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Marco Bardoscia and Raymond Ka-Kay Pang

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Marco Bardoscia[1, 2] and Raymond Ka-Kay Pang[3]

Abstract

Ring-fencing is a reform of the UK banking system that requires large banks to separate their retail services from other activities of the group, such as investment banking. We consider a network of bilateral exposures between banks in which financial contagion can spread because banks incorporate the creditworthiness of their counterparties into the valuation of their assets. Ring-fencing acts as an exogenous shock that impacts the creditworthiness of banks through leverage, depending on how assets are allocated between ring-fenced and non-ring-fenced entities. We find conditions on this allocation that leads to safer ring-fenced entities and less safe non-ring-fenced entities when compared with their groups prior to the implementation of ring-fencing. We also show that ring-fencing can make both the equity of individual banking groups and the aggregate equity of the banking system decrease. When this happens, ring-fenced entities are safer than their groups prior to ring-fencing.

Key words: Ring-fencing, financial networks, systemic risk.

JEL classification: G21, G28.

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(1) Bank of England. Email: marco.bardoscia@bankofengland.co.uk
(2) University College London, Department of Computer Science.
(3) London School of Economics and Political Sciences.

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Bank of England, Threadneedle Street, London, EC2R 8AH
Email: enquiries@bankofengland.co.uk

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1 Introduction

After the Global Financial Crisis, several reforms have been put in place worldwide to limit the propagation of shocks between financial institutions and between those and the real economy. Ring-fencing refers to separating retail services provided by banks, such as taking deposits from households and small businesses, from investment and international banking. According to the Independent Commission on Banking (ICB), which recommended its introduction, ring-fencing would (ICB, 2011): protect the provision of core financial services to retail customers — deposit-taking, making and receiving payments — from shocks that might impact riskier activities; make it easier to resolve banks without taxpayer support; reduce excessive risk-taking driven by the expectation of government guarantees. For example, ICB argues that (ICB, 2011) for the Royal Bank of Scotland:

> *The ring-fence would have isolated its EEA banking operations from its global markets activities where most of its losses arose. Together with more loss-absorbent debt, this would have given the authorities credible alternative options to injecting £45bn of taxpayer funds into the group — e.g. isolating the ring-fenced bank for sale or temporary public ownership and an orderly wind-down of the rest of the group at no public cost.*

Ring-fencing has been introduced in the UK legislation with the Financial Services (Banking Reform) Act 2013 and it has come into effect at the beginning of 2019, affecting banks with more than £25 billion in retail deposits.[1]

We propose a theoretical model to analyse the implications of ring-fencing for systemic risk. In particular, we focus on one channel through which shocks can spread across banks: solvency contagion. When one bank's assets are hit by an exogenous shock, other banks re-evaluate the assets corresponding to their investment in that bank because they expect to recover a smaller proportion of their investment. As a consequence, also the values of *their own* assets (and therefore of their equities) diminish, triggering a second round of re-evaluations. Banks continue to adjust their asset valuations in subsequent rounds until the equilibrium valuation is reached (Bardoscia et al., 2019; Barucca et al., 2020). As a result, shocks propagate from one bank to another when their creditworthiness changes, even in the absence of defaults. This channel was especially active during the Global Financial Crisis. In fact, according to the Basel Committee on Banking Supervision, "roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses[2] and only about one-third were due to actual defaults" (BIS, 2011).

We treat the implementation of ring-fencing as an exogenous shock. In fact, ring-fencing requires some banking groups to split their activities between ring-fenced bodies (RFBs) and non-ring-fenced bodies (nRFBs). In our model, this means that the groups that implement ring-fencing allocate some of their assets and liabilities to their RFB and the remainder to their nRFB. Consistently with the spirit of the reform, RFBs can only hold *external* assets, corresponding for example to mortgages and corporate credit, and *external* liabilities, corresponding to deposits. In addition to those, nRFBs can also hold *interbank* assets, corresponding to investments in other banks, and interbank liabilities, corresponding to funding from other banks.[3] In reality, the allocations of assets and liabilities are partly determined by law and regulations and partly by banks' individual choices. In fact, while

[1] A full list of banking groups that have implemented ring-fencing is available in Bank of England (2022).

[2] Credit Valuation Adjustment (CVA) losses originate from incorporating counterparty credit risk into asset valuations.

[3] As a consequence, RFBs cannot invest or receive funding from any nRFB, including the nRFB within their own group. Therefore, as in Farhi and Tirole (2021), any RFB is not exposed to the nRFB belonging to the same group.

some activities must sit within RFBs and some other activities are "prohibited" and must sit within nRFBs, there are activities that banking groups can assign either to their RFB or to their nRFB, such as lending to large corporates. Within the context of our model, such allocation of assets and liabilities impacts the creditworthiness of RFBs and nRFBs in two ways. First, RFBs are insulated from counterparty credit risk because they do not hold interbank assets. Instead, nRFBs hold interbank assets and are exposed to counterparty credit risk, like banks before ring-fencing. Second, if external assets are allocated to RFBs and nRFBs in different proportions, also their leverage might change. We stress that these effects are mechanical, in the sense that they occur when assets and liabilities are allocated to RFBs and nRFBs, and *no further action is taken* to change either the size or the composition of the balance sheet of RFBs and nRFBs.

We now summarise our main results. First, we find conditions on the allocation of assets and liabilities that lead to a safer RFB, i.e. that make its probability of default smaller than to its group prior to ring-fencing. When the net interbank lending of a bank is zero, this happens when a larger proportion of assets than liabilities is allocated to its RFB, so that its leverage decreases with respect to the external leverage of its group prior to ring-fencing. However, when a bank is a net lender (borrower) in the interbank market, in order to decrease the external leverage of its RFB and therefore make it safer, it needs to allocate proportionally more (fewer) assets or fewer (more) liabilities to their RFB compared to the case in which their net interbank lending is zero.

