, Hoshi and Kashyap (2008), Homar et al. (2015), Acharya et al. (2023), Banerjee and Hofmann (2022), Blattner, Farinha and Rebelo (2023), Adalet McGowan, Andrews and Millot (2018), Schivardi et al. (2022), Storz et al. (2017), and Andrews and Petroulakis (2019).

2 Forbearance lending in Japan: Then and now

2.1 Zombie lending and the Lost Decade

Japan is the most prominent case study for forbearance lending to firms. The latter has often been identified as one of the main drivers behind the elevated number of zombie firms in the Japanese economy. Indeed, Japan's "Lost Decade" following the financial and banking crisis in the late 1990s and early 2000s is where the phenomenon of zombie lending was first widely studied. Peek and Rosengren (2005) find evidence that banks increased loans to financially weaker firms. The study also sheds light on the motives for zombie lending. Troubled banks with reported capital ratios close to the required minimum value were more likely to increase loans to their weaker borrowers. This is evidence that weak banks were practising loan forbearance to hide the extent of their non-performing loans. Moreover, if a bank was in the same business group (keiretsu) as the firm, it was more likely to increase loans to weaker firms. Caballero, Hoshi and Kashyap (2008) document that there was a significant increase in zombie firms in several sectors during the period 1993-2002. They present industry-level evidence that industries with a higher prevalence of zombie firms exhibit more depressed job creation and destruction, and lower productivity. They also present firm-level regressions showing that the increase in zombies depresses the investment and employment growth of non-zombies. Other studies found similar evidence.[3] Finally, Muto, Sudo and Yoneyama (2023) use a DSGE model to show that a large fraction of the TFP growth decline in the early 1990s in Japan was due to the impairment of banks' and firms' balance sheets. Overall, the literature is unequivocal in its finding that forbearance lending contributed to the widespread zombification of the Japanese economy during the Lost Decade.

2.2 The SME Financing Facilitation Act

In November 2009, the Japanese government enacted the SME Financing Facilitation Act[4] to help small and medium-sized enterprises (SMEs) that had fallen into unprofitable conditions. Under this law, financial institutions were required to make their "best effort" to ease repayment conditions for qualifying SMEs that asked for support. Not all SMEs were eligible for forbearance lending under the Act. Article 2 paragraph (2) of the Act defines qualifying SMEs based on their number of employees and stated share capital. Criteria vary by industry. Article 4 then lays out certain types of firms that are not eligible, including financial institutions, and subsidiaries or parent companies of financial institutions. The eligibility criteria are listed in Supplemental Appendix A. At the same time, the JFSA allowed financial institutions to exclude the restructured SME loans from their reported non-performing loans under the condition that they came up with restructuring plans that were expected to make the loans perform again within five years.[5] Although there was no penalty imposed on banks that did not follow the "best efforts" requirement, almost all requests for loan restructurings were

[3] See, e.g., Sekine, Kobayashi and Saita (2003), Ahearne and Shinada (2005), Hamao, Kutsuna and Peek (2012), and Kwon, Narita and Narita (2015).

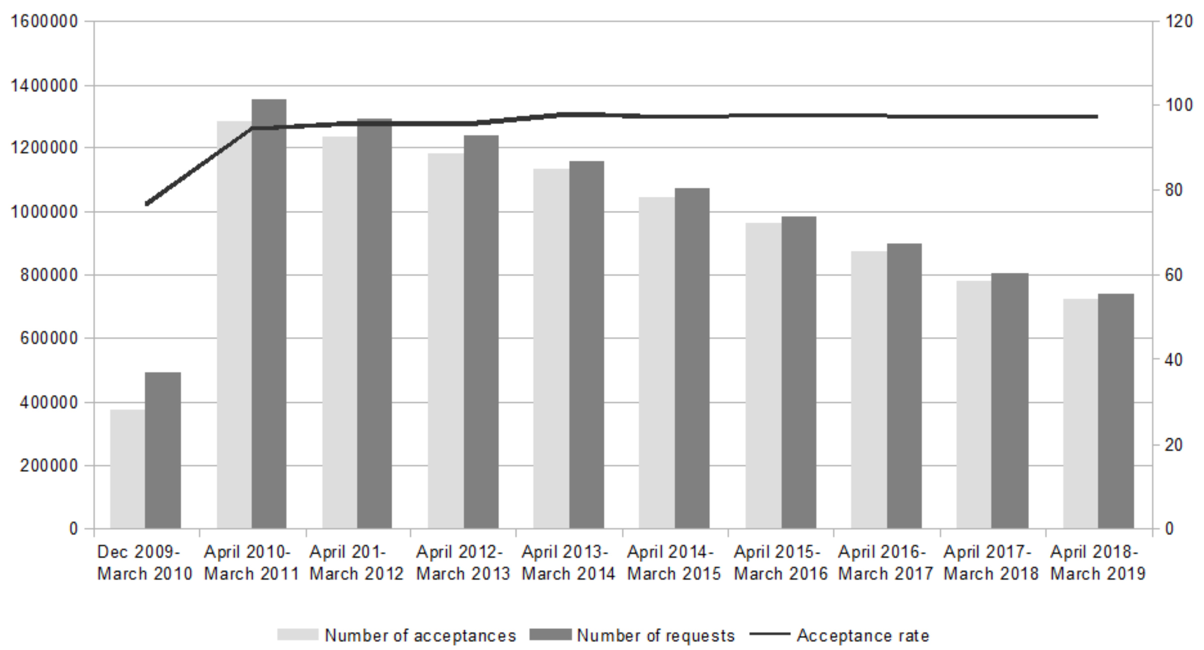
[4] The Act is also referred to as the Debt Moratorium Law.

[5] See Table 12 in Yamori (2019) for examples of business restructuring measures.

accepted (see Yamori, 2019). This reflects that political incentives to bail out struggling firms remain very powerful in Japan.

The Act was originally set to expire at the end of March 2011, but it was extended twice before finally expiring at the end of March 2013. However, the practice seems to have endured after the Act expired. According to Harada et al. (2015), the JFSA did not reverse the rule that allows banks to classify the restructured loans to SMEs as "normal" after the Act expired. As a result, troubled SMEs continued to ask for loan restructuring, and banks continued to grant loan restructuring for almost all who asked. Figure 1 shows the number of requests by SMEs to change loan conditions and the number of requests that banks accepted. The acceptance rates are extremely high, averaging around 91% between December 2009 and March 2013. The acceptance rates averaged 97% between April 2013 and March 2019, even though the law had formally expired.

Figure 1: Number of requests by SMEs and acceptance by banks



Notes: Number of requests by SMEs to restructure their loans and number of accepted requests for each fiscal year. Source: Financial Services Agency of Japan.

There is a concern that the Act might have encouraged banks to roll over loans to zombie firms like during the Lost Decade (see, e.g., Harada et al., 2015; Imai, 2019). However, there are reasons why this time might be different. First, loan forbearance was mandated by law and disclosed by financial institutions. One important characteristic of forbearance during the Lost Decade was that it was a hidden phenomenon driven by banks' incentives to hide the extent of their non-performing loans (see e.g., Peek and Rosengren, 2005). By contrast, forbearance under the SME Financing Facilitation Act was not a hidden phenomenon. Therefore, we should not expect it to be practised solely by weak banks with distorted incentives (see Supplemental Appendix G). Second, banks were only allowed to exclude the restructured SME loans from their reported non-performing loans under the condition that they came up with business turnaround plans that were expected to make the loans perform again within five years. This should have helped limit the extent to which forbearance propped up unviable firms, and mitigate moral hazard whereby firms in receipt of assistance have weak incentives to raise their profitability. Yamori (2019) finds that the Act was successful in that about 60% of the companies whose loan conditions were amended ultimately recovered. Of course, this still leaves a substantial proportion of firms that failed to bounce back. Finally, not all firms were eligible for support. Importantly, the Act excludes financial institutions, as well as their subsidiaries or parent companies. This provision should help address distortions created by business affiliations during the Lost Decade, in particular the membership of non-financial businesses and banks in the same business groups (keiretsu).

Overall, the design of the policy means that loan forbearance under the SME Financing Facilitation Act is more likely to be "good forbearance" than loan forbearance during the Lost Decade. Good forbearance is defined here as any form of temporary debt relief for struggling firms that are otherwise solvent and will recover from a temporary shock. The literature provides a rationale for such interventions. Evidence has shown that deeper and longer recessions can cause lasting damage to the productive capacity of the economy (e.g., Vinci and Licandro, 2021). One of the potential mechanisms for the scarring effect of recessions is that credit frictions can force the exit of productive but constrained firms. Credit market interventions can prevent this. Crouzet and Tourre (2021) show that if a downturn is accompanied by financial market disruptions, interventions in corporate credit markets can initially help forestall inefficient liquidations. This comes at the expense of debt overhang and depressed investment in the long run. However, the short-term benefits quantitatively dominate the long-run overhang costs.

3 A structural model of bank forbearance

In this section, we present a tractable model of bank forbearance. The purpose of our model is to provide an analytical framework for our empirical analysis and to serve as a guide for interpreting our results. The tractability of the model allows us to perform back-of-the-envelope counterfactual exercises using difference-in-differences estimation results. The model economy is populated by two types of agents, entrepreneurs and banks. Credit markets are characterized by search frictions. Entrepreneurs produce a homogeneous final good and search for bank credit to expand their capital stock to produce at their optimal scale. Firms differ in terms of their total factor productivity (TFP), z. Firms receive a new TFP draw in every period from a distribution $H(z)$. In addition, we assume that entrepreneurs can be of J observable types, where a type-j firm is characterised by k_{0j}, its capital stock that is not financed by bank loans, but instead is financed by, e.g., equity capital and publicly traded debt. These types are meant to capture time-invariant or slow-moving characteristics, such as size, which are an important determinant of access to bank loans. The J observable types correspond to J credit markets with different credit market tightness.

Banks search for entrepreneurs to whom they can lend profitably. We assume that each firm with observable properties j enters a different credit market to search for an optimal expansion of its capital stock. Hence, all firms in any credit market j have the same properties when entering the market. A bank opening a new credit line in a market must pay a fixed search cost of κ to attempt to find an entrepreneur on the market in every period. Matching frictions characterize each market. Credit market tightness in each market is denoted with θ_{jt}. It is defined as the ratio of vacant credit lines v_{jt} over unmatched entrepreneurs u_{jt} in credit market j $\left(\theta_{jt} = \frac{v_{jt}}{u_{jt}}\right)$. The probability that a bank is matched with an entrepreneur is denoted with q_{jt}. As long as the expected value of a vacant credit line is greater than zero in a market, banks will open additional credit lines. Available credit lines are matched with searching entrepreneurs in a frictional process summarized by a matching function. Once a vacant credit line is matched with an entrepreneur, the lender and the borrower agree on a contract that determines the interest rate on the desired loan amount. Contracting is the result of a Nash bargaining process. Loan conditions are defined intra-period, so a new loan interest rate and loan amount are determined every period. The benefit of a credit line is subject to idiosyncratic shocks to productivity. As long as the idiosyncratic benefit of the bank-entrepreneur match is sufficiently high, the contract is renewed in the next period. If not, both parties agree to end the relationship. The loan creation and destruction decisions generate entrepreneur flows in and out of the credit market. In terms of timing, we assume that credit markets open and matches are formed. Then productivity draws (z_{ijt}) are observed (productivity can therefore be seen as match-specific), interest rates and loan amounts are determined, and finally production ensues.

In addition to search frictions, each credit market is characterised by forbearance incentives. Incentives for forbearance lending are modelled as a termination cost (or severance cost) τ_{jt} incurred by banks when terminating a lending relationship that is no longer profitable. These abstract forbearance incentives can be interpreted to capture any kind of monetary or reputational cost that the bank incurs when exiting a credit relationship. While this cost could be micro-founded by modelling banks' motives[6], there is no need to do so because forbearance was *mandated* by the SME Financing Facilitation Act. In other words, banks effectively had to offer loan restructurings to struggling SMEs that asked for support. Therefore, banks' characteristics are unlikely to have any causal effect on their decisions to offer loan forbearance (see Supplemental Appendix G).

[6] For example, the cost could be associated with incentives to rationally avoid liquidating non-performing loans, e.g., when realizing loan losses would push the lender's capital below the regulatory threshold.

3.1 Production

Firm i of type j produces output y_{ijt} according to a decreasing returns Cobb-Douglas production function:

$$y_{ijt} = z_{ijt}(k_{ijt}^{\alpha} l_{ijt}^{1-\alpha})^{\gamma} \tag{1}$$

The parameter $\gamma < 1$ describes the degree to which returns per input decrease as firms expand, while the parameter α is the typical exponent capturing returns to factor inputs capital k and labour l. z_{ijt} is the total factor productivity of the firm, l_{ijt} is labour input, and k_{ijt} is the firm's total capital stock. The capital stock is the sum of the firm's initial capital stock (k_{0jt}) and any bank loan that the firm obtains when matched to a lender ($\hat{k}_{ijt}$). Firms pay their workers wages and we assume that labour is fixed. As a result, we define the firm's profits π_{ijt} before interest on bank loans in Equation (2):

$$\pi_{ijt}(z_{ijt}, \hat{k}_{ijt}) = y_{ijt} - w_{ijt} l_{ijt} - r_{0ijt} k_{0jt} \tag{2}$$

$\pi_{ijt}(z_{ijt}, \hat{k}_{ijt})$ corresponds to "profit before interest and taxes" in the profit and loss statements in our financial accounts data set. Upon matching, entrepreneurs seek to expand their capital stock, solving the profit maximisation problem:

$$\max_{\hat{k}_{ijt} \geq 0} \pi_{ijt}(z_{it}, \hat{k}_{ijt}) - \hat{k}_{ijt} r_{ijt}(z_{it}, \hat{k}_{ijt}) \tag{3}$$

$\hat{k}_{ijt} = k_{ijt} - k_{0jt}$ is the optimally chosen size of the credit contract following matching. $\pi_{ijt}(z_{ijt}, \hat{k}_{ijt})$ is the firm's profit before interest payments on bank loans, namely output minus the wage bill and any interest payments on other forms of capital (k_{0jt}).

An unmatched firm of type j has a probability p_{jt} of being matched, and hence receiving the surplus of a matched firm in every period. p_{jt} depends positively on the tightness θ_{jt} in credit market j. The value function of an unmatched type-j entrepreneur i, V_{ijt}^{U}, is given by Equation (4):

$$V_{ijt}^{U} = \pi_{0jt} + p_{jt}\beta \int_{\tilde{z}_{jt+1}}^{\infty} \left(V_{ijt+1}^{E}(z) - V_{ijt+1}^{U}\right) h(z) dz + \beta V_{ijt+1}^{U} \tag{4}$$

$\tilde{z}_{jt+1}$ is the productivity cutoff for new lending relationships[7]. $V_{ijt+1}^{E}(z)$ is the value function of a matched entrepreneur with productivity z, β is the discount factor, π_{0jt} is the profit of an unmatched type-j firm i which produces only with k_{0jt} (i.e. output minus the wage bill and any interest payments on non-bank capital).

Equation (4) says that the value function of an unmatched entrepreneur is equal to the firm's current outside option value π_{i0jt}, the discounted value function of an unmatched entrepreneur in the next period (βV_{ijt+1}^{U}), plus the expected surplus of a new match in the next period, i.e. the probability of

[7] We show the determination of this cutoff in the Supplemental Appendix in section E.

a match (p_{jt}) times the expected surplus from the match (integral over z).

The value function of a matched firm is given by Equation (5):

$$V^E_{ijt}(z_{it}) = \pi^*_{ijt} - \hat{k}_{ijt} r_{ijt} + \beta \int_{\tilde{z}_{jt+1}}^{\infty} \left(V^E_{ijt+1}(z) - V^U_{ijt+1} \right) h(z) dz + \beta V^U_{ijt+1} \tag{5}$$

$\pi^*_{ijt} = \pi_{ijt}(z_{it}, \hat{k}_{ijt})$ denotes the optimally chosen firm profit before interest payments on loans, i.e. the firm's profit at the optimally chosen loan amount ($\hat{k}_{ijt}$). $r_{ijt} = r_{ijt}(z_{it}, \hat{k}_{ijt})$ denotes the interest rate paid on the optimally chosen bank loan. Equation (5) states that the value function of a matched entrepreneur is the firm's current profit minus interest payments on bank loans, plus the discounted value of the surplus from remaining in the match in the next period, plus the discounted value function of an unmatched entrepreneur in the next period (since the match could be terminated).

3.2 Banks

Banks are active in all J credit markets that promise positive expected returns. Entry into a credit market continues until expected profits from entering are zero. The entry condition in credit market j is given by Equation (6):

$$\frac{\kappa}{q_{jt}} \leq \int_{\tilde{z}_{jt}}^{\infty} \left(V^B_{ijt}(z) + \tau_{ijt} \right) h(z) dz - \tau_{ijt} \tag{6}$$

Equation (6) states that banks will enter credit market j as long as the expected cost is smaller than or equal to the expected benefit of doing so. κ is the cost of posting an open credit line in a credit market in every period. q_{jt} is the probability of matching with an entrepreneur in market j. It is a decreasing function of the tightness of the credit market θ_{jt}. $V^B_{ijt}(z)$ is the value for the bank of continuing an existing credit relationship with a type-j firm with productivity z. τ_{ijt} is the cost that the bank incurs when terminating a lending relationship. It is time-varying, relationship-specific and type-dependent. Free entry means that Equation (6) holds with equality for all active credit markets at any time t.

The value of continuing an existing credit relationship for the bank is given by Equation (7):

$$V^B_{ijt}(z) = \hat{k}_{ijt} \left(r_{ijt} - \rho_{jt} \right) - \kappa^B_{jt} + \beta \int_{\tilde{z}_{jt+1}}^{\infty} \left[V^B_{ijt+1} + \tau_{ijt+1} \right] h(z) dz - \beta \tau_{ijt+1} \ . \tag{7}$$

κ^B_{jt} is the cost of maintaining a credit line in every period in credit market j and ρ_{jt} is the bank's funding cost for lending to type j. This cost encompasses all other specific characteristics observable to the bank and outside observers and is best described as the interest rate accounting for the riskiness of lending to firms of type j in credit market j. Equation (7) states that the value of continuing an existing credit relationship for the bank is equal to the loan amount multiplied by the interest margin, plus the discounted value of the surplus from continuing the match in the next period, plus the discounted value of the termination cost in the next period.

3.3 Solving for the interest rate

We assume that interest rates (r_{ijt}) are the result of a Nash bargaining process, in which the surplus of the match of the credit relationship is divided according to the bargaining power of the entrepreneur, η. The surplus split from new and continued credit relationships with a type-j firm with productivity z_{ijt} is given by Equation (8):

$$\eta \left(V_{ijt}^{B}(z) + \tau_{ijt} \right) = (1-\eta) \left(V_{ijt}^{E}(z) - V_{ijt}^{U} \right) \tag{8}$$

The resulting interest rate is given by Equation (9):

$$r_{ijt} = \eta \rho_{jt} + (1-\eta) \frac{(\pi_{ijt}^{*} - \pi_{0jt})}{\hat{k}_{ijt}} - \eta \frac{\kappa}{\hat{k}_{ijt}} \theta_{jt} + \eta \frac{\kappa_{jt}^{B}}{\hat{k}_{ijt}} - \eta \frac{(\tau_{ijt} + p_{jt} \tau_{kjt} - \beta \tau_{ijt+1})}{\hat{k}_{ijt}} \tag{9}$$

Equation (9) shows how firm characteristics, search frictions, and forbearance incentives affect interest rates. First, the interest rate paid by the firm increases in the bank's funding cost (ρ_{jt}). The effect is stronger when the firm has a higher bargaining weight η. A higher funding cost decreases the bank's surplus from a lending relationship. The bank takes a larger hit when the firm has a higher bargaining weight and therefore demands a higher interest rate. Second, the interest rate increases in the per-capital-unit extra profit before interest payments that the loan enables the firm to realise $\left(\frac{(\pi_{ijt}^{*} - \pi_{0jt})}{\hat{k}_{ijt}}\right)$. The higher the bargaining power of the bank, the larger the proportion of that extra per-unit profit the bank is able to appropriate through interest payments. Third, a higher cost of posting credit lines (κ) and higher credit market tightness (θ_{jt}), i.e. competition for unmatched firms, intuitively work to push interest rates downward. Their impact on rates is larger the larger the bargaining weight of the firm. The fourth term is similar to the third, but works in the opposite direction. A higher per-period cost of maintaining a credit line κ^{B} necessitates higher baseline interest rates to finance the bank's operations. Firms with more bargaining power are affected more strongly by this cost as they get a larger share of the surplus generated by the match. The fifth term captures the effect of forbearance incentives through various mechanisms. First, a higher per-capital-unit cost of termination in the current period $\left(\frac{\tau_{ijt}}{\hat{k}_{ijt}}\right)$ drives interest rates down. The effect is larger when the firm has a higher bargaining weight. This is intuitive since the termination cost is incurred by the bank in case of separation of existing relationships, affecting its threat point in the Nash bargaining. The resulting interest rate subsidy can be interpreted as the bargaining effect of the termination cost. Second, interest rates are driven down by the expected per-capital-unit cost of termination associated with a potential new match between the firm and another lender in the current period $\left(\frac{\tau_{kjt}}{\hat{k}_{ijt}}, k \neq i\right)$[8]. We can interpret this as the forbearance effect being magnified when competition among banks for

[8] This term appears in Equation (9) through the value function of an unmatched entrepreneur (which contains within it the possibility of a new match) and the assumption that termination costs are firm-specific.

credit lines is large and forbearance incentives are firm-specific rather than match-specific. Again, this effect is larger when the firm has a higher bargaining weight. Finally, discounted per-capital-unit termination costs on existing relationships in the next period $\left(\frac{\beta\tau_{ijt+1}}{\hat{k}_{ijt}}\right)$ increase interest rates. Banks foresee future termination costs and factor that into their current loan pricing.

Equation (9) leads to the following proposition:

Proposition 1. *The mean interest rate a firm pays on its bank loans is driven by the following empirically observable variables:*

a) A larger benefit generated by the bank loan increases mean interest rates.

b) Higher credit market tightness for a firm of type j reduces mean interest rates.

c) A higher funding rate and higher credit line maintenance cost for the bank results in higher mean interest rates paid by the firm.

d) Higher forbearance incentives in the current period reduce mean interest rates.

e) Higher forbearance incentives in the next period increase mean interest rate rates.

f) Current forbearance incentives are more important when credit market tightness is larger, assuming that incentives are firm-specific.

4 Empirical methodology

4.1 Difference-in-differences specifications

Forbearance programmes allow firms to either temporarily stop making payments on certain debt obligations or to obtain debt forgiveness in the form of, e.g., reduced interest rates. Therefore, a firm in receipt of forbearance will experience a decrease in the average interest rate on its debt obligations. Equation (9) serves as a guide for our empirical analysis of the impact of the SME Financing Facilitation Act on loan interest rates. According to Equation (9), the determinants are the average product of borrowed capital, the lender's funding cost, credit market tightness, and forbearance incentives. To capture the latter, we construct a treatment variable $Post_t \cdot TreatmentIntensity_i$ where $Post_t$ refers to the post-treatment period (starting in 2010) and $TreatmentIntensity_i$ is a measure of the exposure of firm i to treatment under the SME Financing Facilitation Act in 2009, prior to the introduction of the Act. The variable $TreatmentIntensity_i$ is the product of a dummy variable for treatment eligibility and the probability of firm i receiving treatment (any type of forbearance lending) conditional on eligibility, estimated based on relevant characteristics (see Section 5.6). Since almost all requests for loan restructurings were accepted, this probability of treatment can be seen as measuring the probability that firm i applies for support. To identify the treatment effects, we exploit the fact that the lender's decision to offer forbearance conditional on the firm requesting it is plausibly exogenous.

The firm's average interest rate on debt obligations is regressed on the treatment variable as per the following specification:

$$\begin{aligned} ln(r_{it}) = \alpha + \beta \cdot Post_t \cdot TreatmentIntensity_i + \Gamma X_{it} + f_i + f_t + ... \\ ... \sum_c \gamma_c \cdot Post_t \cdot X^c_{it} + \epsilon_{it} \end{aligned} \tag{10}$$

where $ln(r_{it})$ is the natural logarithm of firm i's average interest rate on debt obligations at time t. The variables f_i and f_t are firm and year fixed effects, respectively. The time fixed effects control for time-specific factors that affect all firms equally, including the Bank of Japan's policy rates which affect the lenders' cost of funds. The firm fixed effects control for firm-specific time-invariant characteristics. X_{it} is a vector of control variables, including the firm's average product of borrowed capital and credit market tightness for the size segment to which firm i belongs. It also includes firm i's share of bonds and accounts payable (trade credit) in total debt. These two variables are included because r_{it} measures the firm's average interest rate on all debt obligations (including loans, bonds, and accounts payable[9]) and we aim to isolate the impact of the Act on loan interest rates. Finally, X^c_{it} includes the characteristics of firm i which have been shown to statistically affect a firm's probability of treatment (demand-side determinants). The determinants of treatment exposure include the firm's leverage, credit score, return on assets, the natural logarithm of sales, the number of employees, and the firm's age (see Section 5.6 for details). $Post_t$ interacted with X^c_{it} controls for factors that may affect interest payments specifically in periods after the introduction of the Act, especially potentially confounding covariates that are known to be correlated with the probability of treatment (demand for treatment). Standard errors are clustered at the firm level[10].

Equation (10) delivers an estimate of the average treatment effect over the entire post-intervention period 2010-2018. However, the treatment effects are likely to vary substantially over time. Therefore, a dynamic exploration of the treatment effects is a better starting point for the back-of-the-envelope counterfactual exercises. The dynamic DiD specification is as follows:

$$\begin{aligned} ln(r_{it}) = \alpha + \sum_{t=T_0}^{T} \beta_t \cdot D_t \cdot TreatmentIntensity_i + \Gamma X_{it} + f_i + f_t + ... \\ ... \sum_c \gamma_c \cdot Post_t \cdot X^c_{it} + \epsilon_{it} \end{aligned} \tag{11}$$

where D_t represents year dummies, and the other variables are as described in Equation (10). T_0 is the first post-treatment year (2010) and T is 2018. The estimated annual treatment effects from

[9] The share of bonds and trade credit in total debt for our sample firms are on average 1.8% and 31.6%, respectively. The average share of bank loans in total debt is 53.4%. The share of commercial paper in total debt is on average negligible.

[10] Clustering at the industry level does not affect the results.

Equation (11) form the basis of the counterfactual back-of-the-envelope exercises.

We also build separate treatment exposure variables for two types of forbearance documented in the survey data from RIETI, namely payment deferrals (referred to as "financing") and debt forgiveness (see Section 5.6), and estimate the following model:

$$\begin{aligned} ln(r_{it}) = \alpha + \beta_{fi} \cdot Post_t \cdot TreatmentIntensity_i^{fi} + \beta_{fo} \cdot Post_t \cdot TreatmentIntensity_i^{fo} + ... \\ ...\Gamma X_{it} + f_i + f_t + \sum_c \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it} \end{aligned} \tag{12}$$

where $TreatmentIntensity_i^{fi}$ is a firm's exposure to deferrals and $TreatmentIntensity_i^{fo}$ is its exposure to debt forgiveness. Note that both treatment exposure variables are positively correlated. In addition, payment deferrals and debt forgiveness are not mutually exclusive. Hence, both variables are included concomitantly in the model to avoid omitted variable bias. In addition, the effects of deferrals and forgiveness are expected to differ in terms of dynamics. While payment deferrals are temporary by nature, debt forgiveness is expected to have more persistent effects on interest payments. Therefore, we estimate the following model:

$$\begin{aligned} ln(r_{it}) = \alpha + \sum_{t=T_0}^{T} \beta_{t,fi} \cdot D_t \cdot TreatmentIntensity_i^{fi} + \sum_{t=T_0}^{T} \beta_{t,fo} \cdot D_t \cdot TreatmentIntensity_i^{fo} + ... \\ ...\Gamma X_{it} + f_i + f_t + \sum_c \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it} \end{aligned} \tag{13}$$

While the estimates from Equations (10) and (11) are arguably well identified once we control for demand because banks were not given a choice in granting forbearance, those of Equations (12) and (13) may suffer from omitted variable bias. The reason is that we do not observe the process whereby the lender decides to offer either financing or forgiveness, conditional on a firm requesting support. This decision might be endogenous to unobservable firm characteristics (e.g., bargaining power), lender characteristics (e.g., relationship versus arm's length lender), as well as firm-lender match-specific characteristics (e.g., length of relationship, previous business affiliations). Therefore, we do not use these coefficients for the counterfactual exercises, but rather to shed some qualitative light on the dynamics of the coefficients in Equation (11).

4.2 Parallel trends assumption

A critical assumption in DiD is "parallel trends". The latter says that, if no treatment had occurred, the difference between the treated group and the untreated group would have stayed the same in the post-treatment period as it was in the pre-treatment period. While it is nearly impossible to provide definite proof that this assumption holds, we follow a variety of approaches. First, we visually inspect the trend in average interest rates pre- and post-treatment for the group of eligible and non-eligible firms. Second, we conduct a placebo test using a specification that interacts treatment exposure with a full set of time dummies (see e.g., Tripathy, 2020):

$$\begin{aligned} ln(r_{it}) = \alpha + \sum_{t=T_0^*}^{T} \beta_t \cdot D_t \cdot TreatmentIntensity_i + \Gamma X_{it} + f_i + f_t + ... \\ ... \sum_c \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it} \end{aligned} \tag{14}$$

All the variables are as described previously in Equation (11). T_0^* denotes the first sample year prior to the intervention (2007) and T denotes the last sample year (2018). In this specification, the "treatment effect" is estimated for each year from 2007 until 2018, using the last pre-treatment year (2009) as the benchmark. The β_t estimate for each time period shows whether changes in interest rates captured by exposure to the SME Financing Facilitation Act are driven by time trends or reflect statistically significant changes only in the post-treatment periods (2010-2018). If the parallel trends assumption is not true, the β_t estimated in periods before the introduction of the Act (the placebo years) would be statistically different from zero as well. Note that we conduct the placebo test using a single treatment exposure variable, which measures exposure to *any* type of forbearance (financing or forgiveness). This is because, as explained before, while the mandate for forbearance lending represents a plausibly exogenous treatment for qualifying firms that found themselves in trouble, the type of forbearance a firm receives upon treatment is more likely to be endogenous.

Finally, we test whether there is a differential trend in interest rates between eligible and non-eligible firms. We estimate Equation (15), which controls for the difference in time trend between the eligible and non-eligible groups[11]:

$$\begin{aligned} ln(r_{it}) = \alpha + \sum_{t=T_0}^{T} \beta_t \cdot D_t \cdot TreatmentIntensity_i + \theta \cdot E_i \cdot t + \Gamma X_{it} + f_i + f_t + ... \\ ... + \sum_c \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it} \end{aligned} \tag{15}$$

E_i is a dummy variable equal to one if firm i is eligible for loan forbearance under the Act in 2009, and zero otherwise. θ is the difference in the time trend between treatment and comparison groups. If the differential slope $\theta = 0$, we cannot reject the assumption of parallel trends.

4.3 Counterfactual exercises

We use the structure of the partial equilibrium model presented in Section 3 to conduct back-of-the-envelope counterfactual exercises to estimate the aggregate impact of mandated forbearance. We start with the profit maximization problem for optimal loan size $\hat{k}_{ijt}$ in Equation (3). The first-order condition for this problem defines the optimal capital stock as a function of the interest rate available

[11]See e.g., Muralidharan and Prakash (2017).

to the firm and changes in the interest rate when the capital stock expands:

$$\frac{\partial \pi_{i,j,t}}{\partial \hat{k}_{i,j,t}} = r_{i,j,t} + \hat{k}_{i,j,t} \frac{\partial r_{i,j,t}}{\partial \hat{k}_{i,j,t}} \tag{16}$$

Equation (16) shows that a decrease in the interest rate paid by firms due to forbearance must lead to an increase in the firms' capital stock. If the change in profits is larger than the change in interest rates as the capital stock expands, the profit maximisation problem has an optimal solution as the profits per unit of capital are marginally decreasing in the capital stock. Assuming that the firm treats the interest rate as exogenous when choosing its optimal capital stock, $\hat{k}_{i,j,t} \frac{\partial r_{i,j,t}}{\partial \hat{k}_{i,j,t}} = 0$, and the firm's marginal product is optimally chosen to equal the interest rate. Substituting the production function (1) and the interest rate (9) into Equation (16) then yields Equation (17):

$$k^*_{ijt} = \left(\frac{\alpha \gamma z_{it} l_{ijt}^{(1-\alpha)\gamma}}{r_{i,j,t}} \right)^{\frac{1}{1-\alpha\gamma}} \tag{17}$$

This specification allows us to perform tractable counterfactuals. Assuming labour input is kept constant[12], a change in the interest rate following a change in forbearance incentives affects the firm's capital stock according to Equation (18):

$$\Delta \log(k_{ijt}) = -\frac{1}{1-\alpha\gamma} \Delta \log(r_{i,j,t}) \tag{18}$$

Using Equation (18), we calculate the change in the firm's capital stock that results from the removal of the annual interest rate subsidy estimated using our dynamic DiD model in Equation (11). In doing so, we take into account each firm's exposure to treatment. Specifically, we multiply the estimated change in the interest rate by the firm's treatment intensity. We then calculate the firm's counterfactual capital stock. Using the production function, we then calculate the output each firm would have produced without the policy intervention, after adjusting their capital stock optimally to the interest rate change, given their estimated productivity. Once we have the firm-level counterfactuals, we calculate the counterfactual aggregate capital stock, capital productivity, and output. In doing so, we use sampling weights to ensure that the counterfactual population of firms is representative of the employment and industry distribution in the full TSR data set. We calculate counterfactual capital productivity as counterfactual aggregate output net of labour costs per counterfactual capital unit, namely:

$$Prod^{CF} = \frac{Y_t^{CF} - L_t w_t}{K_t^{CF}} \tag{19}$$

We perform the counterfactuals under two scenarios for capital reallocation. First, we assume that the capital freed up from treated firms by the removal of the policy, $K^{freed} = K_t - K_t^{CF}$, is not

[12] We use this as the baseline assumption as Japanese firms are well known for being reluctant to lay off staff during performance declines (See, e.g., Kang and Shivdasani, 1997). However, we relax the assumption in Section 9.

reallocated to other firms. Second, we assume that the freed-up capital is seamlessly reallocated to firms that produce at counterfactual aggregate capital productivity (i.e., the aggregate productivity of untreated firms). The extra output produced by K^{freed} at $Prod^{CF}$ is added to the counterfactual output without reallocation:

$$Y_{t,reall}^{CF} = Prod_t^{CF}(K_t - K_t^{CF}) + Y_t^{CF}$$

It should be noted that this exercise assumes instant and costless capital redistribution to firms producing with capital productivity at the same level as firms would produce without the policy intervention. This redistribution is unlikely given the usual frictions associated with firm entry and exit, scaling up production, and acquiring credit. Therefore, our two reallocation scenarios provide empirical upper bounds for the output loss or gain from the policy intervention.

5 Data

We draw upon data provided by Tokyo Shoko Research Ltd. (TSR, hereafter), a private market research and credit reporting agency in Japan, which surveys Japanese companies operating in virtually all industries of the economy. TSR data is widely used in empirical studies of the Japanese economy. From this data, we construct a rich firm-level panel data set for both listed and unlisted Japanese firms, covering all sectors of the economy for the years 2007-2018. We focus on the non-financial corporate sector. Our data set contains identifying information, including industry classification (Japanese Standard Industrial Classification 4-digit), location, and a range of data from the firms' income statements and balance sheets. Crucially, the data set contains information on the firms' stocks of interest-bearing debt obligations, interest payments, credit scores, and exits, as well as information on business affiliations, which allows us to determine whether a firm is eligible to request forbearance under the Act.

5.1 Sampling and representativeness

Although the TSR data set does not cover the universe of firms in Japan, it resembles closely the distribution of the Census data in terms of geographic coverage and firm size (see e.g. Hong et al., 2020). Therefore, the TSR data set is representative of aggregate developments in Japan. However, we cannot conduct our empirical analysis on the full TSR data set. Specifically, we can only work with firm-year observations that have all the data items required for the estimation of our difference-in-differences specifications. As described in the following sections, the data requirements are large.

Table 1 displays the number of annual observations in the full TSR data set and our empirical data set, i.e. firm-year observations with all the data requirements for Equation (11). The panel is unbalanced with some firms only appearing in some years, leading to an average of about 960 thousand firms observed annually in the full data set. Unfortunately, most firms do not report the data we need for the empirical analysis. The overall number of observations with these data items for 2007-2018 is about 439 thousand, or about 4% of the total number of observations in the full TSR. Our sample has on average around 36 thousand observations in each year. Out of the total of 439 thousand observations, about 96% are for firms with fewer than 250 employees.

Table 1: TSR and empirical sample

	Sample			TSR		
	All firms	SMEs	Large firms	All firms	SMEs	Large firms
2007	25,143	24,485	658	889,224	873,819	15,405
2008	33,497	32,799	698	902,720	884,830	17,890
2009	66,371	64,198	2,173	930,925	912,979	17,946
2010	47,812	45,893	1,919	955,417	935,247	20,170
2011	42,873	41,038	1,835	967,598	945,224	22,374
2012	39,634	37,871	1,763	988,808	964,741	24,067
2013	38,407	36,655	1,752	997,449	975,163	22,286
2014	36,250	34,526	1,724	977,673	956,737	20,936
2015	33,956	32,239	1,717	960,107	939,825	20,282
2016	30,126	28,442	1,684	966,965	946,376	20,589
2017	29,523	27,861	1,662	963,857	942,396	21,461
2018	14,927	13,449	1,478	974,982	952,836	22,146

Notes: Large firms are defined as having 250 employees or more, and SMEs are defined as having strictly fewer than 250 employees. The TSR numbers exclude Finance and insurance (as we focus on non-financial corporations) and other sectors not covered at all by our sample (real estate and goods rental and leasing; education, learning support; medical, health care and welfare; compound services; and services n.e.c.).

Although our sample only represents a small fraction of the TSR data set, it closely resembles its distribution in terms of geographic coverage, industry coverage, and firm size (See Supplemental Appendix B). To ensure that our counterfactual exercises are representative of the population of Japanese firms, we nevertheless compute sampling weights. We construct sampling cells defined by employment bands (strictly fewer than 250 employees, 250 employees or more), four-digit industries, and prefectures. The weight assigned to each firm in the counterfactuals is the inverse of its cell's sampling probability, defined as the ratio of total cell employment in the population (full TSR data set) and total cell employment in the empirical data set.[13]

5.2 Interest rates on debt obligations

The dependent variable in our DiD equations, $ln(r_{it})$, is the natural logarithm of the firm's average interest rate on all debt obligations. The average interest rate is calculated as the ratio of interest payments on interest-bearing debt obligations and their outstanding amount. While our focus is on

[13]In robustness checks, we use more granular employment cells. Specifically, we construct sampling cells defined by employment bands (fewer than 4 employees, 5-9, 10-19, 20-29, 30-49, 50-99, 100-299, 300-999, 1000-1999, more than 2000 employees), four-digit industries, and prefectures. This robustness check is not reported.

bank loans, interest-bearing debt obligations also encompass bonds, commercial paper, and accounts payable. While the share of commercial paper in total debt is on average negligible, bonds and accounts payable provide significant alternative sources of debt finance. Nevertheless, Japanese firms rely mainly on bank financing. Table 2 presents the sample average share of different types of debt in total debt for different categories of firms. Overall, bank debt represents 53.4% of total debt. The share of bank debt decreases with firm size but remains relatively high for all firm sizes. Firms eligible for forbearance under the Act have a higher share of bank debt than non-eligible firms, consistent with them being classified as SMEs by the Act. Trade credit is the second most important type of debt funding, at 31.6% overall. This share is relatively stable across size categories. Finally, bond debt plays a relatively minor role. These observations are in line with the literature that documents that Japan is a bank-based, rather than a capital market-oriented, economy (see e.g. Antoniou et al., 2008). In order to isolate the effect of the Act on loan interest rates, we control for each firm's debt structure in the DiD estimations. Specifically, the set of control variables X_{it} includes firm i's share of bonds and accounts payable (trade credit) in total debt.