Second, we find conditions that lead to a less safe nRFB with respect to its group prior to ring-fencing. Since nRFBs can hold interbank assets of other nRFBs, their creditworthiness depend on the creditworthiness of other nRFBs. Indeed, a nRFB becomes less safe than its group prior to ring-fencing when *both that group and all the groups to which that group is directly or indirectly exposed* implement ring-fencing so that the leverage of their RFBs is sufficiently below the external leverage of their groups prior to ring-fencing. As a consequence, while those RFBs have a smaller probability of default with respect to their groups prior to ring-fencing, the corresponding nRFBs have a larger external leverage than their groups prior to ring-fencing, and therefore also a larger probability of default.

Third, we compare the equity of the group after the introduction of ring-fencing (i.e. the equity of the RFB plus the equity of the nRFB) with the equity of the group prior to ring-fencing. Interestingly, we find that the equity of the group after the introduction of ring-fencing does not depend on whether that group actually implements ring-fencing (and how), but only on whether the groups to which that group is directly or indirectly exposed implement ring-fencing (and how). We find that when *all the groups to which that group is directly or indirectly* exposed bring the leverage of their RFBs sufficiently below the external leverage of their groups prior to ring-fencing, then the equity of that group is smaller after the implementation of ring-fencing. Intuitively, when sufficiently more assets are allocated to a RFB to lower its external leverage, making it safer, fewer assets are allocated to the corresponding nRFB, thereby increasing its external leverage and making it less safe. But a riskier nRFB has downstream effects on the other nRFBs exposed to it. The lost equity corresponds precisely to the loss in the value of interbank assets held by the nRFB that is exposed to these other riskier nRFBs.[4] We stress that this result does not apply to a group if, for example, one of the groups to which that group is directly or indirectly exposed implements ring-fencing by increasing the external leverage of its RFB, or by not decreasing it sufficiently. In this case, the equity of that group would not necessarily be smaller after the implementation of ring-fencing.

[4] If agents exposed to RFBs marked-to-market their assets by accounting for RFBs creditworthiness, their equity could increase when those RFBs are safer. Here we do not model explicitly the balance sheet of those agents, and therefore we do not investigate to what extent this increase would offset the equity loss of nRFBs.

Finally, we point out that the equity lost by one banking group after the implementation of ring-fencing is not gained by other banking groups. Indeed, if *all* groups implement ring-fencing so that their RFBs have a leverage sufficiently smaller than the external leverage of their groups prior to ring-fencing, then the aggregate equity of the banking system will be smaller after the implementation of ring-fencing.

Our model is necessarily stylised and therefore subject to some limitations. First, we assume that, after assets and liabilities of banking groups are allocated to RFBs and nRFBs, no further action is taken on the size or composition of their balance sheets. However, banking groups can react to the outcome of this allocation — either directly, or by responding to regulation — in several ways. One possibility is to deleverage the nRFB. Another possibility is to narrow the scope for solvency contagion, for example by reducing the exposure of the nRFB to other banks by converting some of its interbank assets into external assets, which corresponds to divesting from other banks and investing in the real economy. The alternative is to reallocate some external assets from the RFB to the nRFB. This reduces the external leverage of the nRFB, but it also increases the external leverage of the RFB, and therefore also its probability of default. Second, while we make the simplifying assumption that RFBs cannot hold interbank assets, in reality they could still be exposed to other RFBs. Those exposures could, everything else equal, lower their creditworthiness. Third, we assume that the intrinsic riskiness of external assets held by RFBs and nRFB is the same. In reality, by holding intrinsically safer assets, RFBs could be made safer also when their leverages are larger than the external leverages of their groups prior to ring-fencing.

Empirical research on ring-fencing in the UK focused on a few key aspects. Erten et al. (2022) find that RFBs face lower funding costs than prior to the implementation of the reform, whereas nRFBs funding costs do not change significantly. To the extent that probabilities of default can be taken as a proxy for funding costs, our model can accommodate that outcome. This could happen, for example, if a RFB had a smaller external leverage than the group prior to ring-fencing, *and* if banks to which the nRFB is exposed had also reduced their external leverage. This could occur either mechanically, as a result of the external assets and liabilities allocated to those nRFBs, or because those nRFBs decided to deleverage after the implementation of ring-fencing. Chavaz and Elliott (2020) find that groups affected by ring-fencing substantially reduce their investment banking activities prior to the implementation of ring-fencing. Moreover, RFBs are able to offer lower interest rates on deposits than prior to ring-fencing due to the change in their funding mix. In contrast, while our model captures the change in liabilities due to the implementation of ring-fencing, it does not capture any difference in banks' funding mix.

A larger literature explores the broader implications of separating retail and investment banking. Caprio et al. (2007) find that banks in countries that impose greater restrictions on bank activities tend to have lower valuations than in countries that impose fewer restrictions. Laeven and Levine (2007) find that valuations of groups that engage both in lending and non-lending activities are lower than if the groups were broken down into specialised entities. Indeed, our model predicts that, when all banking groups implement ring-fencing so that the external leverage (and therefore the riskiness) of RFBs decreases, also the equity valuation of banking groups decreases. Cornett et al. (2002) find that commercial banks with an investment subsidiary have larger cash returns on their assets, without being riskier. A number of studies (Kroszner and Rajan, 1994; Puri, 1994, 1996; Gande et al., 1997; Drucker and Puri, 2005) focuses on whether bundling retail and investment banking leads to synergies or conflicts of interest, and on the implications for their clients (Neuhann and Saidi, 2018; Akiyoshi, 2019). Theoretical studies suggest that separating retail and investment banking could reduce moral hazard and risk-taking (Boyd et al., 1998; Freixas et al., 2007; Farhi and Tirole, 2021).