Table 2: Share of different types of debt funding in total debt (%)

	Bank loans	**Bond debt**	**Trade credit**
All firms	53.4	1.8	31.6
Small firms	61.9	0.2	31.3
Medium-size firms	50.6	1.3	32.8
Large firms	47.2	5.2	29.3
Non-eligible firms	48.1	5.0	30.6
Eligible firms	54.4	1.1	31.8

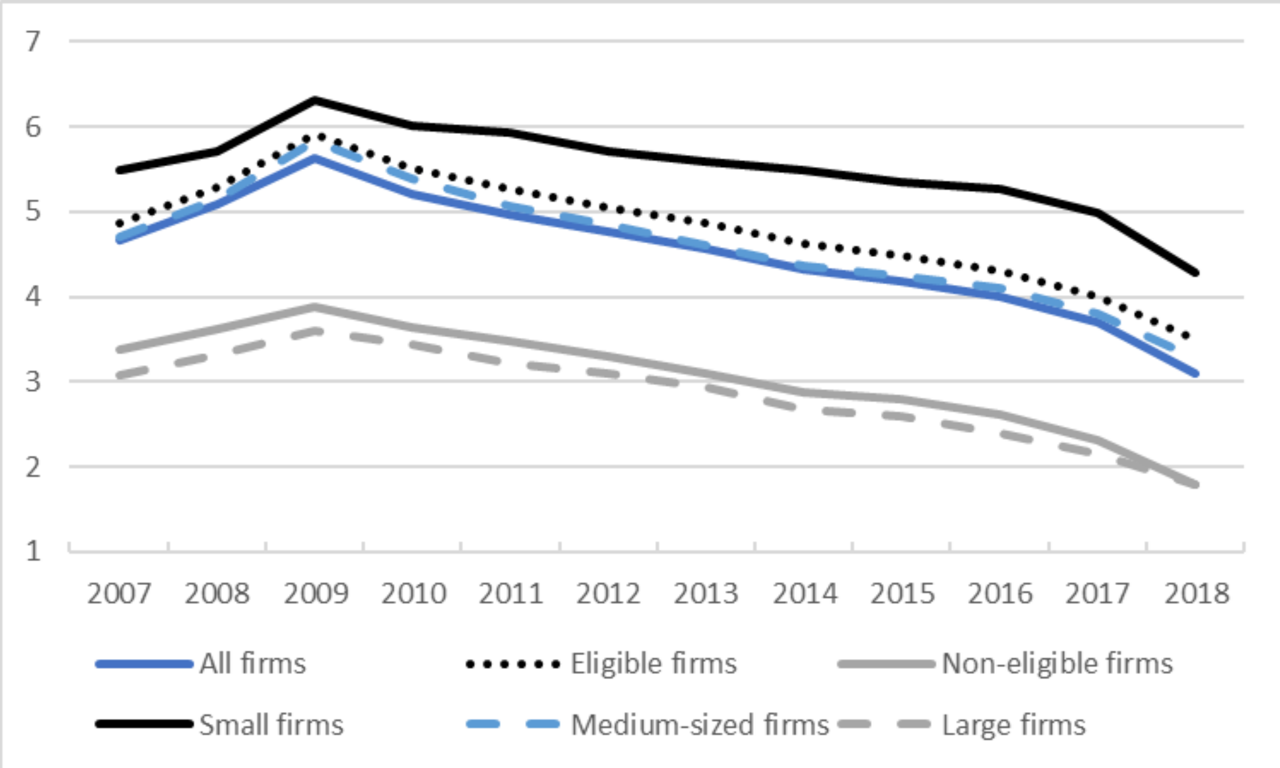
Notes: Share of different types of funding as a percentage of total debt in our sample. Size categories are those of the Tankan Survey of the Bank of Japan. Source: TSR.

Figure 2 below shows the evolution of the average interest rate on debt obligations in our sample. The average interest rate paid by firms that are eligible for loan forbearance under the SME Financing Facilitation Act is systematically higher than that paid by non-eligible firms. This is as expected since eligible firms are SMEs that typically face a higher cost of external finance than large firms, as confirmed by Figure 2. The interest rates paid by eligible and non-eligible firms, however, display very similar dynamics. There is an increase between 2007 and 2009, and then the trend reverses with rates declining from 2010 onwards until the end of the sample period.

5.3 Average product of borrowed capital

We define the average product of borrowed capital as the difference between the profit before interest payments which the firm makes using its entire capital stock (π_{ijt}) and its profit before interest payments when it only employs the part of the capital stock that is not financed by banks (π_{i0jt}), divided by the capital financed by banks ($\hat{k}_{ijt}$):

Figure 2: Average interest rate on debt obligations (%)



Note: Average interest rate on debt obligations in the sample. Size categories are those of the Tankan Survey of the Bank of Japan. Source: TSR.

$$APBK_{it} = \frac{(\pi^*_{ijt} - \pi_{i0jt})}{\hat{k}_{ijt}} = \frac{(z_i(k_i^{\alpha} l_i^{1-\alpha})^{\gamma}) - (z_i(k_{i,0}^{\alpha} l_i^{1-\alpha})^{\gamma})}{\hat{k}_{ijt}} \tag{20}$$

In other words, $APBK_{it}$ measures the average profit (in ¥) before interest payments generated by an extra unit of borrowed capital. Equation (20) assumes that labour input does not adjust in the short run so that the wage bill terms cancel out.[14] In the data, the capital stock not financed by banks ($k_{i,0}$) is calculated as the sum of all assets minus bank loans on the firm's balance sheet. It encompasses shareholder funds and non-bank debt (mainly bonds and trade credit). The parameters of the production function $y = z_i(l^{\alpha-1}k^{\alpha})^{\gamma}$ are estimated. Firm productivity, z, and the parameter α are estimated using the method of Wooldridge (2009). We estimate $\alpha = 0.283$. Further, we estimate that $\gamma = 0.963$ using a regression of productivity z on explained production factors ($l^{\alpha-1}k^{\alpha}$). Supplemental Appendix C.1 presents some descriptive statistics.

5.4 Credit scores

TSR assigns a credit score to each firm. Credit scores are recorded as an integer between 0 and 100. The credit score is the sum of the sub-scores on four aspects of firm performance: management quality

[14] We also estimate our model assuming adjustment of labour input with similar results in section 9.

(0-20 pts), growth measured by sales growth, profit growth, and the firm's product market prospects (0-25 pts), stability measured by balance sheet strength and relationships with lenders, suppliers and client firms (0-45 pts), and transparency and reputation (0-10 pts).[15] The TSR credit scores are known to be positively correlated with actual defaults (Miyakawa, Miyauchi, and Perez, 2017). TSR subscribers use the credit scores to determine the creditworthiness of corporate customers, especially when they provide trade credit to the latter. TSR classifies firms into five groups of creditworthiness according to their credit scores. Scores less than or equal to 29 are classified as "keikai" (caution). Scores between 30 and 49 are classified as "ichio keikai" (somewhat caution). Scores between 50 and 64 are categorized as "tasho chui" (attention). Scores between 65 and 79 are "bunan" (safe), and those between 80 and 100 are considered to be "keikai fuyo" (no risk). The credit scores of eligible firms are systematically lower (at 49 on average) than those of non-eligible firms (at 55.3 on average). However, the time series pattern is identical for eligible and non-eligible firms. Supplemental Appendix C.2 presents some descriptive statistics.

5.5 Credit market tightness

We use the Tankan survey of the Bank of Japan[16] to capture time-varying credit market tightness by size class. The survey is a short-term economic survey of private enterprises. Enterprise size is defined by the firm's amount of capital and divided into three categories. Large enterprises are defined as having a capital of 1 billion yen or more. Firms with capital from 100 million yen to less than 1 billion yen are classified as medium-sized enterprises, and those with capital from 20 million yen to less than 100 million yen are small enterprises. We classify all enterprises with a capital below 100 million yen as small firms. To capture time-varying credit market tightness, we use the indices measuring the lending attitude of banks as reported by private enterprises in the survey. A more positive index means that credit is being offered more easily to firms, which we interpret as higher credit market tightness, namely more credit lines on offer compared to the number of firms searching for credit. Figure 3 presents the indices that proxy for credit market tightness by size category. There is a sharp deterioration in lending conditions for all firms between 2007 and 2009, followed by a sharp recovery to pre-crisis levels until 2016 when the indices stabilise. The time series pattern is common to all firms, but small firms face systematically tighter conditions than large and medium-sized firms, as reflected by lower index values. There is a clear relaxation in lending attitudes following the implementation of the SME Financing Facilitation Act. Since this is likely to be driven by a multitude of factors, including the Act, it is important that we control for credit market tightness in our DiD regressions.

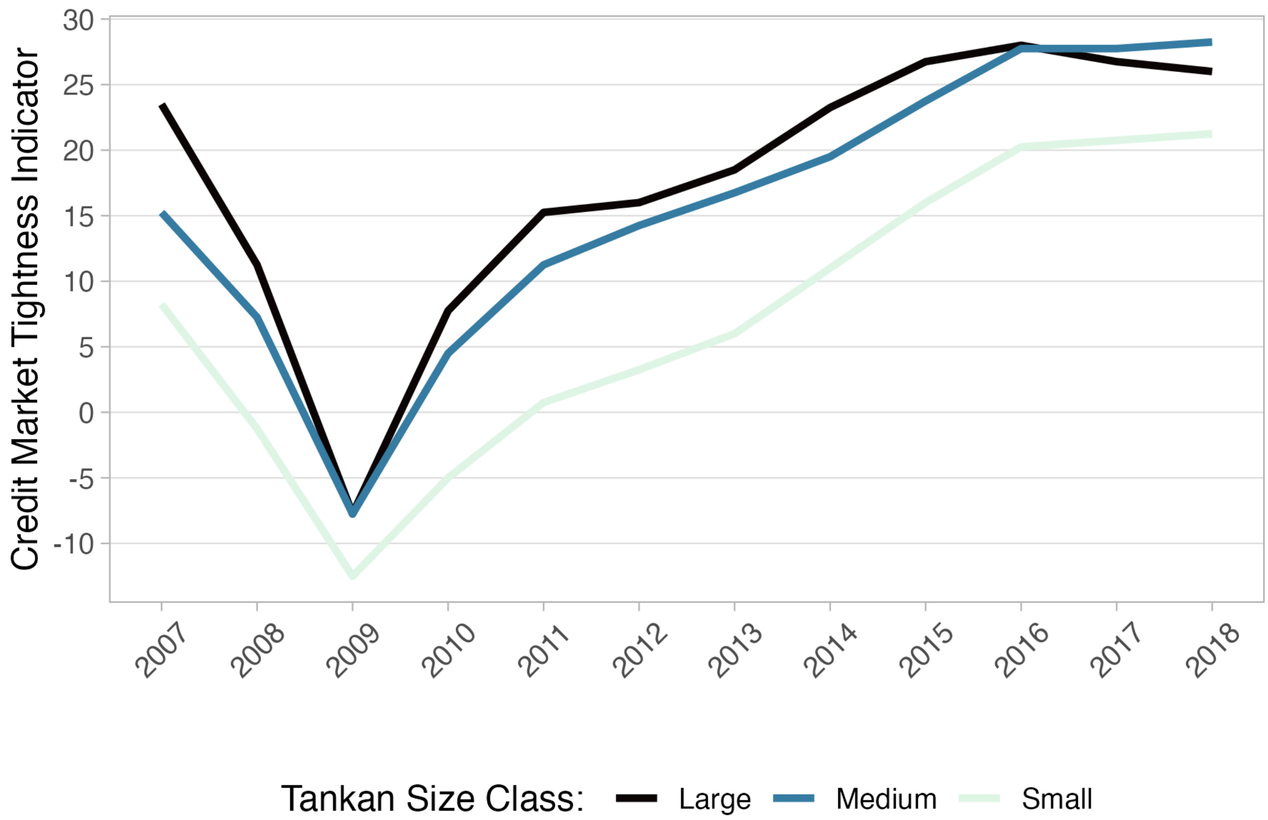
5.6 Treatment intensity

$TreatmentIntensity_i$ is a measure of the exposure of firm i to treatment under the SME Financing Facilitation Act in 2009, prior to the introduction of the Act. It is the product of a dummy variable

[15] An explanation of TSR credit scores is found at: `http://www.tsrnet.co.jp/guide/knowledge/glossary/ha_05.html` (in Japanese only).

[16] See `https://www.boj.or.jp/en/statistics/tk/index.htm`

Figure 3: Credit market tightness, by size category



Notes: Proxy for credit market tightness by size category. Credit market tightness is defined as the supply of credit lines over the demand for credit lines. Source: Tankan survey of the Bank of Japan.

for treatment eligibility and the probability of firm i receiving forbearance of any kind, conditional on eligibility.[17] Since almost all requests for loan restructurings were accepted, this probability of treatment is a good proxy for the probability that firm i applied for and received support. The dummy variable for treatment eligibility is equal to one when the firm was considered eligible for loan forbearance under the SME Financing Facilitation Act in 2009. The eligibility criteria are laid out in Supplemental Appendix A. The probability of firm i receiving treatment conditional on eligibility is calculated using the results of a Probit estimation on a sample of 3,298 eligible firms surveyed by the Research Institute of Economy, Trade and Industry (RIETI) (unpublished results from Ono and Yasuda, 2017).[18] The survey asked qualifying SMEs whether they had received debt forbearance from their lenders since December 2009. The survey also contains information on the type of debt forbearance that the firms received, namely payment deferral or "financing" and payment relief or "debt

[17] $TreatmentIntensity_i$ =P(firm i receives any type of forbearance and is eligible)=P(firm i is eligible)*P(firm i receives any type of forbearance | firm i is eligible). P(firm i is eligible) is either zero or one, and P(firm i receives any type of forbearance | firm i is eligible) is estimated with a Probit model.

[18] In October 2014, RIETI conducted the "Survey on the Aftermath of the SME Financing Facilitation Act" (Kinyuenkatsukaho Shuryogo ni okeru Kinyu Jittai Chosa). For papers discussing this survey, see Uesugi et al. (2015) (in Japanese) and Ono and Yasuda (2017).

forgiveness". Financing includes term extensions and deferrals of principal repayments. Forgiveness includes reductions in interest rates, partial write-offs, debt-equity swaps, and debt-debt swaps (see Table 1 in Ono and Yasuda, 2017). The RIETI survey was sent to eligible SMEs chosen from the TSR database. Respondents included firms that received loan forbearance (treated firms) and firms that did not (control firms). The sample consists of firms that requested forbearance but did not receive it (very small percentage), firms that requested forbearance and received it, and firms that did not apply for support.

Three Probit estimations were performed following the specification below:

$$Prob(Forbearance_i = 1) = \Phi\left(\sum_c \gamma_c \cdot X_i^c + D_j\right) \tag{21}$$

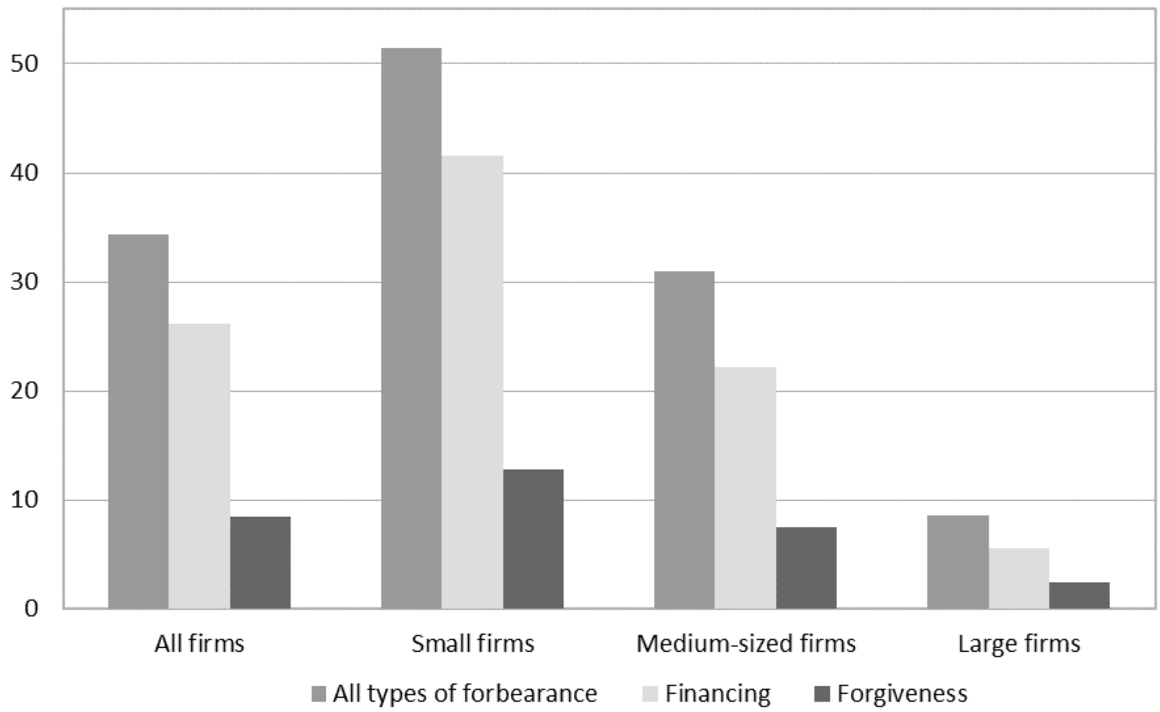
The dependent variable $Forbearance_i$ is, in turn, a dummy equal to 1 if the firm reported having received forbearance of any type, financing, or debt forgiveness. The control variables X_i^c are taken from the TSR database and relate to the financial year 2008-2009 before the SME Financing Facilitation Act applied. They include the firm's leverage, credit score, return on assets, the natural logarithm of sales, the number of employees, and the firm's age. The leverage ratio is defined as a firm's interest-accruing liabilities divided by its total assets. It is a proxy for debt overhang. The firm's credit score is the TSR score (1-100 points) presented in Section 5.4. Return on assets is defined as operating profits divided by total assets. The firm's score and ROA are proxies for a firm's net present value. The natural logarithm of gross annual sales and the number of employees are measures of size. It is important to control for firm size, since the transaction costs involved in debt renegotiations may be higher for smaller firms. Finally, firm age is measured as the number of years since a firm was established. D_j are industry fixed effects. The results are in Table 9 in Appendix A (unpublished results from Ono and Yasuda, 2017). In general, SMEs with higher leverage, ROA, and (marginally) sales are more likely to receive loan forbearance. By contrast, firms with a higher credit score and older firms are less likely to receive any kind of forbearance. There are some noticeable differences between some coefficients on financing and forgiveness. The coefficient on ROA is only significant for forgiveness, indicating that banks choose debt forgiveness for more profitable firms. In addition, age is only significant for financing, indicating that banks choose debt forgiveness for older firms.

We use the Probit coefficients from Table 9 to estimate firm-level probabilities of treatment conditional on eligibility (forbearance of any type, financing, and forgiveness) using 2009 TSR data on the explanatory variables in the Probit model. These probabilities are multiplied with the dummy for eligibility based on the firm's characteristics in 2009.[19] Firms that are not eligible have a treatment intensity of zero, while eligible firms have a treatment intensity that is equal to the probability of treatment from the Probit model. Figure 4 shows how average treatment intensities vary by Tankan size class for any type of forbearance, financing, and forgiveness. Note that an SME according to the

[19] In other words, a firm's treatment intensity is constant in the empirical analysis as it is based on 2009 data.

SME Financing Facilitation Act is not necessarily an SME according to the Tankan surveys as they use different definitions. Hence the average treatment intensity for large firms is not zero.

Figure 4: Average treatment intensity, by Tankan size category



Notes: Average treatment intensity for different size classes. Size categories are those of the Tankan Survey of the Bank of Japan. Source: TSR, unpublished results from Ono and Yasuda (2017).

The average treatment intensity for all firms is 34.3% overall, 26.1% for financing and 8.5% for forgiveness. Treatment intensities are on average much larger for financing in line with the fact that financing was much more prevalent than forgiveness (see Table 1 in Ono and Yasuda, 2017). Debt forgiveness only constitutes 25.1% of all instances of forbearance lending recorded in the RIETI survey, with reductions in interest rate representing 16.3% of the total. As expected, treatment intensities increase as firm size decreases. The average treatment intensity for small firms is 51.4% overall. For medium-sized and large firms, it is 31% and 8.5% respectively.

5.7 Zombie firms

Zombie firms are generally defined as non-viable firms which, in the absence of financial support measures, would default on their debt obligations and fail. This definition hinges on two criteria: The firm (i) is in financial distress and (ii) receives financial support that keeps it alive (see, e.g., Acharya et al., 2022; Álvarez et al., 2023). In this paper, we construct a zombie dummy variable based on the definition of Fukuda and Nakamura (2013) (henceforth, FN (2013)) because it allows us to precisely combine the two criteria of subsidized credit and financial distress. FN (2013) combine the subsidized credit criterion of Caballero et al. (2008) (henceforth CHK (2008)) with measures of financial distress (low operating income and high leverage) as well as new debt issuance.