We model solvency contagion due to the re-evaluation of interbank assets as in Bardoscia et al. (2019) and Barucca et al. (2020).[5] This means that in contrast with models of contagion on default (Eisenberg and Noe, 2001; Rogers and Veraart, 2013), shocks propagate to counterparties even in the absence of defaults. Risks stemming from this channel appear to have peaked during the GFC and sharply decreased since (Bardoscia et al., 2019), but to be concentrated (Fink et al., 2016). Several studies have investigated how structural features of the financial network impact its stability and resilience to shocks (Allen and Gale, 2000; Freixas et al., 2007; Nier et al., 2007; Gai and Kapadia, 2010; Battiston et al., 2012a,b; Elliott et al., 2014; Acemoglu et al., 2015; Bardoscia et al., 2017). Here, even though the implementation of ring-fencing changes affects banks' balance sheets, it does not change the underlying structure of the financial network. In fact, RFBs are fully isolated from the rest of the network and the network of nRFBs is identical to the network of banking groups prior to ring-fencing.

The paper is organised as follows. In Section 2 we discuss the institutional details of ring-fencing in the UK and briefly compare it to some other jurisdictions, in Section 3 we introduce a simple model of ring-fencing for one bank, whereas in Section 4 we extend the model to the case of multiple banks interconnected in a financial network and derive our main results. We draw our conclusions in Section 5.

2 Institutional details

The UK ring-fencing regime has required banks with more than £25 billion in retail deposits to separate their retail and investment activities by the beginning of 2019. In practice, this means that such banks must create a new legal entity, the ring-fenced bank (RFB). The legislation specifies "core activities", which can be provided only by RFBs, and "prohibited activities", which cannot be provided by RFBs. The following lists closely follow Britton et al. (2016). Core activities include taking deposits from retail and SMEs in the UK, whereas prohibited activities include: Trading and selling securities, commodities and derivatives; having exposures to financial institutions other than other RFBs; having operations outside the EEA; underwriting securities; buying securitisations of other financial institutions. Prohibited activities can be provided by other entities within the same banking group. For convenience, we collectively refer to those entities as the non-ring-fenced bank (nRFB). Some other activities can be provided both by RFBs and by nRFBs: deposit-taking activities for large corporates and other RFBs; lending to individuals and corporates; transactions with central banks; holding own securitisations; trade finance; payment services; hedging liquidity, interest rate, currency, commodity and credit risks; selling simple derivatives to corporates and other RFBs.

Importantly, the RFB is required to be independent of the nRFB within the same banking group. This means, for example, that the RFB governance and management should allow the RFB to make decisions in its own interest, independently of the rest of the group. It also means that the RFB is subject to capital and liquidity requirements separately from other entities within the banking group. Finally, the financial relationships between the RFB and the nRFB within the same group, if any, should not be privileged when compared to those between the RFB and other financial institutions.

An independent review of the ring-fencing regime in the UK has been recently published (RFPT, 2022). The final report acknowledges that the regime has made the UK banking system safer because RFBs are easier to supervise. However, it also recognises that the

[5] For alternative valuation frameworks, see Elsinger et al. (2006); Fischer (2014); Veraart (2020). For earlier work on pre-default contagion, see Bardoscia et al. (2015, 2016); Fink et al. (2016).

definition of critical functions provided by banks has broadened since the original recommendation by the ICB to include activities that fall within nRFBs (see PRA (2014)). Moreover, it suggests that the resolution regime might be sufficient for implicit government guarantees to too-big-to-fail banks and that ring-fencing might be redundant in this respect. The report includes several recommendations, on which the UK government has announced the intention to consult (HMT, 2022; HMT, 2023).

A similar structural separation was initially proposed (Liikanen, 2012) and then rejected in the EU. In the US the 1933 Glass-Stegall Act prohibited banks that took insured deposits from providing investment banking activities. This separation was stronger than the current UK ring-fencing regime, as the deposit-taker and the entity providing investment banking activities could not be part of the same banking group. The provisions of the 1933 Glass-Stegall Act were gradually relaxed over time, eventually allowing those entities to be part of the same group (for more details, see Appendix A in Chavaz and Elliott (2020)). This remains the case also after the GFC when the Dodd-Frank Act imposed further restrictions on the relationships between the two entities.

3 Ring-fencing one bank

We start by discussing the mechanics of ring-fencing one bank. We consider one bank that, prior to ring-fencing, has assets A and liabilities (debt) L.[6] After ring-fencing, the bank is split into two separate entities: the ring-fenced bank (RFB) with assets A^{RF} and liabilities L^{RF}, and the non-ring-fenced bank (nRFB) with assets A^{nRF} and liabilities L^{nRF}. Total assets and liabilities of the bank prior to ring-fencing are simply $A = A^{\mathrm{RF}} + A^{\mathrm{nRF}}$ and $L = L^{\mathrm{RF}} + L^{\mathrm{nRF}}$. A and L can be also interpreted as the total assets and liabilities of the banking group that consolidates the balance sheets of the RFB and nRFB after the implementation of ring-fencing.

Let $\psi^A \in [0,1]$ and $\psi^L \in [0,1]$ be respectively the fraction of total assets and liabilities of the RFB. Therefore, $1-\psi^A$ and $1-\psi^L$ are the fraction of the total assets and liabilities of the nRFB:

$$A^{\mathrm{RF}} = \psi^A A, \qquad A^{\mathrm{nRF}} = (1-\psi^A)A, \tag{1a}$$
$$L^{\mathrm{RF}} = \psi^L L, \qquad L^{\mathrm{nRF}} = (1-\psi^L)L\,. \tag{1b}$$

For all entities, equities are defined as the difference between assets and liabilities:

$$E = A - L, \tag{2a}$$
$$E^{\mathrm{RF}} = A^{\mathrm{RF}} - L^{\mathrm{RF}} = \psi^A A - \psi^L L, \tag{2b}$$
$$E^{\mathrm{nRF}} = A^{\mathrm{nRF}} - L^{\mathrm{nRF}} = (1-\psi^A)A - (1-\psi^L)L\,. \tag{2c}$$

Therefore, we have that the equity of the bank prior to ring-fencing (or of the consolidated group) is equal to the sum of the equities of the RFB and of the nRFB:

$$E = E^{\mathrm{RF}} + E^{\mathrm{nRF}}\,. \tag{3}$$

Eq. (3) may seem very intuitive, but we anticipate that this identity will not necessarily hold for banks embedded in a financial network.