Following CHK (2008), we first compare each firm's interest payments to a benchmark level of interest payments that is calculated assuming the lowest possible rate of interest for a healthy borrower (lower bound interest rate). If a firm's interest payments are below this lower bound, a firm is understood as receiving subsidized credit. The calculation of this lower bound is described in Supplemental Appendix D. We classify a firm as a zombie in period t if (i) it receives subsidized credit and the three-year moving average of its ROA is below the three-year moving average of the lower bound interest rate[20], or (ii) it does not receive subsidized credit in period t, but the three-year moving average of its ROA is below the three-year moving average of the lower bound interest rate in period t, its leverage (total debt over total assets) is above the sample's 90th percentile in period $t-1$[21], and its bank borrowing increased between $t-1$ and t. This means that profitable firms are excluded from being categorized as zombies, even if they might be receiving subsidized credit. It also means that financially distressed firms that do not receive subsidized credit, but nevertheless receive new credit from their lenders, are classified as zombies. Our measure therefore goes a long way in avoiding misclassifications.

Figure 5 displays the percentage of zombie firms over time in our sample. The proportion of zombies among firms that were eligible for loan forbearance in 2009 is on average about 11% over the sample period, while it is about 9% for non-eligible firms. Both eligible and non-eligible firms experienced a decrease in the percentage of zombie firms over the sample period - by 4.5pp for eligible firms and 2.8pp for non-eligible ones. The proportion of zombie firms is highest for small firms, and lowest for medium-sized firms. Perhaps surprisingly, the proportion of zombie firms is on average 10% for large firms. Firms of all sizes experienced a sharp decrease in the percentage of zombie firms between 2008 and 2017[22]. Firms classified as zombies in 2009 also have a slightly higher average treatment intensity than non-zombies - with an average probability of being treated conditional on eligibility (irrespective of the type of forbearance) of 33.7% for zombies versus 30.5% for non-zombies.[23]

For comparison, we also use the definitions of Schivardi et al. (2022) and Faria-e-Castro et al. (2024). Both measures classify a firm as a zombie if it has high leverage and low productivity. Importantly, they do not rely at all on interest rates.[24] We classify a firm as a zombie if the three-year moving average

[20]The criterion in FN (2013) is: earnings before interest and taxes (EBIT) are below the hypothetical risk-free interest payments in period t. We work with moving averages to avoid misclassifying a firm due to temporary decreases or increases in profits, in the spirit of Imai (2016).

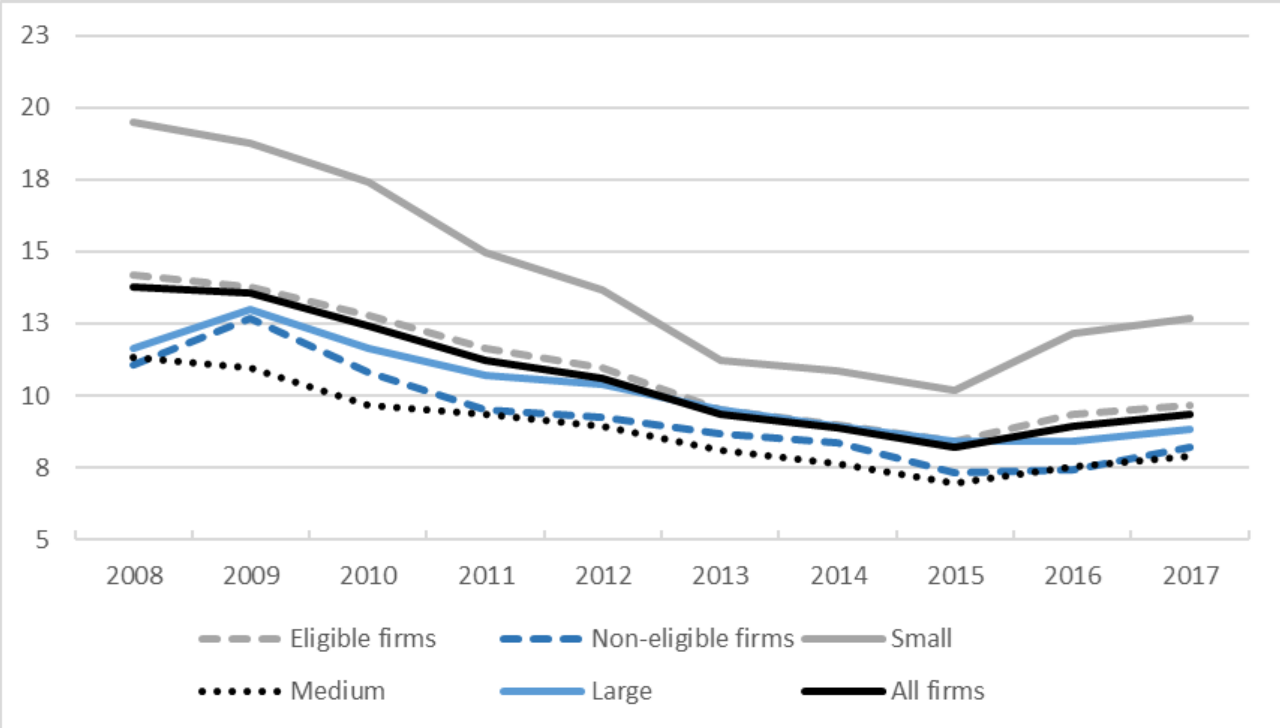
[21]FN (2013) use a 20% leverage threshold. We use the sample's 90th percentile to make the threshold specific to our sample.

[22]This is in line with existing evidence on the zombie share according to the FN definition (see, e.g., Honda et al., 2023).

[23]A similar pattern is found for financing - with an average probability of being treated conditional on eligibility of 26.4% for zombies versus 22.7% for non-zombies. The average treatment intensities for debt forgiveness were 9% for zombies and 7.5% for non-zombies.

[24]Schivardi et al. (2022) classify a firm as a zombie in any given year if its return on assets (ROA), defined as

Figure 5: Proportion of zombie firms over time (%)



Notes: Percentage of zombie firms over time. Zombies are defined as described above, based on FN (2013). Source: TSR.

of its ROA is below the three-year moving average of the lower bound interest rate and its leverage is above the 90th percentile for that year. The fraction of zombies is systematically higher according to our definition as there is a large number of firms with subsidized credit and low productivity, but not necessarily high leverage (above the 90th percentile). However, both definitions indicate a declining incidence of zombies in our sample (See Supplemental Appendix C.4).

6 The effect of the SME Financing Facilitation Act

6.1 Difference-in-differences estimation results

Table 3 presents the results of our DiD estimations. The first row presents the estimates of the average treatment effects from Equations (10) and (12), while the remaining rows present the annual treatment effects estimated according to Equations (11) and (13). Column (1) examines all types of loan forbearance, whereas columns (2) and (3) zoom in on financing and forgiveness. The average treatment effect for 2010-2018 is negative and highly significant overall, and for both financing and

the three-year moving average of earnings before interest and taxes (EBIT) over total assets, is below the cost of capital for the safest borrowers, defined as the three-year moving average prime rate, and its leverage exceeds 40%. Faria-e-Castro (2024) classify a firm as a zombie if it has a leverage ratio above the 90th percentile and ROA below the 10th percentile for that year.

forgiveness in isolation. We estimate that loan forbearance under the SME Financing Facilitation Act predicts an average decrease in the interest rate on bank loans of 18.5% for treated firms. While financing predicts an average decrease of around 11% for treated firms, debt forgiveness is associated with a much larger subsidy of 39%. We would indeed expect a much larger effect of debt forgiveness as it encompasses permanent reductions in principal and interest rates. Financing is mainly deferrals, which are temporary by nature. These coefficients represent the average effect of the Act for treated firms, i.e. firms with a treatment exposure equal to one. However, the average treatment exposures in the sample are 34% for all types of forbearance, 26% for financing, and 8% for forgiveness. At average treatment intensities, forbearance of any type is found to decrease loan interest rates by about 6%, while financing and forgiveness in isolation both reduce interest rates by about 3%.

The remaining rows of column (1) in Table 3 present the annual treatment effects used in the counterfactuals. The treatment effect is significantly negative in every post-intervention year from 2010 to 2015 for all types of forbearance. The effect is larger in the years closer to the implementation of the Act and fades away over time. It becomes insignificant in 2016, and turns significantly positive in 2017-2018. Since most forbearance is granted in the form of deferrals (see Table 1 in Ono and Yasuda, 2017), it is intuitive that the overall effect should decrease in magnitude over time and ultimately become positive. While payment deferrals have a large impact on interest payments in the short run, interest payments will kick in again after a while at the previously prevailing interest rate, showing up as a positive treatment effect. This is confirmed when we look at the results on financing in column (2). The treatment effect is significantly negative from 2010 to 2013 (when the Act expired), and fades away over time. It is insignificant in 2014-15, and turns significantly positive in 2016-2018. This indicates that firms that received payment deferrals experienced a period of subsidized credit before returning to higher interest rates around 2016. The results on forgiveness in column (3) show that forgiveness had a significantly negative effect on interest rates in every post-treatment period, except 2011. The magnitude of the treatment effect increases over time. Finally, Table 3 shows that the average product of borrowed capital is positively correlated with interest rates - although the coefficients are not significant. The model suggests that the higher the bargaining power of the bank, the larger the proportion of the average product of borrowed capital the bank is able to appropriate through interest payments. Therefore, the results indicate that the bargaining power of Japanese banks was limited during the sample period, in line with strong political pressure on banks to keep lending to struggling SMEs on favourable terms. By contrast, credit market tightness is significantly negatively correlated with interest rates - as predicted by the model.

6.2 Test of the parallel trends assumption

We start with a visual examination of the parallel trends assumption. The latter says that, if no treatment had occurred, the difference between the treated group and the untreated group would have stayed the same in the post-treatment period as it was in the pre-treatment period. Figure 6

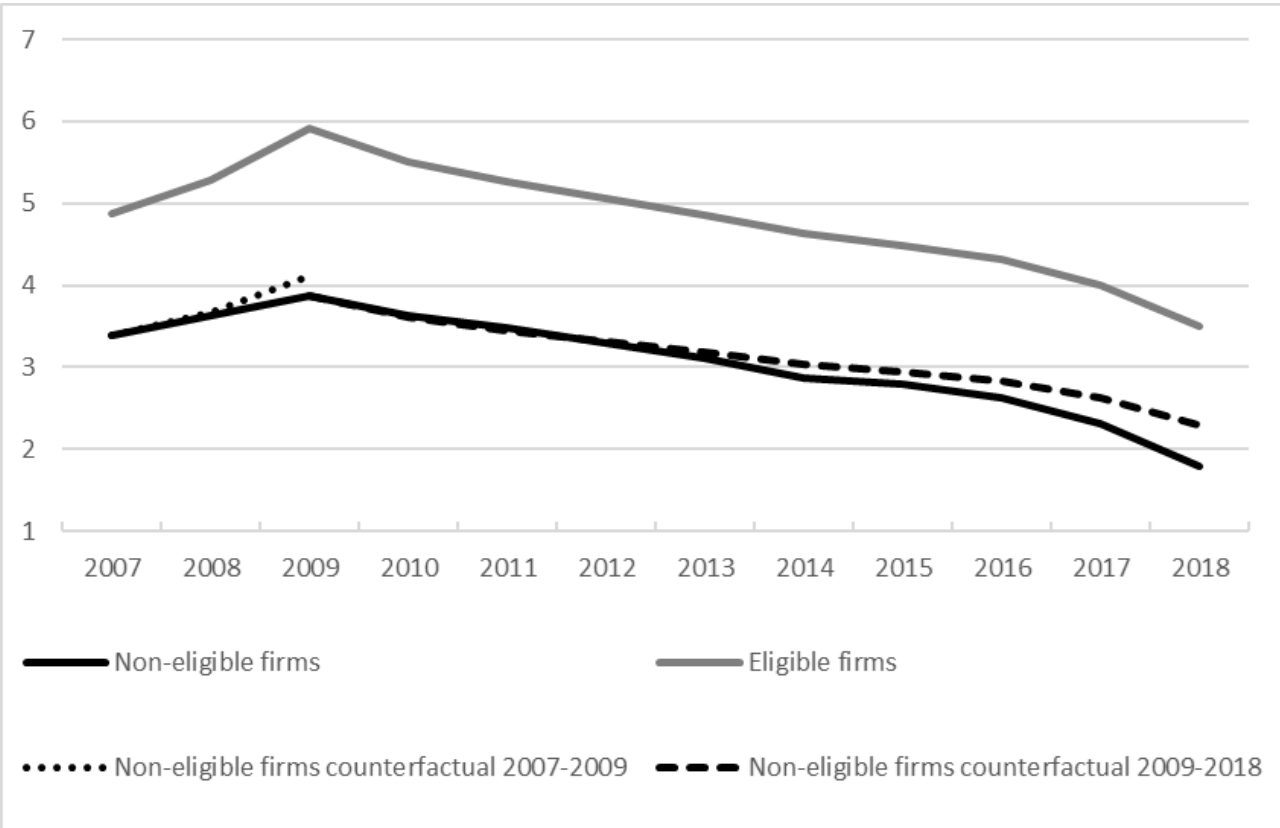
Table 3: Treatment effects on interest rates

	(1) Any forbearance	(2) Financing	(3) Forgiveness	(4) Placebo test	(5) Trend differential
2010-2018	-0.185*** (0.0212)	-0.114*** (0.0344)	-0.394*** (0.0941)	NA	NA
2007	NA	NA	NA	-0.0441** (0.0216)	NA
2008	NA	NA	NA	-0.0262 (0.0169)	NA
2010	-0.426*** (0.0207)	-0.432*** (0.0334)	-0.206** (0.0920)	-0.441*** (0.0206)	-0.430*** (0.0209)
2011	-0.328*** (0.0225)	-0.341*** (0.0373)	-0.156 (0.105)	-0.344*** (0.0226)	-0.336*** (0.0230)
2012	-0.277*** (0.0236)	-0.237*** (0.0406)	-0.337*** (0.119)	-0.292*** (0.0235)	-0.288*** (0.0245)
2013	-0.227*** (0.0246)	-0.177*** (0.0422)	-0.355*** (0.119)	-0.243*** (0.0244)	-0.242*** (0.0261)
2014	-0.114*** (0.0250)	-0.00916 (0.0472)	-0.462*** (0.139)	-0.129*** (0.0249)	-0.133*** (0.0273)
2015	-0.0745*** (0.0260)	0.0227 (0.0494)	-0.430*** (0.146)	-0.0892*** (0.0259)	-0.0971*** (0.0290)
2016	0.0139 (0.0277)	0.177*** (0.0527)	-0.631*** (0.155)	0.0000814 (0.0274)	-0.0125 (0.0314)
2017	0.136*** (0.0288)	0.291*** (0.0560)	-0.557*** (0.170)	0.123*** (0.0285)	0.106*** (0.0332)
2018	0.239*** (0.0357)	0.451*** (0.0810)	-0.726*** (0.263)	0.227*** (0.0354)	0.203*** (0.0405)
Average product of borrowed capital	0.0406 (0.0280)	0.0388 (0.0271)		0.0405 (0.0279)	0.0405 (0.0279)
Credit market tightness	-0.00709*** (0.000720)	-0.00675*** (0.000717)		-0.00740*** (0.000757)	-0.00751*** (0.000732)
Trend	NA	NA	NA	NA	0.00460** (0.00184)
Observations	437,219	437,219	437,219	437,219	437,219
Firm fixed effects	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes
Post · Confounders	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. Standard errors clustered at the firm level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

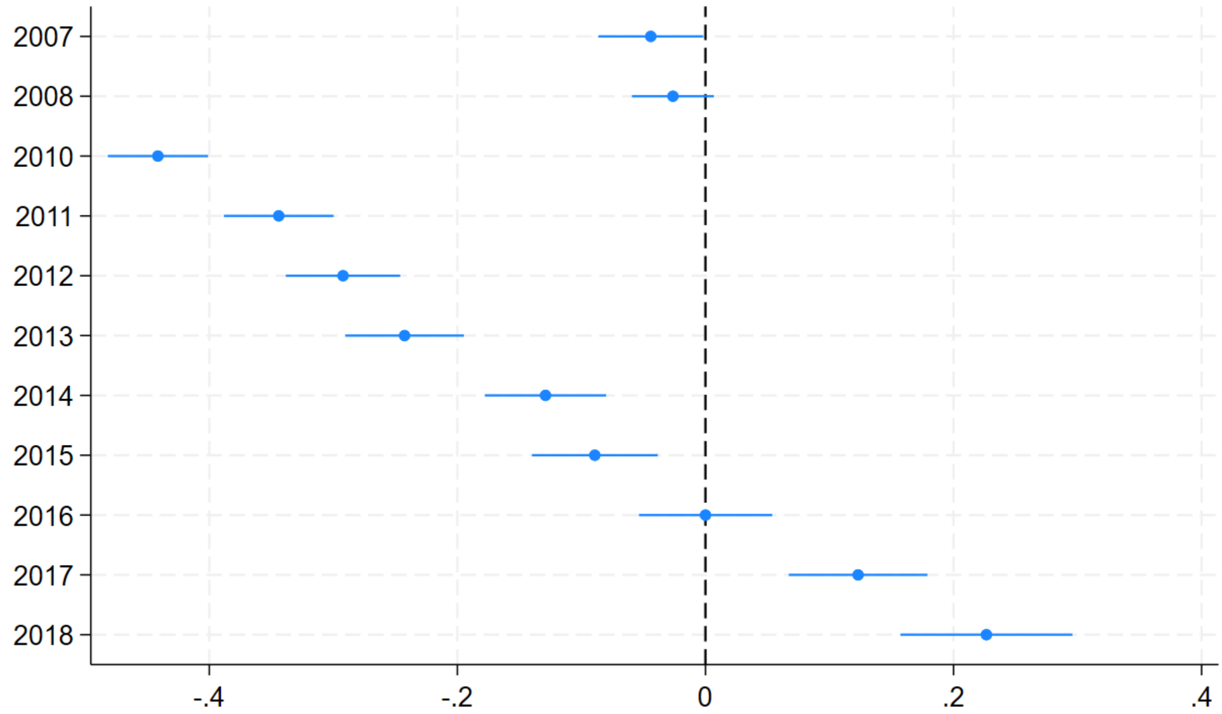
shows the average interest rate on debt obligations for eligible firms (solid grey line) and non-eligible firms (solid black line) over time. The average interest rate paid by firms that are eligible for loan forbearance under the Act is systematically higher than that paid by non-eligible firms. This is as expected since eligible firms are SMEs that typically face a higher cost of external finance than large firms. The interest rates paid by eligible and non-eligible firms, however, display very similar dynamics. There is a sharp increase between 2007 and 2009, and then the trend reverses with rates declining from 2010 onwards until the end of the sample period. Importantly, the pre-treatment trends look very similar. A closer inspection, however, points to a small trend differential which might bias our results *against* finding significantly negative treatment effects. The dotted black line shows the counterfactual path of the average interest rate for non-eligible firms if they had experienced the same year-on-year percentage change as eligible firms in the pre-treatment period. This shows that eligible firms experienced a sharper increase in interest rates on average in the pre-treatment period. The dashed black line shows the counterfactual path of the average interest rate for non-eligible firms if they had experienced the same year-on-year percentage change as eligible firms in the post-treatment period. This shows that eligible firms experienced a smaller decrease in interest rates on average in the post-treatment period. This indicates that eligible firms might have a positive trend differential relative to non-eligible firms, which predates treatment and continues thereafter. If not accounted for, this might create a downward bias on our estimated negative treatment effects, something we investigate below.

Figure 6: Average interest rates over time, by eligibility status (in %)



Next, we turn to the placebo test in Equation (14). The results are in column (4) in Table 3 and plotted in Figure 7. Here, the "treatment effect" is estimated for each year from 2007 until 2018, using the last pre-treatment year (2009) as the benchmark. If the parallel trends assumption were true, the β_t estimated in 2007 and 2008, before the introduction of the Act, would not be statistically different from zero. The results show that while there is no significant "treatment effect" in 2008, there is a small significantly negative effect in 2007. While this is evidence against the parallel trends assumption, the effect is very small in magnitude in comparison to the negative treatment effects in 2010-2015. Crucially, a Wald test for the joint significance of the "treatment effects" in 2007 and 2008 indicates that the null hypothesis that the latter are jointly equal to zero cannot be rejected[25]. This is in contrast to the significantly negative average treatment effect for the post-intervention period in the first row of Table 3.

Figure 7: Event-study plot



Notes: While the "treatment effect" is significant in 2007, a Wald test for the joint significance of the "treatment effects" in 2007 and 2008 indicates that the null hypothesis that the latter are jointly equal to zero cannot be rejected.

Finally, column (5) in Table 3 presents the results of estimating Equation (15). There is a small and significantly positive differential trend (in line with Figure 6). The inclusion of the trend does not affect the qualitative results. However, the treatment effects are estimated to be slightly larger (more negative). While a positive trend differential is a violation of the parallel trends assumption, it is a violation that can bias the results *against* finding significantly negative treatment effects. Hence, the results of column (1) provide a lower bound estimate of the credit subsidy provided by the Act.

7 Counterfactual exercises

In this section, we present the results of the back-of-the-envelope counterfactual exercises described in Section 4.3. We multiply the estimated change in the interest rate resulting from removing the

[25] $F(2, 66224) = 2.25$ and $Prob > F = 0.1050$.

policy by each firm's treatment intensity, and compute each firm's counterfactual capital stock using Equation (18). In doing so, we truncate changes in the capital stock at 15%. This is to allow only for realistic changes in the capital stock. We calculate each firm's counterfactual output using the production function and estimated firm-level TFP. In aggregating, we use sampling weights to ensure that the counterfactual population of firms is representative of the full TSR data set.

Table 4 presents the results from removing the annual treatment effects in column (1) of Table 3.[26] The percentages show the annual deviations between counterfactual aggregate output, capital stock, and capital productivity and their observed equivalents. The first row of Table 4 shows the percentage deviation between the aggregate counterfactual capital stock and the actual aggregate capital stock. The results point to a substantial effect of removing the interest rate subsidy generated by the Act. The aggregate capital stock would have declined by 4.22% in 2010 if the policy had not been enacted. On average, the policy boosted the aggregate capital stock by about 1.4% during 2010-2018. The gains from the policy fade over time and turn into losses from 2016 onward. We interpret these dynamics as resulting from the fact that payment deferrals only generate a temporary subsidy, after which firms return to higher interest rates (possibly higher than pre-treatment).[27] In terms of the model, this indicates a weakening of current forbearance incentives relative to the future. Equation (9) shows that forbearance incentives can generate a tax, as opposed to a subsidy, when the per-capital-unit termination cost in the current period is small relative to the discounted per-capital-unit termination cost in the next period. In other words, a tax can arise if banks have weak forbearance incentives in the present but price in high severance costs in the future.

Table 4: Effect of removing the estimated forbearance incentives (in %) - fixed labour

Counterfactuals - % change	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Capital stock	-4.22 %	-3.76 %	-3.46 %	-3.43 %	-1.58 %	-1.07 %	0.20 %	2.09 %	2.97 %	-1.36 %
Capital productivity	1.47 %	1.38 %	1.12 %	1.23 %	0.53 %	0.36 %	-0.07 %	-0.76 %	-0.87 %	0.49 %
Output, without reallocation	-8.30 %	-6.64 %	-5.86 %	-5.59 %	-2.44 %	-1.66 %	0.30 %	2.89 %	4.42 %	-2.54 %
Output, with reallocation	4.78 %	5.86 %	2.91 %	1.53 %	-0.36 %	-1.64 %	0.00 %	0.00 %	0.00 %	1.45 %

The second row of Table 4 shows that a higher capital stock came at the expense of lower capital productivity. On average, the Act depressed productivity by about 0.5%. The productivity losses decrease over time as the interest rate subsidy and its impact on the capital stock fade. In addition,

[26] Individual counterfactuals for forgiveness and financing (columns (2) and (3) of Table 4) are in Supplemental Appendix F.2.

[27] Supplemental Appendix F.2 clearly shows that this reversal is driven by financing.

treated firms became more productive in later years (see also Table 5). The third row indicates that the policy intervention protected output, which would have otherwise dropped by about 8.3% in 2010. On average, the policy is estimated to have boosted output by about 2.5%. This assumes that the capital freed up by the policy's hypothetical removal is not reallocated to other firms. If we assume that the freed up capital is costlessly and instantaneously reallocated to new firms similar to the ones that were not eligible for treatment, the policy is estimated to have decreased output by around 4.8% in 2010 and by about 1.5% on average.

It should be noted that the reallocation results assume instant and costless capital redistribution to firms producing with capital productivity at the same level as firms would produce without the policy intervention. Such a redistribution is unlikely given the usual frictions in the cost of firm entry, firm exit, scaling up output and acquiring credit usually observed. Furthermore, capital reallocation is procyclical, i.e. depressed during recessions (e.g., Caballero and Hammour, 2005; Eisfeldt and Rampini, 2006). Empirical evidence suggests that restructuring is lower during recessions than during normal times (Caballero and Hammour, 2005). As expected, liquidations are higher during downturns but they only result in an increased level of restructuring if they are followed by more creation during the recovery. Instead, the empirical evidence suggests that recoveries tend to occur through a lower rate of destruction. Caballero and Hammour (2005) suggest that during a recession and its aftermath less financing is available, which limits creation, so that the recovery must occur through less destruction. Similarly, Eisfeldt and Rampini (2006) provide evidence that capital reallocation is procyclical, i.e. depressed during recessions. Our reallocation counterfactuals therefore provide an upper-bound estimate of output losses from the policy intervention.

8 Zombification

In this section, we examine whether the SME Financing Facilitation Act encouraged zombification. Using a DiD approach, we examine whether the Act affected the probability that a firm is classified as a zombie according to the two definitions presented in Section 5.7. We then turn to firm-level performance, specifically TFP, the interest coverage ratio, and the probability of exit. If banks granted forbearance to unviable firms, we should observe a positive treatment effect on the probability of zombieness. We should also expect a negative treatment effect on TFP. If, on the other hand, banks granted forbearance to illiquid but viable firms, which put in place successful restructuring plans, we should observe a negative treatment effect on the probability of zombieness and a positive treatment effect on TFP. Irrespective of whether forbearance was granted to viable or unviable firms, we expect a negative effect on exit and a positive impact on the interest coverage ratio.

8.1 Loan forbearance and zombie status

We explore whether a higher treatment intensity in 2009 resulted in a higher probability of being classified as a zombie in the post-intervention years. We perform a Logit DiD estimation with firm and year fixed effects where the natural logarithm of the odds that the zombie dummy variable equals one are modelled as follows:

$$\begin{aligned} ln\left(\frac{P_{it}}{1-P_{it}}\right) = \alpha + \sum_{t=T_0}^{T} \beta_t \cdot D_t \cdot TreatmentIntensity_i + f_i + f_t + ... \\ ... + \sum_c \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it} \end{aligned} \quad (22)$$

where P_{it} is the probability that the zombie dummy equals one for firm i at time t. The sample is restricted to 2008-2017 due to data requirements. Standard errors are robust. The results (alongside the corresponding placebo tests) are in Table 5. Column (1) uses the zombie definition based on FN (2013) (Distress & interest rates), while column (3) uses the definition based on Schivardi et al. (2022) and Faria-e-Castro et al. (2024) (Distress only). The logistic regression coefficients give the change in the log odds of the outcome for a one unit increase in the predictor variable. For ease of interpretation, we convert those logit scale coefficients to a probability scale instead, and report the average marginal effects (AME)[28].

The estimated AME are significantly negative in every post-treatment period for both zombie definitions. For example, an AME of -0.3991 for 2010 in column (1) means that on average, a treated firm is about 40 percentage points less likely to become a zombie according to the definition of FN (2013). The treatment effects are generally larger in magnitude when we use the zombie dummy based solely on financial distress. This indicates that exposure to the policy had a strong positive impact on several dimensions of firm performance. Our results are in stark contrast with the findings of the literature on the Lost Decade. Despite the fact that zombie firms were more likely to receive loan forbearance, we find that loan forbearance is associated with a lower probability of zombieness in the post-treatment period.

This echoes the findings of Fukuda and Nakamura (2011), which highlight that most zombie firms during the Lost Decade recovered during the first half of the 2000s, thanks in part to successful corporate restructuring strategies. The fact that banks were only allowed to exclude the restructured loans from their reported NPLs if restructuring plans were put in place suggests that we should expect firms in receipt of forbearance to have implemented business turnaround plans. These either prevented them from becoming zombies or lifted them out of zombieness. The placebo test for the zombie dummy based on FN (2013) shows that there is a positive "treatment effect" in 2008. In addition, there is no significant "treatment effect" for the zombie variable based on financial distress alone. Crucially, the treatment effects become significantly negative from 2010 onward. The estimated "treatment effect" for 2008 picks up the fact that treated firms were significantly more likely to be zombies in the pre-intervention period. While this potentially indicates a violation of the parallel trends assumption, this should at worst bias our results against finding a significantly negative treatment effect in the

[28]The AME can be thought of as a numerical derivative that computes the derivative of the probability of zombieness for $D_t \cdot TreatmentIntensity_i$ for each observation using the observed values of the covariates and takes the average over the observations.

Table 5: Loan forbearance and zombification

	(1) Zombie - Ditress & interest rates	(2) Placebo	(3) Zombie - Distress only	(4) Placebo
2008		0.0857*** (0.0326)		-0.0717 (0.0441)
2010	-0.3991*** (0.0298)	-0.3677*** (0.0304)	-0.5040*** (0.0594)	-0.5306*** (0.0625)
2011	-0.4591*** 0.0329	-0.4276*** (0.0330)	-0.5259*** (0.0635)	-0.5527*** (0.0667)
2012	-0.4139*** (0.0322)	-0.3815*** (0.0327)	-0.3842*** (0.0533)	-0.4118*** (0.0562)
2013	-0.4580*** (0.0347)	-0.4255*** (0.0348)	-0.5615*** (0.0693)	-0.5889*** (0.0724)
2014	-0.4007*** (0.0326)	-0.3683*** (0.0332)	-0.5538*** (0.0681)	-0.5811*** (0.0713)
2015	-0.3722*** (0.0333)	-0.3400*** (0.0342)	-0.7187*** (0.0864)	-0.7459*** (0.0896)
2016	-0.2868*** (0.0329)	-0.2546*** (0.0343)	-0.6942*** (0.0835)	-0.7214*** (0.0868)
2017	-0.3385*** (0.0350)	-0.3063*** (0.0360)	-0.7639*** (0.0915)	-0.7911*** (0.0947)
Observations	63,367	63,367	25,072	25,072
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Post · Confounders	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. Standard errors are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1. Zombies are defined as described above, either based on FN (2013) (Distress & interest rates) or on Schivardi et al. (2022) and Faria-e-Castro et al. (2024) (Distress only).

post-treatment periods.

8.2 Loan forbearance and firm performance

In this section, we examine whether exposure to the policy affected further dimensions of firm performance, namely TFP, debt sustainability (proxied by a firm's interest coverage ratio or ICR), and the probability of exit in the post-intervention years. For exit, we perform the same Logit DiD estimation with firm and year fixed effects as in Equation (22) where the dependent variable is replaced with a dummy equal to one if firm i goes bankrupt at time t.[29] For TFP and ICRs, we estimate the following

[29]The TSR data set contains information on firm exits. Exits can be categorized into three groups: *tosan* (bankruptcy), *gappei* (merger), and *kaihaikyu* (voluntary exit). The year of exit in TSR is recorded from October (current year) to September (following year). When merging the exit data with our firm-level data set, we define the year of exit as the TSR year of exit minus one.

model:

$$
\begin{aligned}
ln(Y_{it}) = \alpha + \sum_{t=T_0}^{T} \beta_t \cdot D_t \cdot TreatmentIntensity_i + f_i + f_t + ... \\
... + \sum_{c} \gamma_c \cdot Post_t \cdot X_{it}^c + \epsilon_{it}
\end{aligned}
\tag{23}
$$

where Y_{it} is either a firm's TFP, estimated following Wooldridge (2009), or its ICR, defined as the ratio of EBIT over interest expenses at time t. The results are in Table 6, alongside the corresponding placebo tests. The reported coefficients for exit are average marginal effects.

The results of column (1) indicate that the SME Financing Facilitation Act improved the TFP of treated firms. The treatment effects are significantly positive in all the post-treatment years. For example, a coefficient of 0.263 in 2010 suggests that treated firms experienced a 26% increase in TFP in that year. The magnitude of the treatment effects is quite stable over time. The placebo test in column (2) indicates that there is a significantly negative treatment effect in 2008 - which picks up the fact that treated firms were significantly less productive in the pre-intervention period. While this potentially indicates a violation of the parallel trends assumption, this should at worst bias our results against finding a significantly positive treatment effect in the post-treatment periods.

Column (3) indicates, as expected, that treated firms experienced an increase in their interest coverage ratios. In other words, the policy reduced debt-servicing pressures. For example, a coefficient of 1.045 in 2010 suggests that treated firms on average experienced a doubling of their ICR in that year. The magnitude of the treatment effects decreases over time, in line with the treatment effects on interest rates in Table 3. This means that subsidized credit improved debt sustainability. The placebo test in column (4) indicates that there is a significantly negative effect in 2007 and 2008. The estimated "treatment effects" pick up the fact that treated firms had significantly lower ICRs in the pre-intervention period. While this potentially indicates a violation of the parallel trends assumption, this should at worst bias our results against finding a significantly positive treatment effect in the post-treatment periods.

Finally, column (5) provides evidence that forbearance reduced the incidence of bankruptcies. The estimated treatment effect is significantly negative in the early post-interventions years, from 2010 to 2012. For example, an AME of -0.0015 in 2010 means that a treated firm was 0.15 percentage points less likely to declare bankruptcy in that year. This is an economically significant effect because the average bankruptcy rate is very low in Japan (0.2% in 2018). The placebo test of column (6) shows that treated firms were more likely to exit in the pre-intervention period.

Table 6: Loan forbearance and firm performance

	(1) ln(TFP)	(2) Placebo	(3) ln(ICR)	(4) Placebo	(5) Exit	(6) Placebo
2007	NA	-0.0101 (0.0165)	NA	-0.354*** (0.0565)	NA	0.0043 (0.0127)
2008	NA	- 0.0490*** (0.0149)	NA	-0.365*** (0.0493)	NA	0.0044** (0.0017)
2010	0.263*** (0.0207)	0.249*** (0.0215)	1.045*** (0.126)	0.881*** (0.125)	-0.0015* (0.0008)	-0.0015* (0.0008)
2011	0.276*** (0.0247)	0.262*** (0.0254)	0.917*** (0.143)	0.750*** (0.141)	-0.0019* (0.0010)	-0.0019* (0.0010)
2012	0.300*** (0.0269)	0.286*** (0.0274)	0.907*** (0.151)	0.739*** (0.150)	-0.0029** (0.0012)	-0.0029** (0.0012)
2013	0.299*** (0.0282)	0.285*** (0.0288)	0.920*** (0.157)	0.752*** (0.155)	-0.0005 (0.0016)	-0.0005 (0.0016)
2014	0.278*** (0.0298)	0.263*** (0.0303)	0.837*** (0.152)	0.669*** (0.151)	0.0003 (0.0015)	0.0003 (0.0015)
2015	0.259*** (0.0271)	0.245*** (0.0277)	0.784*** (0.142)	0.616*** (0.141)	0.0016 (0.0021)	0.0016 (0.0021)
2016	0.263*** (0.0256)	0.249*** (0.0263)	0.617*** (0.140)	0.449*** (0.139)	-0.0020 (0.0019)	-0.0020 (0.0019)
2017	0.247*** (0.0267)	0.233*** (0.0272)	0.538*** (0.137)	0.370*** (0.135)	0.0005 (0.0021)	0.0005 (0.0021)
2018	0.246*** (0.0281)	0.232*** (0.0286)	0.475*** (0.136)	0.307** (0.135)	NA	NA
Observations	437,219	327,100	437,219	327,100	423,592	423,592
Firm fixed effects	Yes	Yes	Yes	Yes	No	No
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Post · Confounders	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

9 Counterfactuals with flexible labour input

Table 8 mirrors Table 3 and presents the results of our DiD estimation of the treatment effects of the Act on interest rates, assuming flexible labour input. The only difference is that we are now allowing labour input to vary in the definition of the $APBK$:

$$APBK_{it} = \frac{(\pi_{ijt} - \pi_{i0jt})}{\hat{k}_{ijt}} = \frac{(z_{ijt}(k_{ijt}^{\alpha} l_{ijt}^{1-\alpha})^{\gamma} - wl_{ijt}) - (z_{ijt}(k_{i,0}^{\alpha} l_{ijt,0}^{1-\alpha})^{\gamma} - wl_{ijt,0})}{\hat{k}_{ijt}} \tag{24}$$

where l_{ijt} is the observed labour input when the firm uses both bank-funded capital and alternative resources and $l_{ijt,0}$ is the optimal labour input when the firm only uses non-bank-funded capital ($k_{ijt,0}$).

As expected, the results of the DiD estimation do not change much when we allow for flexible labour because the only variable that is affected by this assumption is $APBK_{it}$. We estimate that loan forbearance under the SME Financing Facilitation Act predicts an average decrease in the interest rate on bank loans of 18.9% for treated firms (compared to 18.5% in Table 3). Financing predicts an average decrease of around 10% for treated firms and debt forgiveness is associated with a much larger decrease of 44% (versus 11% and 39% in Table 3). The remaining rows of Table 8 present the annual treatment effects used in the counterfactuals. These are of a similar magnitude as in Table 3 and follow the same pattern. The average product of capital receives a coefficient of zero (suggesting again that Japanese banks had limited bargaining power during the sample period), while credit market tightness receives a significantly negative coefficient - as predicted by the model. The results of the placebo test in column (4) show significant "treatment effects" in 2007 and 2008, but they are very small in magnitude in comparison to the negative treatment effects in 2010-2015 and not jointly significant[30].

The counterfactuals, by contrast, are affected by the assumption of flexible labour. Equation (18) changes as firms now choose their optimal capital stock given the optimal adjustment in labour input. Without loss of generality, we normalise wages to 1. Optimal labour input is where the marginal product of labour is equalised with the marginal cost, i.e. $l_{ijt} = \left[(1-\alpha)\gamma z_{ijt} k_{ijt}^{\alpha\gamma}\right]^{\frac{1}{1-\gamma(1-\alpha)}}$. Optimal capital input is then given by $k_{ijt} = \left(\frac{\alpha\gamma z_{ijt} l_{ijt}^{\gamma(1-\alpha)}}{r_{ijt}}\right)^{\frac{1}{1-\alpha\gamma}}$. Combining these two equations, a change in the interest rate following a change in forbearance incentives affects the firm's capital stock according to Equation (25):

$$\Delta \log(k_{ijt}) = -\left(1 - \frac{\alpha\gamma}{1-\gamma(1-\alpha)}\right)^{-1} \Delta \log(r_{i,j,t}) \tag{25}$$

Furthermore, the change in optimal labour input is given by:

$$\Delta \log(l_{ijt}) = -\frac{\alpha\gamma}{1-\gamma(1-\alpha)} \Delta \log(k_{i,j,t}) \tag{26}$$

We compute the counterfactuals presented in Table 7 using Equations (25) and (26). As in the baseline exercise in Section 7, we restrict our computation of the counterfactual to downward adjustments in the capital stock as a result of interest rate increases and cap movements in the capital stock at 15 % to avoid unrealistically large adjustments. The results of Table 7 are qualitatively similar to those of Table 4, but the magnitudes are slightly affected by adjustments in labour input. The aggregate capital stock would have declined by 5.20% (vs. 4.22%) in 2010 and by about 2% (vs. 1.4%) on average if the policy had not been enacted. On average, the Act depressed productivity by about 0.6% (vs 0.5%). In the counterfactual without reallocation, output would have declined by 3.7% (vs.

[30] A Wald test for the joint significance of the "treatment effects" in 2007 and 2008 indicates that the null hypothesis that the latter are jointly equal to zero cannot be rejected at the 5% significance level. F(2, 66224) = 2.58 and Prob > F = 0.0757.

8.3%) in 2010 and 1.6% (vs. 2.5%) on average. Once we assume seamless reallocation, the policy is estimated to have decreased output by around 7.3% (vs. 4.8%) in 2010 and by about 3% (vs. 1.5%) on average.