The corner cases $\psi^A = 0$ and $\psi^A = 1$ correspond to transferring all assets to the RFB or to the nRFB. $\psi^L = 0$ means that the RFB is fully funded by equity ($E^{\mathrm{RF}} = \psi^A A$), whereas

[6] We refer to the entity prior to the implementation of ring-fencing as to the "bank" or to the "banking group" interchangeably.

$\psi^L = 1$ means that the nRFB is fully funded by equity ($E^{\text{nRF}} = (1 - \psi^A)A$). Starting from a solvent bank ($E > 0$) it is certainly possible to implement ring-fencing, i.e. to pick ψ^A and ψ^L so that either the RFB or the nRFB are not solvent. We rule out those cases as there would be no point in implementing ring-fencing if one of the two entities were not solvent. More precisely, we say that ring-fencing with the pair (ψ^A, ψ^L) is *feasible* or that the bank *implements ring-fencing feasibly* if $E^{\text{RF}} > 0$ and $E^{\text{nRF}} > 0$. In the following, we will assume that (ψ^A, ψ^L) is feasible. We note that if (ψ^A, ψ^L) is feasible, also the bank prior to ring-fencing is solvent ($E > 0$).

We now introduce the *leverage* of the bank prior to ring-fencing and of the RFB and nRFB. Provided that they are solvent (i.e. that their equities are strictly positive), the leverages of the bank prior to ring-fencing, the RFB, and the nRFB are the ratios between their assets and equity:

$$\lambda = \frac{A}{E} \tag{4a}$$

$$\lambda^{\text{RF}} = \frac{A^{\text{RF}}}{E^{\text{RF}}} = \frac{\psi^A A}{E^{\text{RF}}} \tag{4b}$$

$$\lambda^{\text{nRF}} = \frac{A^{\text{nRF}}}{E^{\text{nRF}}} = \frac{(1 - \psi^A)A}{E^{\text{nRF}}} . \tag{4c}$$

The relationship between the leverages of the RFB and the nRFB depends on ψ^A and ψ^L. We have the following result.

Proposition 1. *Let the bank implement ring-fencing feasibly, i.e. let the RFB and nRFB be solvent. The following statements are equivalent:*

- *A larger proportion of assets than liabilities is allocated to the RFB:*

$$\psi^A \geq \psi^L ,$$

- *The leverage of the RFB is smaller than the leverage of the bank prior to ring-fencing:*

$$\lambda^{RF} \leq \lambda ,$$

- *The leverage of the nRFB bank is larger than the leverage of the bank prior to ring-fencing:*

$$\lambda \leq \lambda^{nRF} .$$

Similar equivalences hold with reversed inequalities.

One immediate implication of Proposition 1 is that there are three possibilities. First, $\psi^A > \psi^L$, in which case we have that $\lambda^{\text{RF}} < \lambda < \lambda^{\text{nRF}}$. Second, $\psi^A < \psi^L$, in which case we have that $\lambda^{\text{RF}} > \lambda > \lambda^{\text{nRF}}$. Third, $\psi^A = \psi^L$, in which case we have that $\lambda^{\text{RF}} = \lambda = \lambda^{\text{nRF}}$. As a consequence, if the RFB has a smaller leverage than the bank prior to ring-fencing, the nRFB must necessarily have a larger leverage.

So far we have made no explicit assumptions on the intrinsic riskiness of the assets. Let us assume that assets A follow a geometric Brownian motion: $dA(t) = \mu A(t)dt + \sigma A(t)dW(t)$, where μ is the drift and σ the volatility. In order to introduce probabilities of default, we assume that investors are risk-neutral. In this context, the probability of default in the interval $[t, T]$ is defined as the probability that the equity of the bank is negative at time T, conditional on the information available at time t. When there are no arbitrage opportunities and the market is complete, it can be computed as the conditional risk-neutral expectation $p_{[t,T]} = \mathbb{E}^{\mathbb{Q}} \left[\mathbb{1}_{E(T)<0} | A(t) \right]$, where $A(t)$ is the value of assets observed at time t. By further assuming that no dividends are distributed and that the risk-free rate is equal to zero, we have (Merton, 1974):

$$p_{[t,T]} = 1 - \mathcal{N}\left[\frac{\log\frac{A(t)}{A(t)-E(t)} - \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}\right], \tag{5}$$

where $\mathcal{N}$ is the cumulative distribution function of the normal distribution. For convenience, in the following, we drop the dependence on time, as we consider all quantities to be observed at time t. Eq. (5) can be re-written as:

$$p_{[t,T]} = 1 - \mathcal{N}\left[\frac{\log\frac{\lambda}{\lambda-1} - \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}\right],$$

which is a function only of the leverage λ and the volatility σ. More precisely, $p_{[t,T]}$ is an increasing function of both λ and σ. This property is shared also by other credit structural models, such as Black and Cox (1976), in which banks default as soon as their equity becomes negative.

By multiplying both sides of $dA(t)$ by ψ^A and $1-\psi^A$, we have that: $dA^{\mathrm{RF}}(t) = \mu A^{\mathrm{RF}}(t)dt + \sigma A^{\mathrm{RF}}(t)dW(t)$ and that $dA^{\mathrm{nRF}}(t) = \mu A^{\mathrm{nRF}}(t)dt + \sigma A^{\mathrm{nRF}}(t)dW(t)$, meaning that both A^{RF} and A^{nRF} follow the same geometric Brownian motion as A. By combining Proposition 1 with the fact that $p_{[t,T]}$ is increasing with λ we have the following result.