Table 7: Effect of removing the estimated forbearance incentives (in %) - flexible labour

Counterfactuals - % Change	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Capital stock	-5.20 %	-5.17 %	-5.19 %	-5.14 %	-4.72 %	-4.43 %	1.11 %	5.07 %	4.50 %	-2.13 %
Capital productivity	1.56 %	1.51 %	1.29 %	1.09 %	0.74 %	0.62 %	-0.15 %	-0.88 %	-0.48 %	0.59 %
Output, without reallocation	-3.72 %	-3.73 %	-3.97 %	-4.10 %	-4.02 %	-3.83 %	0.96 %	4.14 %	4.00 %	-1.59 %
Output, with reallocation	7.30 %	7.07 %	5.50 %	4.19 %	1.48 %	0.82 %	0.00 %	0.00 %	0.00 %	2.93 %

10 Conclusions

There has been scepticism about forbearance lending in academic and policy circles. It acquired a bad reputation as one of the potential drivers behind the post-GFC slowdown in European countries, not least because of the analysis of forbearance lending in Japan during the Lost Decade. While it is true that forbearance lending can contribute to zombification and reduce aggregate productivity, an evaluation of Japan's 2009 SME Financing Facilitation Act suggests that a well-designed credit market intervention based on forbearance lending can help viable firms weather difficult times. We use the Act as a semi-experimental setting to quantify the aggregate consequences of loan forbearance using a DiD approach combined with back-of-the-envelope counterfactual exercises. We find that the Act worked as an interest rate subsidy, boosting the aggregate capital stock but depressing aggregate productivity. In the more plausible scenario of subdued credit reallocation, the Act is estimated to have boosted output. Importantly, we find that the Act did not contribute to the creation of zombie firms. On the contrary, we find that greater exposure to the policy increased firm-level TFP and reduced the probability that a firm becomes a zombie. This suggests that the business restructuring plans, which troubled SMEs in receipt of forbearance were requested to submit and adhere to, allowed them to resurrect. Exploring how targeted interventions based on forbearance lending could be designed to respond to future shocks is an interesting avenue for policymakers.

Table 8: Treatment effects on interest rates - flexible labor

	(1) Any forbearance	(2) Financing	(3) Forgiveness	(4) Placebo
2010-2018	-0.189*** (0.0208)	-0.103*** (0.0334)	-0.445*** (0.0859)	NA
2007	NA	NA	NA	-0.0468** (0.0215)
2008	NA	NA	NA	-0.0284* (0.0168)
2010	-0.430*** (0.0204)	-0.420*** (0.0323)	-0.259*** (0.0838)	-0.446*** (0.0202)
2011	-0.333*** (0.0221)	-0.330*** (0.0364)	-0.210** (0.0971)	-0.349*** (0.0220)
2012	-0.280*** (0.0232)	-0.227*** (0.0399)	-0.385*** (0.113)	-0.297*** (0.0231)
2013	-0.232*** (0.0241)	-0.168*** (0.0415)	-0.405*** (0.112)	-0.248*** (0.0239)
2014	-0.117*** (0.0247)	0.00117 (0.0466)	-0.511*** (0.135)	-0.133*** (0.0245)
2015	-0.0785*** (0.0257)	0.0316 (0.0485)	-0.476*** (0.140)	-0.0942*** (0.0255)
2016	0.0123 (0.0275)	0.187*** (0.0521)	-0.674*** (0.152)	-0.00246 (0.0272)
2017	0.135*** (0.0286)	0.300*** (0.0555)	-0.596*** (0.166)	0.121*** (0.0282)
2018	0.238*** (0.0355)	0.464*** (0.0805)	-0.776*** (0.260)	0.224*** (0.0351)
Average product of borrowed capital	-0.0000*** (0.0000)	-0.0000*** (0.0000)		-0.0000*** (0.0000)
Credit market tightness	-0.00737*** (0.000704)	-0.00700*** (0.000704)		-0.00770*** (0.000739)
Observations	437,219	437,219	437,219	437,219
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Post · Confounders	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. Standard errors clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1

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Appendix

A Probability of treatment: Probit estimations

Table 9: Probit estimations for the probability of treatment

	Any forbearance	Financing	Forgiveness
Leverage	0.926***	0.777***	0.343***
	(0.0828)	(0.0833)	(0.0916)
Credit score	-0.076***	-0.076***	-0.037***
	(0.0059)	(0.0064)	(0.0072)
ROA	0.531**	0.150	1.087***
	(0.2285)	(0.2275)	(0.3787)
ln(Sales)	0.043*	0.051*	0.002
	(0.0255)	(0.0286)	(0.0326)
Employment	-0.0004	-0.001*	0.000
	(0.0003)	(0.0004)	(0.0003)
Firm age	-0.004**	-0.006***	0.001
	(0.0017)	(0.0018)	(0.0022)
	(0.145)		
Constant	2.275***	1.977***	0.233
	(0.4043)	(0.449)	(0.5018)
Observations	3,298	3,298	3,298
Industry fixed effects	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Source: Unpublished results from Ono and Yasuda (2017) using RIETI survey data.

Supplemental Appendix for "Forbearance lending as a crisis management tool: Evidence from Japan"

Isabelle Roland   Yukiko Saito   Philip Schnattinger

This is the Supplemental Appendix for "Forbearance lending as a crisis management tool: Evidence from Japan". Section A gives details on the eligibility criteria for firms to benefit from loan forbearance under the SME Financing Facilitation Act. Section B provides further information on the representativeness of our sample in comparison to the full TSR dataset. Section C provides further descriptive statistics to complement Section 5 in the paper. Section D describes the determination of the lower bound interest rate used in the construction of the zombie dummy variable based on Fukuda and Nakamura (2013). Section E derives the loan termination condition of the model. Section F presents additional details on the counterfactuals. Finally section G examines whether there is a relationship between loan forbearance and lender characteristics.

A Eligibility under the SME Financing Facilitation Act

Under the SME Financing Facilitation Act, financial institutions providing loans to SMEs which were experiencing, or were expected to experience, difficulties repaying their debts, were obliged to alter the loan conditions and alleviate the payment burden of the borrowers to the best of their ability. Struggling SMEs were required to submit an application to their respective financial institutions, but as described in section 2.2 most applications were accepted. The eligibility dummy was created in accordance with the SME Financing Facilitation Act, specifically Article 2, paragraph (2) items (i) through (ix) and Article 4. To begin, we identify firms that qualify as SMEs as laid out in Article 2, paragraph (2). The eligibility criteria are based on the firm's number of employees and stated share capital. We then exclude firms identified by Article 4 of the SME Financing Facilitation Act, which specifies certain types of SMEs that are not eligible for financing under the Act. Article 2 Paragraph 2 of the SME Financing Facilitation Act defines qualifying SMEs as follows:

- Companies with stated capital of 300 million yen or less, engaged in general business activities, excluding financial services and other exclusions designated by the Cabinet Order. For companies operating primarily a retail or services business, the stated capital threshold is 50 million yen or less. For companies operating primarily a wholesale business, the stated capital threshold is 100 million yen or less.[31]

[31] Japanese SME legislation groups retail and wholesale industries together as the "commerce" sector, which is different from the "services" industry. In terms of the Japan Standard Industrial Classification, the "services" industries refer to the following: Division G – Information and Communications, 38 Broadcasting, 39

- Companies with fewer than 300 regular employees, engaged in general business activities, excluding financial services and other exclusions designated by the Cabinet Order. For companies operating primarily a retail business, the maximum number of employees is 50. For companies operating primarily a wholesale or services business, the maximum number of employees is 100.
- SME cooperatives, agricultural cooperatives, federations of agricultural cooperatives, fishery cooperatives, forest owners' cooperatives, forestry production associations, federation of forestry cooperatives, consumer cooperatives and federations of consumer cooperatives that are engaged in general business activities, or that have at least two-thirds of their members engaged in general business activities, excluding financial services and other exclusions designated by the Cabinet Order.
- Cooperative partnerships engaged in general business activities.
- Corporations that operate a medical business as their principal business, and whose number of regular employees is not more than 300.
- Commercial and industrial partnerships, federations of commercial and industrial partnerships that are engaged in general business activities or whose members are engaged in general business activities.
- Shopping district promotion cooperatives and federations of shopping district promotion cooperatives that are engaged in general business activities or whose members are engaged in general business activities.
- Environmental health industry associations, minor environmental health industry cooperatives, and federations of environmental health industry associations that are engaged in general business activities or whose members are engaged in general business activities, and that have at least two-thirds of their direct and indirect members reporting a stated capital of 50 million yen or less or a number of regular employees of maximum 50. For associations whose members operate primarily a retail business, the stated capital threshold is 100 million yen. For associations whose members operate primarily a wholesale or services business, the employee threshold is 100.
- Sake brewers' associations, federations of Sake Brewers' Associations and Japan Sake Brewers' Association that have at least two-thirds of their direct and indirect members reporting a stated capital of 300 million yen or less or a number of regular employees of maximum 300.

Information services, 411 Video picture information production and distribution, 412 Sound information production, 415 Commercial art and graphic design, 416 Services incidental to video picture information, sound information, character information production and distribution, Division K - Real Estate and Goods Rental and Leasing, 693 Automobile parking, 70 Goods rental and leasing, Division L - Scientific research, professional and technical services, Division M - Accommodations, Eating and Drinking Services, 75 Accommodations, Division N - Living-related and personal services and amusement services, Excluding 791 Travel Agency, Division O - Education, learning support Division P - Medical, health care and welfare, Division Q -Compound Services, Division R -Services, N.E.C.

- Liquor merchants' associations, federations of liquor merchants' associations and All Japan Liquor Merchants' Association that have at least two-thirds of their direct and indirect members reporting a stated capital of 50 million yen or less or a number of regular employees of maximum 50. For the liquor wholesale industry, the stated capital threshold is 100 million yen and the employee threshold is 100.
- Coastal shipping associations or federations of coastal shipping associations that have at least two-thirds of their direct and indirect members reporting a stated capital of 300 million yen or less or a number of regular employees of maximum 300.
- Other firms designed by the Cabinet Order.

Article 4 excludes the following firms due to their affiliations to large firms or financial institutions:

- Financial institutions.
- Subsidiaries or parent companies of financial institutions.
- Large firms, i.e. firms with stated capital of more than 500 million yen or liabilities of more than 200 million yen.
- Subsidiaries of large firms and firms designated by the Cabinet Order as having a special relationship with a large firm.

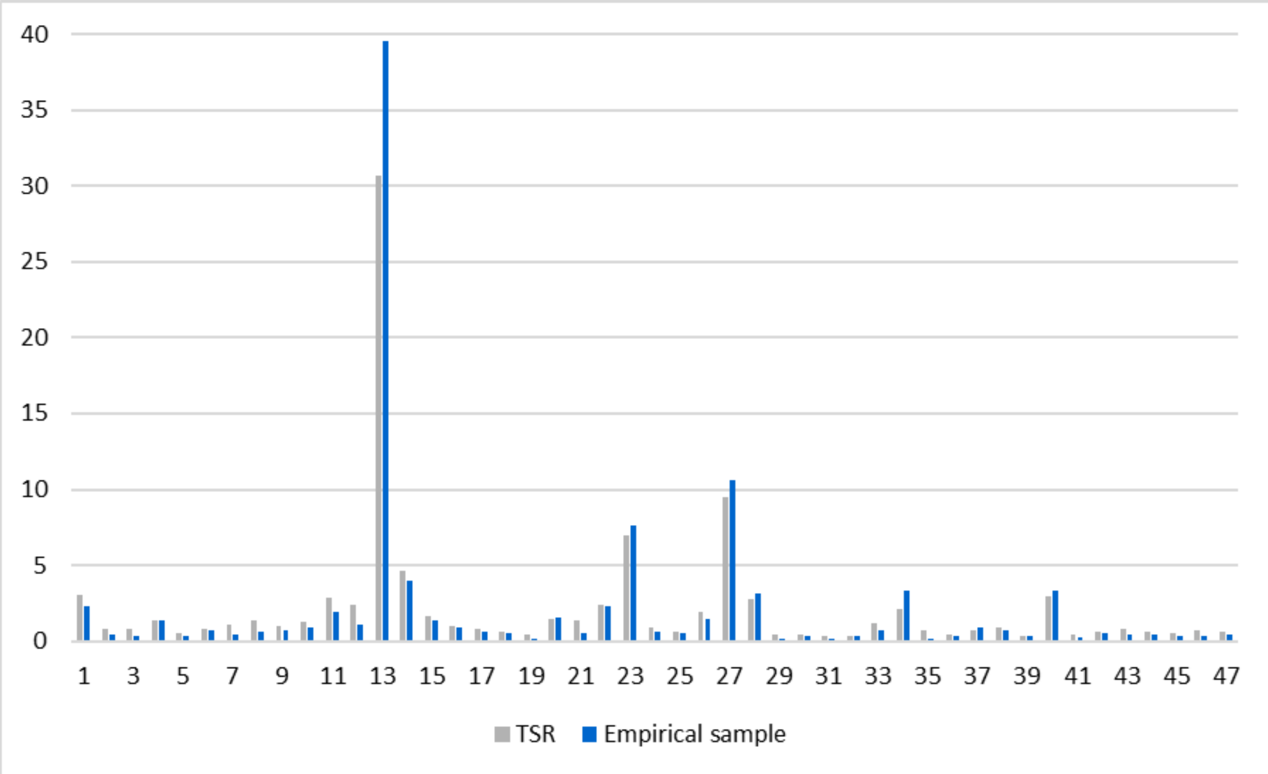
B Comparison of TSR and empirical data set

In this section, we compare our empirical sample to the full TSR data set in terms of georgraphical coverage, and size and industry distributions. The full TSR numbers exclude Finance and insurance (as we focus on non-financial corporations) and other sectors not covered at all by our sample (real estate and goods rental and leasing; education, learning support; medical, health care and welfare; compound services; and services n.e.c.). Figure 1 displays the fraction of firms in each of the 47 prefectures in 2016 in the TSR data set and in our empirical data set. The percentage figures based on the TSR data set are very close to those based on the empirical data set for most prefectures. The picture looks similar in other sample years.

Figure 2 compares the distribution of firms by size in 2016 in the TSR data set and our empirical data set. The firm size distribution of the empirical data closely resembles that of the TSR data. The largest gaps are found for micro-enterprises with fewer than 10 employees (which are under-represented in the empirical data) and for very large enterprises with more than 2000 employees (which are over-represented in the empirical data). The picture looks similar in other sample years.

Figure 3 compares the distribution of firms by industry division in 2016 in the TSR data set and our empirical data set. The firm industry distribution of the empirical data closely resembles that of the TSR data. The picture looks similar in other sample years.

Figure 1: Comparison of geographical distribution: TSR versus empirical data set



Notes: The 47 prefectures are: 1. Hokkaidō, 2. Aomori, 3. Iwate, 4. Miyagi, 5. Akita, 6. Yamagata, 7. Fukushima, 8. Ibaraki, 9. Tochigi, 10. Gunma, 11. Saitama ,12. Chiba, 13. Tōkyō, 14. Kanagawa, 15. Niigata, 16. Toyama, 17. Ishikawa, 18. Fukui, 19. Yamanashi, 20. Nagano, 21. Gifu, 22. Shizuoka, 23. Aichi, 24. Mie, 25. Shiga, 26. Kyōto, 27. Ōsaka, 28. Hyōgo, 29. Nara, 30. Wakayama, 31. Tottori, 32. Shimane, 33. Okayama, 34. Hiroshima, 35. Yamaguchi, 36. Tokushima, 37. Kagawa, 38. Ehime, 39. Kōchi, 40. Fukuoka, 41. Saga, 42. Nagasaki, 43. Kumamoto, 44. Ōita, 45. Miyazaki, 46. Kagoshima, 47. Okinawa.

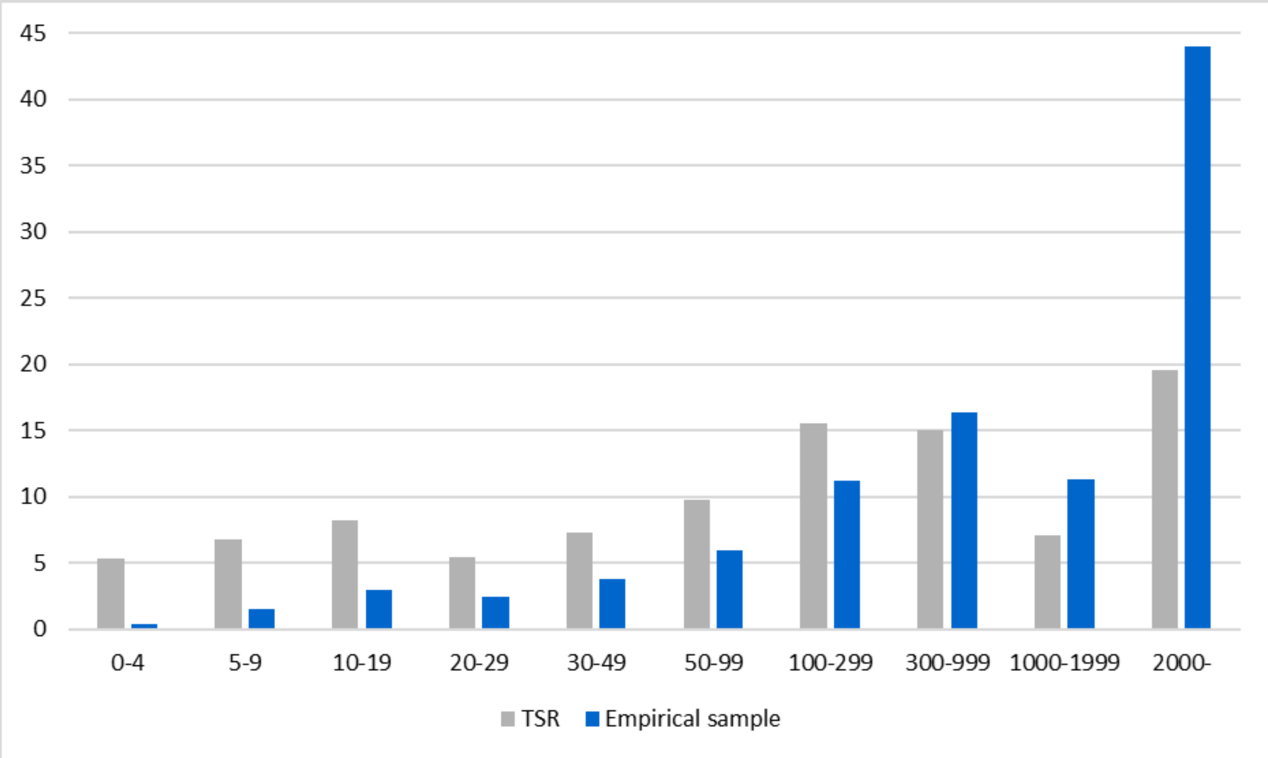
C Supplemental descriptive statistics

This section provides further descriptive statistics to complement Section 5 in the paper.

C.1 Average product of borrowed capital

Figure 4 below shows the sample mean average product of borrowed capital ($APBK$, defined in Equation (20) in the paper) for different types of firms. The $APBK$ is systematically highest for small firms (with one extra unit of bank credit generating an extra 45 ¥ in profit before interest, on average) and lowest for large firms (with one extra unit of bank credit generating an extra 16 ¥ in profit before interest, on average). To the extent that marginal and average costs move together, this is consistent with the fact that small firms face higher marginal costs of borrowed capital, i.e. higher loan interest rates. The $APBK$ is very stable across time for all types of firms.