Corollary 1. *Let the bank implement ring-fencing feasibly, i.e. let the RFB and nRFB be solvent, and let probabilities of default be increasing functions of leverage.*

If a larger proportion of external assets than external liabilities is allocated to the RFB:

$$\psi^A \geq \psi^L,$$

then the probability of default of the RFB is smaller than or equal to the probability of default of the bank prior to ring-fencing, which is, in turn, smaller than or equal to the probability of default of the nRFB:

$$p^{RF}_{[t,T]} \leq p_{[t,T]} \leq p^{nRF}_{[t,T]}.$$

Vice versa, if a smaller proportion of external assets than external liabilities is allocated to the RFB:

$$\psi^A \leq \psi^L,$$

then the probability of default of the RFB is larger than or equal to the probability of default of the bank prior to ring-fencing, which is, in turn, larger than or equal to the probability of default of the nRFB:

$$p^{RF}_{[t,T]} \geq p_{[t,T]} \geq p^{nRF}_{[t,T]}.$$

This result is intuitive: if $\psi^A \geq \psi^L$ the RFB is *less* leveraged than the bank prior to ring-fencing, which is less leveraged than the nRFB. Therefore, the probability of default of the RFB is smaller than the probability of default of the bank prior to ring-fencing, which is, in turn, smaller than the probability of default of the nRFB. Corollary 1 will serve as the blueprint of similar results in the case in which banks are embedded in a financial network.

4 Ring-fencing banks in a financial network

Now we consider a set of N banks, prior to the implementation of ring-fencing, denoted with $\mathcal{N} = \{1, \ldots, N\}$. In this framework, we assume banks are able to lend to each other. When bank i lends to bank j, bank i holds an interbank asset A_{ij} and bank j holds a matching interbank liability $L_{ji} = A_{ij}$. We denote the total interbank assets and liabilities of bank i with $\bar{A}_i = \sum_{j=1}^{N} A_{ij}$ and $\bar{L}_i = \sum_{j=1}^{N} L_{ij}$ respectively. The matrices of interbank assets $\mathbf{A}$ (or equivalently of interbank liabilities $\mathbf{L}$) can be thought of as the weighted adjacency matrix of the interbank network. It will be useful to denote with $\mathcal{G}^A$ and $\mathcal{G}^L$ respectively the graphs defined by the weighted adjacency matrices $\mathbf{A}$ and $\mathbf{L}$. In addition to interbank assets and liabilities, each bank i also holds external assets A_i^e and external liabilities L_i^e. In order to avoid corner cases we assume that $A_i^e > 0$ for all i, that is that all banks prior to ring-fencing hold some (possibly very small amount of) external assets.

Ring-fencing is implemented similarly to Section 3, with the additional assumption that RFBs can only hold external assets and liabilities, or equivalently that only nRFBs can hold interbank assets and liabilities. Indeed, interbank assets and liabilities appear to constitute a negligible proportion of RFBs balance sheets at the end of 2020 (see Figures 3.3 and 3.4 in RFPT, 2022). External assets are investments in entities outside the financial network, such as mortgages or corporate lending. External liabilities correspond to funding provided by entities outside the financial network, such as deposits or bonds. In practice, deposit-taking from households and SMEs must be carried out by RFBs, but deposit-taking from large corporates and lending to households and corporates can be carried out either by RFBs or by nRFBs. Therefore, external assets and liabilities can be held both by RFBs and nRFBs.[7]

Let $\mathbf{\Psi}^A$ and $\mathbf{\Psi}^L$, with $\psi_i^A \in [0,1]$ and $\psi_i^L \in [0,1]$, for $i = 1, \ldots, N$ be respectively the vectors of the fractions of *external* assets and liabilities of the RFB. External assets and liabilities of the RFB i are equal to $\psi_i^A A_i^e$ and $\psi_i^L L_i^e$, whereas external assets and liabilities of the nRFB i are equal to $(1-\psi_i^A)A_i^e$ and $(1-\psi_i^L)L_i^e$. If $\psi_i^A = 0 = \psi_i^L$ no assets or liabilities are transferred to the RFB i, meaning that in practice no ring-fenced entity is created from bank i. Formally, in our model, this corresponds to the nRFB i being equal to bank i prior to the implementation of ring-fencing. For this reason, we also refer to banks that after the implementation of ring-fencing have not actually created any ring-fenced entity as nRFBs. As a consequence, we can split banks into two sets. Let:

$$\overline{\mathcal{R}} = \left\{ i \in \mathcal{N} : \psi_i^A = 0 \quad \text{and} \quad \psi_i^L = 0 \right\} , \tag{6}$$

be the set of banks that do not implement ring-fencing i.e. that do not transfer any assets or liabilities to a RFB. Let:

$$\mathcal{R} = \mathcal{N} \setminus \overline{\mathcal{R}} \tag{7}$$

be a set of banks that implement ring-fencing, i.e. that transfer some assets or liabilities to a RFB. Clearly, $\mathcal{R} \cup \overline{\mathcal{R}} = \mathcal{N}$. In order to avoid corner cases, we assume that $\psi_i^A < 1$ for all i, that is that all nRFBs hold some (possibly very small amount of) external assets.

As already mentioned, in reality, ψ_i^A and ψ_i^L are partly determined by law and regulation and partly by banks' choices. For example, banks that are not required to implement ring-fencing are unlikely to implement it purely by choice and will therefore be part of $\overline{\mathcal{R}}$. Banks that are required to implement ring-fencing must allocate assets and liabilities corresponding to core activities to their RFB and assets and liabilities corresponding to prohibited activities to their nRFB but can choose where to allocate assets and liabilities corresponding to neither core nor prohibited activities. Therefore, banks that implement ring-fencing can choose, to a certain extent, both ψ_i^A and ψ_i^L.