Figure 2: Comparison of size distribution: TSR versus empirical data set



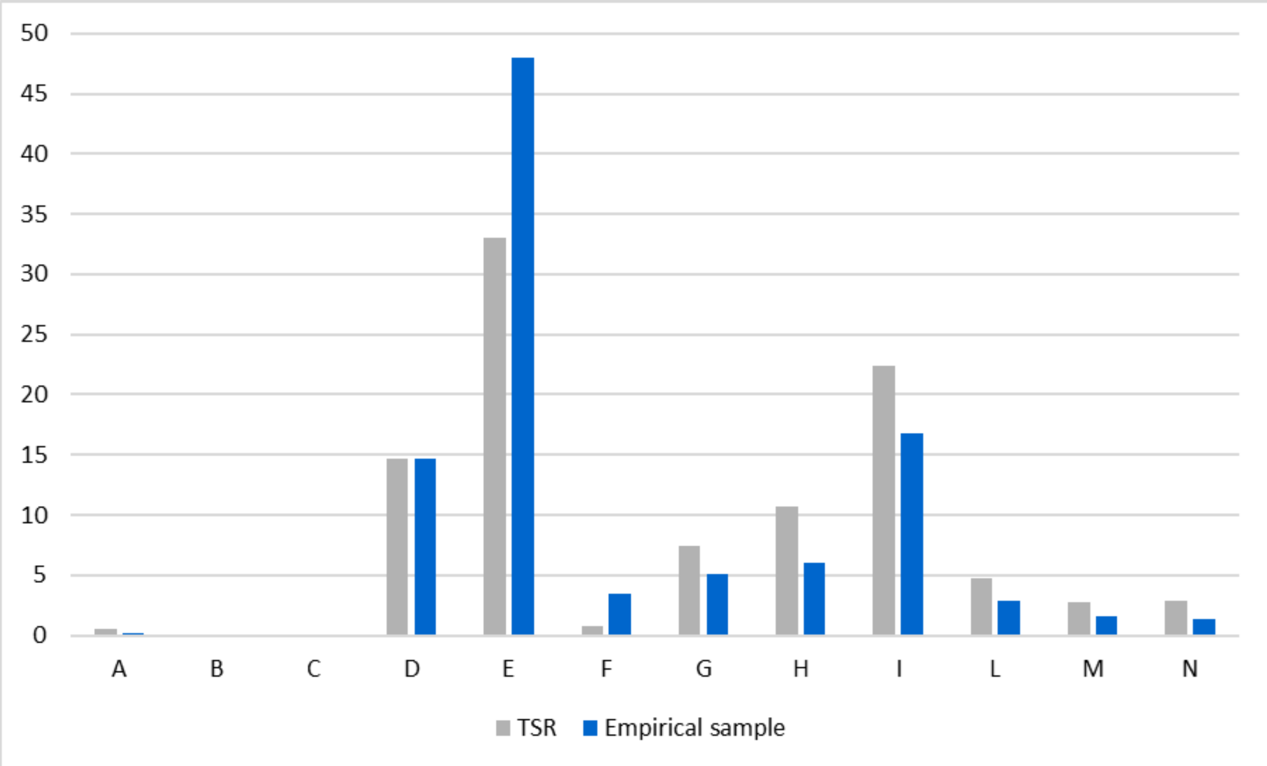
Notes: The size bands are: 0-4 employees, 5-9 employees, 10-19 employees, 20-29 employees, 30-49 employees, 50-99 employees, 100-299 employees, 300-999 employees, 1000-1999 employees, and 2000 employees or more.

C.2 Credit scores

TSR classifies firms into five groups of creditworthiness according to their credit scores. Scores less than or equal to 29 are classified as "keikai" (caution). Scores between 30 and 49 are classified as "ichio keikai" (somewhat caution). Scores between 50 and 64 are categorized as "tasho chui" (attention). Scores between 65 and 79 are "bunan" (safe), and those between 80 and 100 are considered to be "keikai fuyo" (no risk). The average credit score in our sample is 50.1 over 2007-2018, on the fence between "ichio keikai" and "tasho chui" (Figure 5). The credit scores of eligible firms are systematically lower (at 49 on average) than those of non-eligible firms (at 55.3 on average) - consistent with SMEs being riskier borrowers. However, the time series pattern is similar for eligible and non-eligible firms. There is a slight deterioration from a pre-crisis peak in 2007 to a low in 2012, followed by a recovery by 2018.

Table 1 shows the percentage of firms in each creditworthiness category in the sample for the whole sample period, and the years 2007, 2012, and 2018. There are very few firms in the worst (keikai) and best (keikai fuyo) creditworthiness categories. The deterioration in the average credit score between 2007 and 2012 apparent in Figure 5 is driven by a transfer of firms from "tasho chui" to "ichio keikai". This is reversed between 2012 and 2018.

Figure 3: Comparison of industry distribution: TSR versus empirical data set



Notes: The industry divisions follow the Industrial Classification used in the 2016 Economic Census for Business Activity (See `https://www.stat.go.jp/english/data/e-census/2016/industry.html`): A. Agriculture and forestry, B. Fisheries, C. Mining and quarrying of stone and gravel, D. Construction, E. Manufacturing, F. Electricity, gas, heat supply and water, G. Information and communications, H. Transport and postal services, I. Wholesale and retail trade, L. Scientific research, professional and technical services, M. Accommodation, eating and drinking services, N. Living-related and personal services and amusement services.

Table 1: Distribution of firms by creditworthiness category (%)

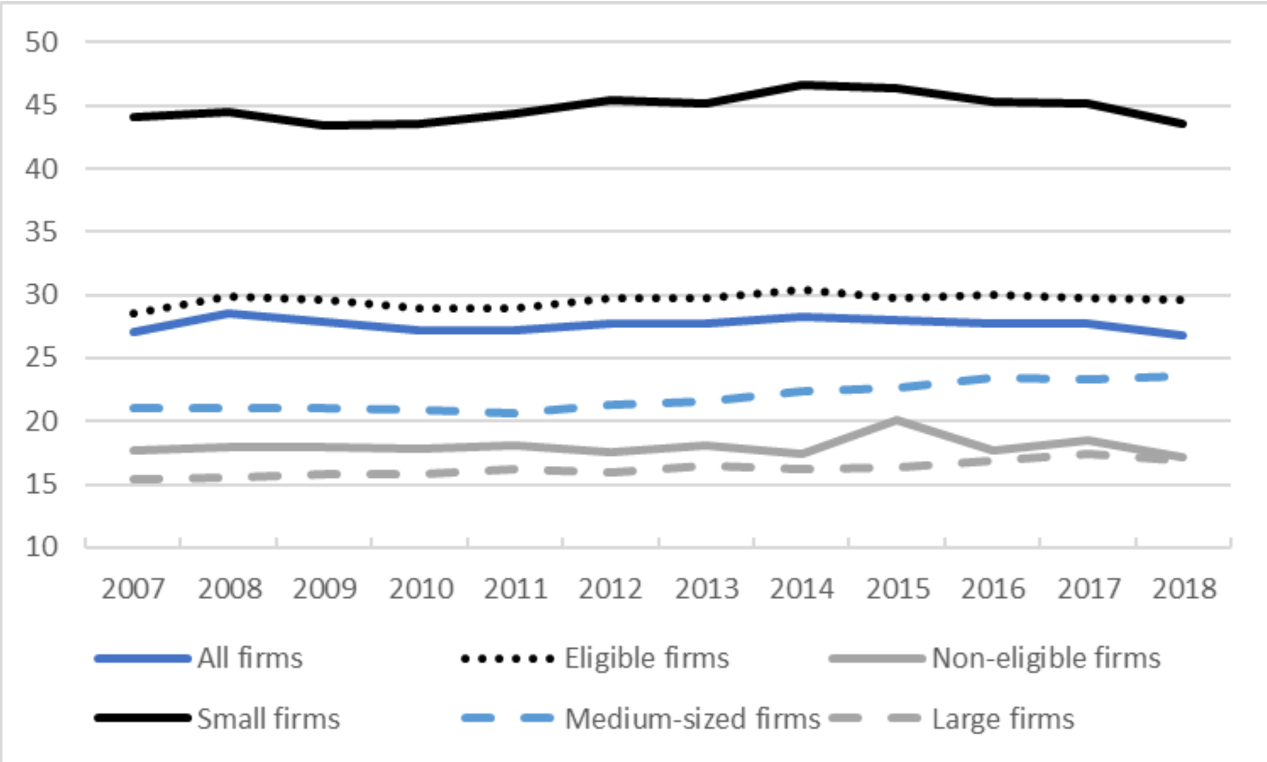
	2007-2018	**2007**	**2012**	**2018**
keikai	0.08	0.06	0.09	0.02
ichio keikai	50.29	43.83	53.49	40.26
tasho chui	46.4	53.01	43.35	53.27
bunan	3.21	3.06	3.07	6.41
keikai fuyo	0.02	0.03	0.01	0.05

Notes: Share of firms in each of the five creditworthiness categories in our sample. Source: TSR.

C.3 Exit

The TSR data set contains information on firm exits. For the firms that exited, TSR provides information about the reasons for exit. Exits can be categorized into three groups: *tosan* (bankruptcy), *gappei* (merger), and *kaihaikyu* (voluntary exit). TSR distinguishes between three different types of voluntary exits: *kyugyo* (temporary suspension of business), *haigyo* (business closure), and *kaisan* (dissolution of company). Figure 6 shows the exit rates over time for all types of exit and for each type

Figure 4: Average product of borrowed capital (in ¥)



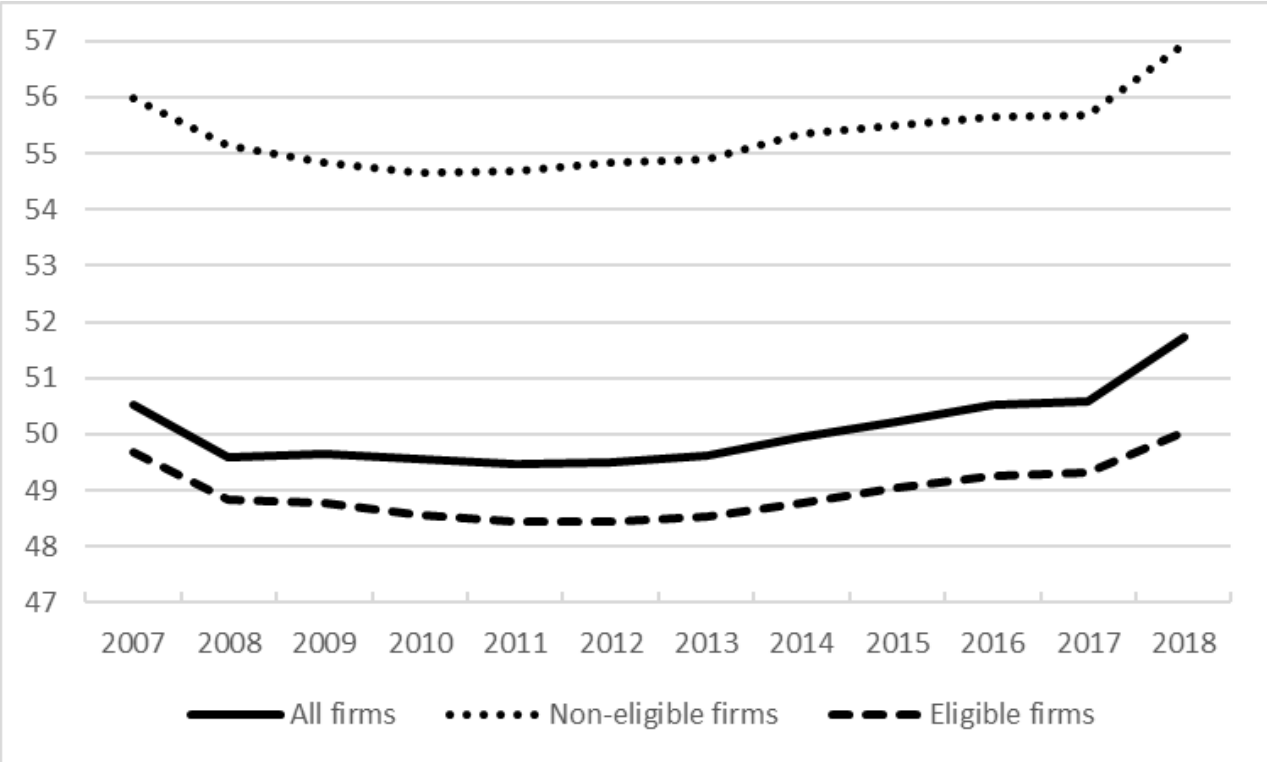
Note: Average product of borrowed capital in the sample, as defined in Equation (20) in the paper, assuming fixed labor input. Size categories are those of the Tankan Survey of the Bank of Japan. Source: TSR.

separately. The year of exit in TSR is recorded from October (current year) to September (following year). When merging the exit data with our firm-level data set, we define the year of exit as the TSR year of exit minus one. In other words, we define the exit rate in a given year as the number of firms that exited between October of the previous year and September of the current year as a percentage of the total number of firms in the previous year. Figure 6 shows that exit rates are very low according to the TSR data, a finding which is well-documented (see, e.g., Hong et al., 2020). In addition, exit rates have declined substantially over our sample period. The overall exit rate declined from 2.07% in 2008 to 1.66% in 2018. The decline is mainly driven by a drop in the rate of bankruptcies, from 0.52% in 2008 to 0.2% in 2018.

C.4 Zombie firms

Figure 7 shows the fraction of zombie firms in our sample according to the two definitions used in the paper. The first definition is based on Fukuda and Nakamura (2013) and combines a subsidized credit criterion with indicators of financial distress (solid line). The second definition is based on Schivardi et al. (2022) and Faria-e-Castro et al. (2024). It classifies a firm as a zombie if it has high leverage and low productivity (dashed line). The fraction of zombies is systematically higher according to our definition as there is a large number of firms with subsidized credit and low productivity, but not

Figure 5: Average credit score



Notes: Average credit score in the sample. Source: TSR.

necessarily high leverage (above the 90th percentile). However, both definitions indicate a declining incidence of zombies in our sample.

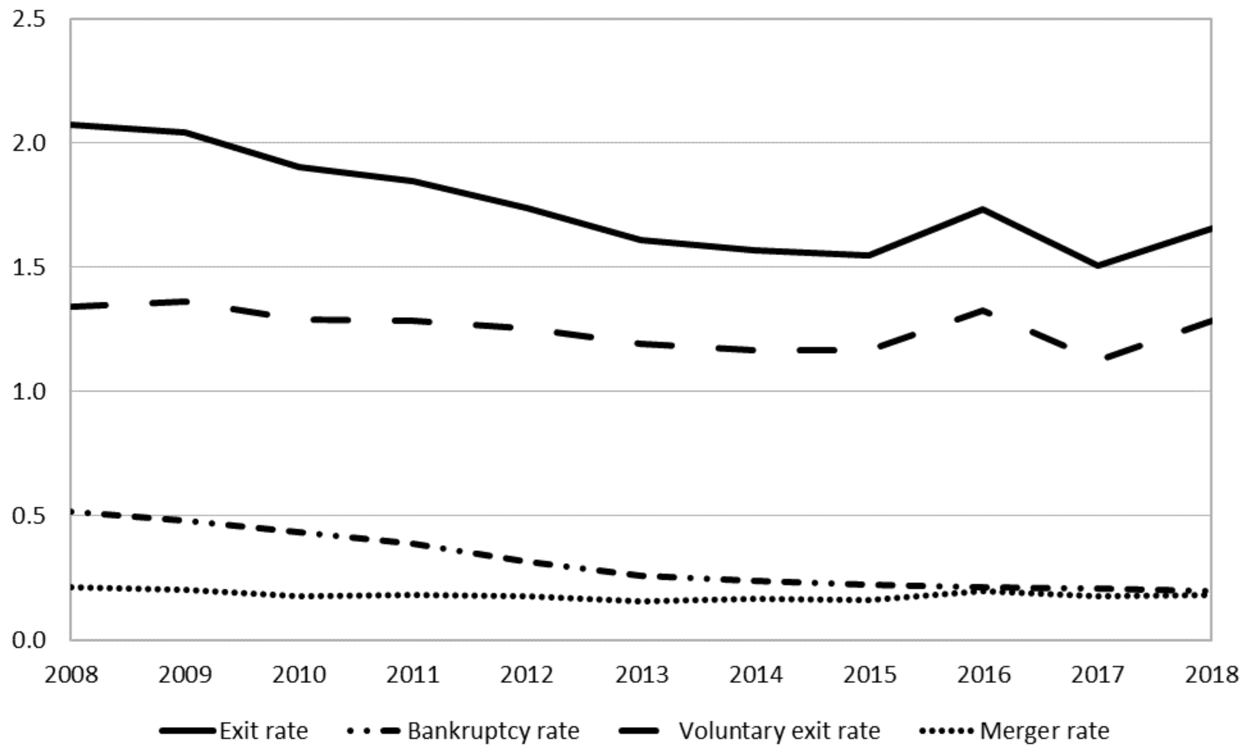
D Measurement of lower bound interest rate

The guiding principle to construct a lower bound interest rate ($R^*_{i,t}$) is to select interest rates that are extremely advantageous for the borrower, so that the lower bound is in fact less than most firms would pay in the absence of forbearance. The lower bound $R^*_{i,t}$ is constructed following the methodology of Caballero, Hoshi and Kashyap (2008):

$$R^*_{i,t} = rs_{t-1}BS_{i,t-1} + \left(\frac{1}{5}\sum_{j=1}^{5} rl_{t-j}\right) BL_{i,t-1} + rb_{t-1} \times Bonds_{i,t-1} \tag{27}$$

Where $BS_{i,t}$, $BL_{i,t}$ and $Bonds_{i,t}$ are short-term bank loans (maturity of less than one year), long-term bank loans (more than one year) and total bonds outstanding from firm i in year t; and rs_t, rl_t and rb_t are interest rates paid on short-term bank loans, long-term bank loans and corporate bonds, respectively.

Figure 6: Exit rates by exit category



Notes: Exit rates for different exit categories. Source: TSR (full TSR sample).

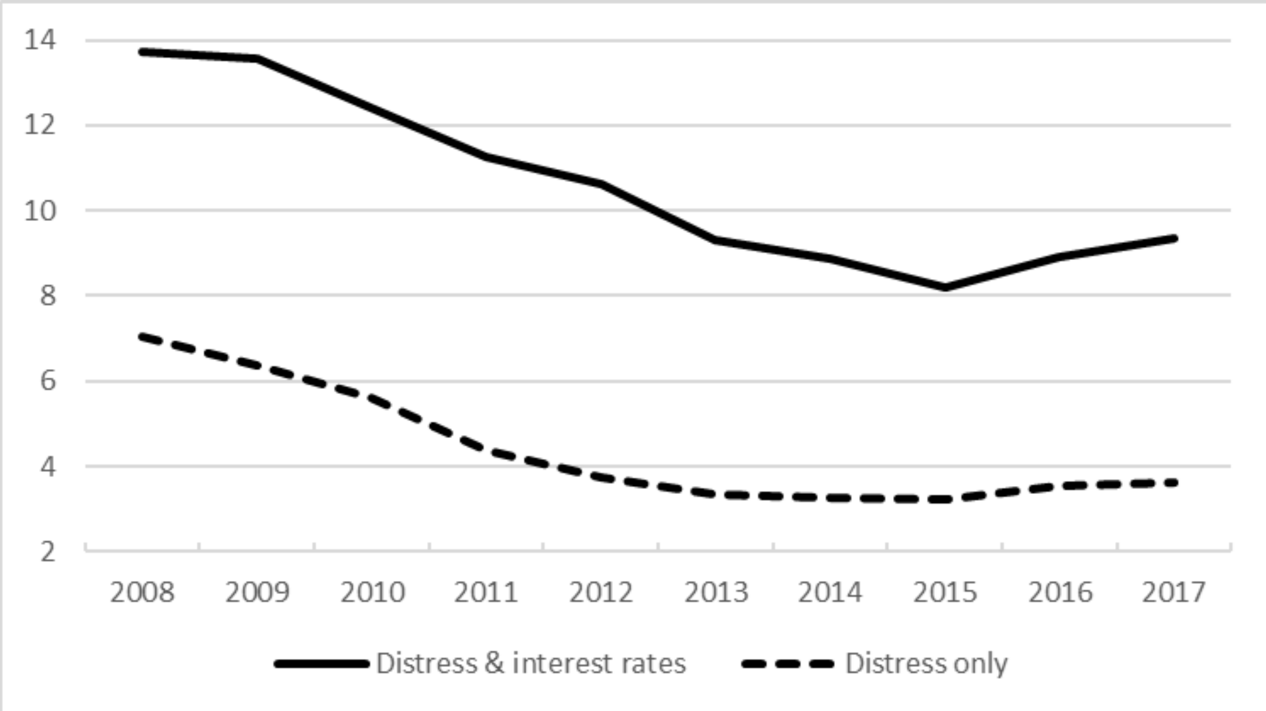
We measure the lower bound interest rate on short-term loans (rs_t) with the Bank of Japan's short-term prime rate. With regards to long-term loans, we follow a similar approach and use the Bank of Japan's long-term prime rate[32]. However, TSR only reports the stock of long-term bank loans outstanding, without information on the exact maturity of the loans. Therefore, we follow Caballero, Hoshi and Kashyap (2008) and assume that each firm's long-term loans have an average maturity of 2.5 years and with one-fifth of them having been originated in each year for five years[33] This implies that the lower bound interest rate on long-term loans is an equally weighted average of the last five years of the long-term prime rates. Thus, we calculate the minimum required interest payment on long-term loans by multiplying the outstanding long-term loans of all maturities with the five-year average of the long-term prime rates. In our sample, short-term bank loans account for around 50% of total debt and long-term bank loans (due in more than 1 year) account for around 3.3% of total debt.

For bonds, we adopt an extremely conservative approach that assumes the minimum required interest rate is zero for the entire sample period. This rather extreme assumption does not affect our results, however. First, bonds account on average for less than 1.8% of total debt in our sample.