[7]Because we assume RFBs cannot hold interbank assets or liabilities, they are completely disconnected from the financial network, including from the nRFB within their own group. In reality, while RFBs cannot be exposed to nRFBs, they could still be exposed to other RFBs, and they could be funded by nRFBs. In other words, RFBs could still hold interbank assets (towards other RFBs) and interbank liabilities (from other RFBs and nRFBs). Accounting for this possibility would, however, considerably complicate the analysis.

4.1 Naive equity and external leverage

Banks that hold interbank assets (i.e. banks prior to ring-fencing and nRFBs) perform a valuation of their interbank assets based on the creditworthiness of their counterparties. As explained in Section 4.2, these valuations will impact their equity. For the moment, we introduce *naive equities*, which do *not* incorporate valuations of interbank assets. Therefore, they are defined as the difference between total assets and liabilities computed by taking interbank assets at their face value and are denoted with the superscript 0. For banks prior to ring-fencing and nRFBs, we have, for all i:

$$E_i^0 = A_i^e + \bar{A}_i - L_i^e - \bar{L}_i \tag{8a}$$

$$= A_i^e + \sum_{j=1}^{N} A_{ij} - L_i^e - \sum_{j=1}^{N} L_{ij} \tag{8b}$$

$$E_i^{\text{nRF},0} = (1-\psi_i^A)A_i^e + \bar{A}_i - (1-\psi_i^L)L_i^e - \bar{L}_i, \tag{8c}$$

$$= (1-\psi_i^A)A_i^e + \sum_{j=1}^{N} A_{ij} - (1-\psi_i^L)L_i^e - \sum_{j=1}^{N} L_{ij}\,. \tag{8d}$$

RFBs do not hold interbank assets, and therefore we do not distinguish between their naive equities ($E_i^{\text{RF},0}$) and the equities that incorporates the valuation of interbank assets (E_i^{RF}). For all i:

$$E_i^{\text{RF},0} = E_i^{\text{RF}} = \psi_i^A A_i^e - \psi_i^L L_i^e\,. \tag{8e}$$

Analogously to Section 3, we have that the naive equity of the bank i prior to ring-fencing (or of the consolidated group) is equal to the sum of the equity of the RFB and of the naive equity of the nRFB:

$$E_i^0 = E_i^{\text{RF}} + E_i^{\text{nRF},0}\,. \tag{9}$$

Similarly to Section 3, we rule out cases in which either at least one RFB is not solvent (there exists i such that $E_i^{\text{RF}} \le 0$) or in which at least one nRFB is not *naively* solvent (there exists i such that $E_i^{\text{nRF},0} \le 0$). We define the pair $(\mathbf{\Psi}^A, \mathbf{\Psi}^L)$ to be *feasible* if all pairs (ψ_i^A, ψ_i^L) are feasible, i.e. if $E_i^{\text{RF}} > 0$ and $E_i^{\text{nRF},0} > 0$, for all i. When the pair (ψ_i^A, ψ_i^L) is feasible we also say that bank i implements ring-fencing feasibly and when the pair $(\mathbf{\Psi}^A, \mathbf{\Psi}^L)$ is feasible that all banks implement ring-fencing feasibly. In the following we will assume that $(\mathbf{\Psi}^A, \mathbf{\Psi}^L)$ is always feasible. If $(\mathbf{\Psi}^A, \mathbf{\Psi}^L)$ is feasible, also all banks prior to ring-fencing are naively solvent ($E_i^0 > 0$, for all i).

When banks are embedded in a financial network an important role is played by the external leverage, that is the leverage restricted to external assets. More precisely, we introduce the *naive external leverage*, which is computed with naive equities. Provided that they are naively solvent (i.e. that their naive equities are strictly positive), the naive external leverages of bank i prior to ring-fencing and of RFB i and nRFB i are the ratios between their external assets and naive equity:

$$B_i^0 = \frac{A_i^e}{E_i^0} \tag{10a}$$

$$B_i^{\text{RF},0} = \frac{\psi_i^A A_i^e}{E_i^{\text{RF},0}} \tag{10b}$$

$$B_i^{\text{nRF},0} = \frac{(1-\psi_i^A)A_i^e}{E_i^{\text{nRF},0}}\,. \tag{10c}$$

As RFBs do not hold interbank assets, their assets are only external and their equities are equal to their naive equities. Hence, the naive external leverage of a RFB is equal to its leverage, i.e. for all i:

$$B_i^{\mathrm{RF},0} = \frac{\psi_i^A A_i^e}{E_i^{\mathrm{RF},0}} = \frac{\psi_i^A A_i^e}{E_i^{\mathrm{RF}}} = \lambda_i^{\mathrm{RF}} .$$

It is possible to extend Proposition 1 to the case in which banks are embedded in a financial network.

Proposition 2. *Let bank i implement ring-fencing feasibly, i.e. let RFB i and nRFB i be naively solvent. The following statements are equivalent:*

- *The allocation of assets and liabilities to the RFB is such that:*

$$\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i) \right) \geq \psi_i^L L_i^e .$$

- *The leverage of the RFB is smaller than the naive external leverage of the bank prior to ring-fencing:*

$$\lambda_i^{RF} \leq B_i^0 .$$

- *The naive external leverage of the nRFB bank is larger than the naive external leverage of the bank prior to ring-fencing:*

$$B_i^0 \leq B_i^{nRF,0} .$$

Similar equivalences hold with reversed inequalities.

Proposition 1 means that, in the case of one bank, the knowledge ψ^A and ψ^L is sufficient to determine whether implementing ring-fencing decreases the leverage of the RFB and increases the leverage of the nRFB with respect to the bank prior to ring-fencing. Instead, when bank i is embedded in a financial network, the knowledge ψ_i^A and ψ_i^L is not sufficient anymore, and one needs to know also *net* interbank assets $\bar{A}_i - \bar{L}_i$ and external liabilities L_i^e. In particular, we have that:

Corollary 2. *Let bank i implement ring-fencing feasibly, i.e. let RFB i and nRFB i be naively solvent. If:*

$$\bar{A}_i - \bar{L}_i \geq L_i^e ,$$

then:

$$\lambda_i^{RF} \geq B_i^0 \geq B_i^{nRF,0} .$$

If *net* interbank assets of bank i are larger than (or equal to) its external liabilities, then implementing ring-fencing increases the leverage of the RFB i and decreases the naive external leverage of the nRFB i with respect to bank i prior to ring-fencing.

By comparing Proposition 2 with Proposition 1 we can assess the impact of interbank lending and borrowing on naive external leverage. Let us recall from Proposition 1 that, when a bank is not embedded in a financial network, that is when that bank does not lend or borrow from other banks, then allocating a larger proportion of assets than liabilities to the RFB ($\psi^A > \psi^L$) has the effect of lowering the leverage of the RFB ($\lambda^{\mathrm{RF}} < \lambda$). When the net interbank lending of bank i is equal to zero ($\bar{A}_i = \bar{L}_i$), then the condition to lower the naive external leverage of the RFB is the same as when bank i does not lend to and borrow from other banks ($\psi_i^A > \psi_i^L$).

However, when bank i is a net lender to other banks ($\bar{A}_i > \bar{L}_i$), in order to lower the leverage of its RFB ($\lambda_i^{\mathrm{RF}} < B_i^0$), bank i needs to allocate proportionally more assets (or fewer liabilities) to the RFB with respect to the case in which its net interbank lending is zero. Similarly, when bank i is a net borrower from other banks ($\bar{A}_i < \bar{L}_i$) in order to lower the leverage of its RFB ($\lambda_i^{\text{RF}} < B_i^0$), bank i needs to allocate proportionally fewer assets (or more liabilities) to the RFB with respect to the case in which its net interbank lending is zero.

4.2 Valuation framework

Banks that hold interbank assets perform the valuation of their interbank assets by applying a discount factor known as *valuation function*.[8] Intuitively, valuation functions quantify banks' creditworthiness. When the valuation function of bank i is equal to one, other banks that have invested in it expect to recover their investment fully and therefore take their investment at face value. Conversely, when the valuation function of bank i is equal to zero, other banks expect to recover nothing, and therefore fully write off the corresponding asset.

Following Barucca et al. (2020), it is convenient to introduce valuation functions by isolating their dependence on equity (E) from their dependence on additional quantities ($\mathcal{C}$).[9]

Definition 1 (Valuation function, Barucca et al. (2020))**.** *A function $\mathbb{V} : \mathbb{R} \to [0, 1]$ is called a feasible valuation function if and only if:*

- *It is non-decreasing $E \leq E' \implies \mathbb{V}(E|\mathcal{C}) \leq \mathbb{V}(E'|\mathcal{C}) \quad \forall E, E' \in \mathbb{R}$*
- *It is continuous from above.*

Valuation functions are decreasing with the equity because, all other things being equal, a smaller equity indicates a deterioration of the creditworthiness and therefore a smaller discount factor. Continuity from above is a technical requirement and its usefulness will be clarified later.

The valuation of interbank assets feeds into equity valuations. The equity valuation of bank i will depend on the valuation of bank i's interbank assets and, via valuation functions, on the equity valuations of bank i's counterparties. In equilibrium, equity valuations $\mathbf{E}^*$ of all banks are self-consistent and, for all i, satisfy:

$$E_i^* = A_i^e + \sum_{j=1}^{N} A_{ij} \mathbb{V}(E_j^*|\mathcal{C}_j) - L_i^e - \sum_{j=1}^{N} L_{ij}, \tag{11a}$$

$$E_i^{\text{nRF},*} = (1 - \psi_i^A) A_i^e + \sum_{j=1}^{N} A_{ij} \mathbb{V}(E_j^{\text{nRF},*}|\mathcal{C}_j^{\text{nRF}}) - (1 - \psi_i^L) L_i^e - \sum_{j=1}^{N} L_{ij}\,, \tag{11b}$$

which is analogous to (8), but with valuation functions. As anticipated, with $\mathcal{C}_j$ and $\mathcal{C}_j^{\text{nRF}}$ we denote quantities on which valuation functions depend, in addition to equity. We note that $\mathbf{E}^*$ can be negative. Therefore, those should be interpreted as valuations of banks' net worth and the valuations of shares held by investors that enjoy limited liability.

In general, the set of equations (11a) and (11b) admit more than one solution, but in Barucca et al. (2020) it is shown that they always admit one *greatest solution*, i.e. one solution in which the equity of each bank is not smaller than its equity in any other solution. In other

[8] Clearly such valuation is performed only by banks that hold interbank assets, that is by banks prior to ring-fencing or by nRFBs.

[9] In Barucca et al. (2020) the valuation function of bank i can in principle depend on the vector of equities of all banks. Here we simplify the exposition by focusing on the case in which it depends explicitly only on the quantities relative to bank i.

words, there are no solutions in which any bank is better off than the greatest solution. Moreover, the greatest solution can be easily computed iteratively by starting from naive equities $\mathbf{E}^0$ and $\mathbf{E}^{\mathrm{nRF},0}$ and by iterating (11a) and (11b) until convergence. Further details on the existence and convergence of the greatest solution is provided in Appendix A. From this point onwards we will focus on equities corresponding to the greatest solution, which we will denote with $\mathbf{E}^*$ for banks prior to ring-fencing and with $\mathbf{E}^{\mathrm{nRF},*}$ for nRFBs.