[32]Short-term and long-term prime rates are obtained from the Bank of Japan database: https://www.boj.or.jp/en/statistics/dl/loan/prime/prime.htm/ The most frequent short-term prime rate is used. For every year, we take the minimum of the available short and long-term prime rates.

[33]Five years corresponds to the average maturity of bank loans according to Smith (2003).

Figure 7: Proportion of zombie firms over time (%)



Notes: Percentage of zombie firms over time. Zombies are defined as described in the paper, either based on FN (2013) (solid line) or on Schivardi et al. (2022) and Faria-e-Castro et al. (2024) (dashed line). Source: TSR.

Therefore, the lower bound interest rate calculations are very insensitive to the assumption we make on the minimum required interest rate on bonds. Moreover, the assumption pushes down the lower bound interest rate $R^*_{i,t}$, and therefore would lead us to identify fewer zombie firms. Some papers in the literature (Caballero, Hoshi and Kashyap, 2008) have also assumed that bond financing uses only convertible bonds, which, by their nature, have lower yields. By assuming such low required interest rates on bonds, the approach reduces the risk of misclassifying creditworthy companies as zombies.

E Loan termination condition

Substituting Equation (9) in Equation (7), we can find the cutoff productivity values at which loan relationships are terminated. There will be one cutoff value for each market j. The credit line termination condition for market j is given by Equation (28),

$$(1-\eta)(\pi_{ijt}(z_s)^*-\pi_{0jt}(z_s))-(1-\eta)\rho_{jt}-\eta\kappa\theta_{jt}+\eta\kappa^B_{jt}-\eta[\tau_{ijt}+p_{jt}\tau_{kjt}-\beta\tau_{ijt+1}]+\beta\frac{\kappa}{q_{jt+1}}-\beta\tau_{ijt+1}=-\tau_{ijt}. \tag{28}$$

The productivity cutoff $\tilde{z}_j$ is found by equating the value of a continuing relationship for the bank at the negotiated interest rate equal to zero.[34] Termination costs reduce the productivity cut-off. In other words, loan relationships that should be terminated are kept alive. We can rewrite this as Equation (29):

$$\pi_{ijt}(z_s)^* - \pi_{0jt}(z_s) = \hat{k}_{ijt}\rho_t + \eta\kappa^B_{jt} - \tau_{ijt} + \beta\tau_{ijt+1} - \frac{1}{1-\eta}\left[\eta\kappa\theta_{jt} + \eta p_{jt}\tau_{kjt} + \beta\frac{\kappa}{q_{jt+1}}\right] \tag{29}$$

Equation (29) shows that the cutoff productivity value is reached when the period surplus of production equals the outside value of the match for the bank minus the continuation value of the match. When the capital stock of the credit line $\hat{k}_{ijt}$ is freely adjustable the cost of maintaining a credit line $\eta\kappa^B_{jt}$ does not exist for the bank then the cutoff value cannot be reached when the loan size can be adjusted freely, as the match can be maintained costlessly by reducing $\hat{k}_{ijt}$ to 0.

Proposition 2. *When the capital of the credit line $\hat{k}_{ijt}$ is freely adjustable, maintaining a credit line is costless for the bank, and forbearance incentives are weakly positive and non-increasing over time, credit lines will not be severed for any productivity realisation z_{ijt}.*

Proof: When $\hat{k}_{ijt}$ is freely adjustable and forbearance incentives are stable then for any productivity realisation $z_{ijt} \in \mathbb{R}$, $\hat{k}_{ijt} = 0$ is an option. In this case $\pi_{ijt}(z_s)^* - \pi_{0jt}(z_s) = 0$. Hence, Equation (29) becomes $0 > -\tau_{ij}(1-\beta) - \frac{1}{1-\eta}\left[\eta\kappa\theta_{jt} + \eta p_{jt}\tau_{kjt} + \beta\frac{\kappa}{q_{jt+1}}\right]$. This is automatically satisfied when $\tau_{ij} > 0$.

Therefore, exit from credit lines can only happen either when τ_{ij} is increasing strongly in the next period when capital is not freely adjustable for the firm, or κ^B_{jt} is sufficiently large. In this case, the match surplus may become negative as a low productivity realisation may result in the average marginal value created by the loan producing negative profit. We assume that the cost of maintaining a credit line is sufficiently large that the bank wants to sever credit lines below a certain productivity level, though sluggish adjustment in the size of the lent capital stock could deliver a similar result.

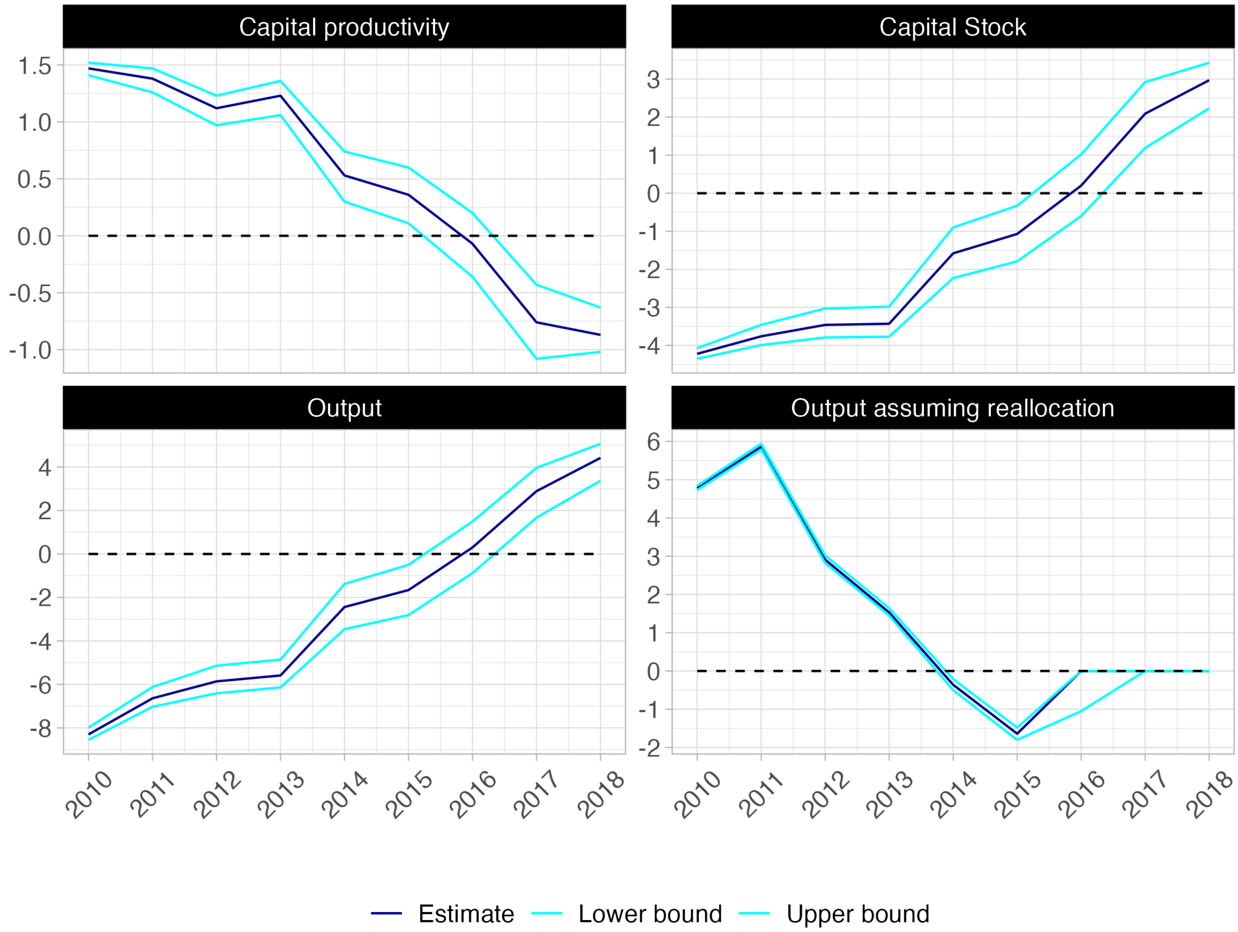
[34] Equivalently, we could set the value of a continuing match for the firm equal to zero, or the total surplus (firm + bank) equal to zero.

F Additional counterfactuals

F.1 Plotting the counterfactuals

Figure 8 plots the counterfactuals from Table 4 in the paper for a better illustration of the time series.

Figure 8: Effect of removing the estimated forbearance incentives (in %) - fixed labour



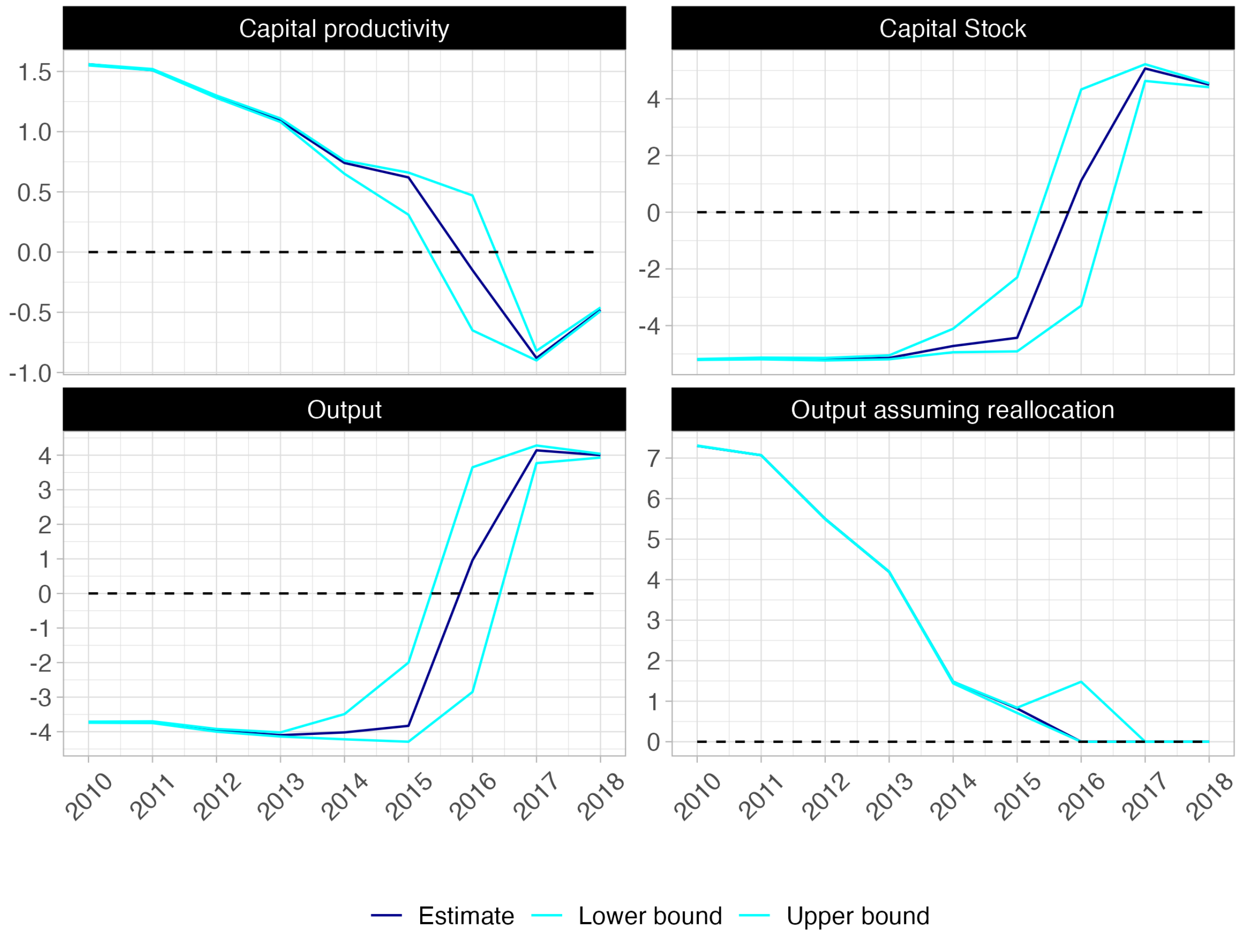
Notes: The lower and upper bounds of the counterfactuals are constructed using two standard deviations of the estimated effects on the interest rate and otherwise making the same assumptions as for the baseline estimate.

Figure 9 plots the counterfactuals from Table 7 in the paper for a better illustration of the time series of counterfactuals with flexible labour.

F.2 Counterfactuals for specific types of forbearance

The counterfactuals presented here show the estimated effects for the two types of forbearance policies which banks enacted, namely debt forgiveness and financing. While the choice to grant forbearance was arguably exogenously imposed by the government on banks, the choice of which type of loan

Figure 9: Effect of removing the estimated forbearance incentives (in %) - fixed labour



Notes: The lower and upper bounds of the counterfactuals are constructed using two standard deviations of the estimated effects on the interest rate and otherwise making the same assumptions as for the baseline estimate.

forbearance to provide to firms was not. The decision process to provide loan forgiveness or payment deferrals might depend on unobservable factors such as characteristics of the individual firm-bank relationship. For this reason, we do not claim that these separated effects of forbearance are well-identified. Nevertheless, the direction of the estimated effects is intuitive. Furthermore, the estimated separate counterfactuals for each policy approximately add up to the overall observed effect of the policy intervention in column (1) in Table 3 in the paper, which gives us some confidence that they are reasonable.

The counterfactuals show that firms that were granted forgiveness experienced longer-lasting reductions in interest rates. This meant that the effects of the policy were longer lasting than for firms that were provided with payment deferrals. These had a strong effect immediately after the policy was enacted, but the effect quickly ebbed off and even reversed. The reversal suggests that this type of forbearance may have been associated with firms which failed to recover by the time temporary support was cut back.

Table 2: Effect of removing the estimated forgiveness incentives (in %) - fixed labour

Counterfactuals - % change	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Capital stock	-0.74 %	-0.57 %	-1.21 %	-1.26 %	-1.60 %	-1.52 %	-2.15 %	-1.95 %	-2.20 %	-1.47 %
Capital productivity	0.28 %	0.23 %	0.45 %	0.51 %	0.62 %	0.60 %	0.86 %	0.79 %	0.74 %	0.56 %
Output, without reallocation	-1.29 %	-0.90 %	-1.75 %	-1.79 %	-2.16 %	-2.01 %	-2.74 %	-2.44 %	-2.93 %	-2.00 %
Output, with reallocation	6.56 %	6.42 %	4.83 %	3.61 %	1.01 %	0.42 %	1.10 %	2.26 %	-0.62 %	2.84 %

Table 3: Effect of removing the estimated financing incentives (in %) - fixed labour

Counterfactuals - % change	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Capital stock	-3.38 %	-2.91 %	-2.19 %	-1.67 %	-0.09 %	0.23 %	1.81 %	3.17 %	4.43 %	-0.07 %
Capital productivity	1.16 %	1.04 %	0.71 %	0.59 %	0.03 %	-0.08 %	-0.66 %	-1.21 %	-1.43 %	0.02 %
Output, without reallocation	-6.34 %	-4.99 %	-3.46 %	-2.60 %	-0.13 %	0.32 %	2.38 %	3.96 %	5.87 %	-0.55 %
Output, with reallocation	5.67 %	5.78 %	4.56 %	3.46 %	1.20 %	0.00 %	0.00 %	0.00 %	0.00 %	2.30 %

Tables 5 and 4 confirm the same narrative of these estimates for the case where labour input is assumed to be flexible and to adjust. Naturally, the response to the policies is stronger when labour input can adjust optimally.

Table 4: Effect of removing the estimated forgiveness incentives (in %) - flexible labour

Counterfactuals - Percent Change	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
Capital Stock	-4.37 %	-3.97 %	-4.92 %	-4.99 %	-5.07 %	-5.10 %	-5.23 %	-5.25 %	-4.58 %	-4.83 %
Output	-3.04 %	-2.80 %	-3.74 %	-3.95 %	-4.30 %	-4.41 %	-4.46 %	-4.36 %	-4.14 %	-3.91 %
Output assuming reallocation	7.24 %	6.96 %	5.48 %	4.20 %	1.50 %	0.86 %	1.61 %	2.79 %	-0.25 %	3.38 %
Capital productivity	1.38 %	1.22 %	1.25 %	1.10 %	0.81 %	0.73 %	0.81 %	0.95 %	0.45 %	0.97 %

Table 5: Effect of removing the estimated financing incentives (in %) - flexible labour

Counterfactuals - Percent Change	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**Average**
Capital Stock	-5.02 %	-4.96 %	-4.82 %	-4.59 %	0.10 %	2.00 %	4.78 %	5.15 %	4.49 %	-0.32 %
Output	-3.55 %	-3.54 %	-3.65 %	-3.63 %	0.09 %	1.74 %	4.06 %	4.22 %	3.99 %	-0.03 %
Output assuming reallocation	7.30 %	7.06 %	5.48 %	4.16 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	2.67 %
Capital productivity	1.55 %	1.49 %	1.24 %	1.00 %	-0.01 %	-0.25 %	-0.69 %	-0.88 %	-0.48 %	0.33 %

G Loan forbearance and lender characteristics

In this section, we explore the correlation between bank health and forbearance incentives in cross-sectional analysis. We measure bank health with the capital ratio and the non-performing loans (NPL) ratio as defined in Ogura and Uchida (2014) and Ogura (2018). Since the SME Financing Facilitation Act represented a plausibly exogenous policy shock from the point of view of lenders, we do not expect to find any significant correlation between forbearance incentives and a bank's capital ratio. By contrast, we expect banks with a higher NPL ratio to have stronger incentives to forbear. This is because the regulator allowed financial institutions to exclude the restructured SME loans from their reported non-performing loans under the condition that they came up with business restructuring plans.

We proxy forbearance incentives at the bank level with the average treatment intensity of its borrowers by taking the arithmetic average of its borrower's treatment intensity in 2009 for all types of forbearance. Each TSR borrower can be linked to up to 10 lenders. The average number of banks reported by firms is under 3.5. Therefore, we limit ourselves to three lenders per firm. In other words, a bank's borrowers are firms that report the bank as either the first, second, or third lender. We regress the average treatment intensity of the bank's borrowers in 2009 on the bank's capital ratio and non-performing loans ratio in 2008. Standard errors are clustered at the bank level. The results are in Table 6.

The correlation between average treatment intensity and the capital ratio is positive but barely significant in all the specifications. By contrast, there is a significantly positive correlation (at the 1% level) between the average treatment intensity of a bank's borrowers and its ratio of non-performing loans in the previous year. This correlation survives the inclusion of the lagged capital ratio. The coefficient on the lagged NPL ratio is 0.00732, indicating that a 1pp increase in the NPL ratio is associated with an increase of 0.7pp in the average treatment intensity of a bank's borrowers in 2009, just before the intervention. This is relatively small compared to an average treatment intensity of 0.35 across all banks in 2009, but nevertheless, an indication that a high NPL ratio might have been an incentive for

Table 6: Loan forbearance and lender characteristics

	(1)	(2)	(3)
Non-performing loans ratio$_{t-1}$	0.00730***		0.00732***
	(0.000437)		(0.000436)
Capital ratio$_{t-1}$		0.000638	0.000728*
		(0.000335)	(0.000336)
Constant	0.307***	0.340***	0.299***
	(0.00267)	(0.00393)	(0.00460)
Observations	12,654	12,654	12,654

Notes: Standard errors in parentheses. Standard errors are clustered at the bank level. *** p<0.01, ** p<0.05, * p<0.1.

banks to offer forbearance.

This analysis cannot shed any light on whether weak banks had more incentives to forbear because of reverse causation between the health of a bank's borrowers (and hence their treatment intensity) and the bank's own health. However, it indicates the absence of a strong correlation between low capitalisation and the practice of forbearance. This is in contrast to the finding that especially low-capitalised banks had an incentive to lend to zombies during the Lost Decade and more recently in Europe (e.g., Peek and Rosengren, 2005; Caballero et al., 2008; Watanabe, 2010; Bruche and Llobet, 2014; Schivardi et al., 2022; Acharya et al., 2020). The positive correlation between the NPL ratio and forbearance, on the other hand, indicates that banks might have had strong incentives to design successful restructuring plans for their borrowers in receipt of forbearance. Indeed, this would have allowed them to exclude the restructured SME loans from their reported non-performing loans under the rules set by the Japanese Financial Services Agency. In turn, these restructuring plans might help explain the positive treatment effect on TFP and the negative treatment effect on the probability of zombieness reported in the paper.

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