We define the (non-naive) *external leverage* of each bank prior to ring-fencing and nRFB as the ratio between their external assets and their equity valuations. Since the greatest solution for equities is smaller than or equal to naive equities (both for banks prior to ring-fencing and for nRFBs), assuming that (ψ_i^A, ψ_i^L) is feasible does not ensure that $E_i^* > 0$ or that $E_i^{\mathrm{nRF}*} > 0$. Therefore, in order to introduce the external leverage, we need to explicitly assume that the equity valuations of banks prior to ring-fencing and of nRFBs are strictly positive. Provided that they are solvent (i.e. that their equity valuations are strictly positive), the external leverages of bank i prior to ring-fencing and of nRFB i are the ratios between their external assets and equity valuations:

$$B_i^* = \frac{A_i^e}{E_i^*} \tag{12a}$$

$$B_i^{\mathrm{nRF},*} = \frac{(1-\psi_i^A)A_i^e}{E_i^{\mathrm{nRF},*}} . \tag{12b}$$

In the following, we will consider valuation functions that depend explicitly only on external leverage and on the volatility of external assets. Before introducing formally such a class of valuation functions, we motivate this choice by following the approach in Bardoscia et al. (2019). In order to keep the notation light, let us focus for a moment on banks prior to ring-fencing. The same line of reasoning applies to nRFBs.

Bardoscia et al. (2019) derive the functional form for valuation functions under the following assumptions: (i) the recovery rate on defaulted interbank assets is equal to zero[10]; (ii) the external assets of banks follow independent geometric Brownian motions with drifts $\boldsymbol{\mu}$ and volatilities $\boldsymbol{\sigma}$; (iii) banks perform a risk-neutral valuation of interbank assets; (iv) there are no arbitrage opportunities and the market is complete; (iv) banks do not distribute dividends; (v) the risk-free rate is equal to zero. Bardoscia et al. (2019) consider, two different definitions of banks' default. If banks default when their equities are smaller than or equal to zero at time T (as in Merton (1974)), then the valuation functions at time $t \le T$ are:

$$\mathbb{V}\left(E(t)|\mathcal{C}(t)\right) = \begin{cases} 1 & \text{if} \quad E(t) \ge A^e(t), \\ \mathcal{N}\left[\frac{\log \frac{A^e(t)}{A^e(t)-E(t)} - \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}\right] & \text{if} \quad E(t) < A^e(t). \end{cases} \tag{13a}$$

If banks default as soon as their equities become smaller than or equal to zero between times t and T (as in Black and Cox (1976)), then the valuation functions at time $t \le T$ are:

$$\mathbb{V}\left(E(t)|\mathcal{C}(t)\right) = \begin{cases} 1 & \text{if} \quad E(t) \ge A^e(t), \\ \mathcal{N}\left[\frac{\log \frac{A^e(t)}{A^e(t)-E(t)} - \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}\right] & \\ \qquad - \frac{A^e(t)}{A^e(t)-E(t)}\mathcal{N}\left[\frac{-\log \frac{A^e(t)}{A^e(t)-E(t)} - \frac{\sigma^2(T-t)}{2}}{\sigma\sqrt{T-t}}\right] & \text{if} \quad 0 \le E(t) < A^e(t), \\ 0 & \text{if} \quad E(t) < 0. \end{cases} \tag{13b}$$

[10] Valuation functions can still be larger than zero because there is uncertainty on whether defaults will occur or not in the future.

In both cases, Bardoscia et al. (2019) show that probabilities of default at time t are:

$$p_{[t,T]} = 1 - \mathbb{V}\left(E(t)|\mathcal{C}(t)\right) . \tag{14}$$

We note that valuation functions (13) depend explicitly only on $A^e(t)/E(t)$, which is the external leverage at time t, and on $\sigma\sqrt{T-t}$, which is the volatility of external assets over a period $T-t$. As one would expect, those are decreasing functions of the external leverage and of the volatility of external assets. Intuitively, if one bank has a smaller external leverage or if the volatility of its external assets is smaller, its counterparties will deem it to be safer. Therefore, they will expect to recover a larger proposition of their investments in that bank, which corresponds to a larger valuation function for that bank.

We can use (13) and (14) also to compute valuation functions and probabilities of default for nRFBs, simply by plugging in the appropriate quantities: $\mathcal{C}_i^{\mathrm{nRF}}(t) = \{(1-\psi_i^A)A_i^e(t), \sigma_i\sqrt{T-t}\}$. We point out that it makes sense to compute (13) and (14) also in the case in which banks do not hold any interbank assets or interbank liabilities. In this case, external assets are equal to total assets and the volatility of external assets is equal to the volatility of total assets, and indeed one recovers the original results in Merton (1974) and Black and Cox (1976). In particular, this means that one can use (13) and (14) to compute the probability of default of RFBs, again by plugging in the appropriate quantities: $\mathcal{C}_i^{\mathrm{RF}}(t) = \{\psi_i^A A_i^e(t), \sigma_i\sqrt{T-t}\}$.

For the sake of brevity, and with a slight abuse of notation, in the following, we drop the dependence of all quantities on t and denote the volatility of external assets over a period $T-t$ simply with σ. Let us now introduce the following class of valuation functions.

Definition 2 (Simple ex-ante valuation functions). *A function $\mathbb{V}(E|\mathcal{C})$ with $\mathcal{C} = \{A^e, \sigma\}$, $A^e > 0$, $\sigma > 0$ is a simple ex-ante valuation function if it depends explicitly only on the inverse of external leverage $\tilde{B} = E/A^e$ and on the volatility of external assets σ:*

$$\mathbb{V}(E|\mathcal{C}) = f(\tilde{B}, \sigma) ,$$

where $f : \mathbb{R} \times \mathbb{R}^+ \to [0,1]$ satisfies the following properties:

- *it is non-decreasing in $\tilde{B}$,*
- *it is right-continuous in $\tilde{B}$.*

One can check that simple ex-ante valuation functions satisfy the hypotheses of Definition 1, and therefore are also feasible valuation functions. Moreover, it is easy to see that both valuation functions in (13) are simple ex-ante valuation functions and are also non-increasing in σ.[11] Having assumed that $A_i^e > 0$ and $\psi_i^A < 1$ for all i ensures that simple ex-ante valuation functions are well-defined for all banks prior to ring-fencing and for all nRFBs.

All our results apply to simple ex-ante valuation functions, to the extent that one interprets valuation functions to be equal to one minus probabilities of default, as in equilibrium — that is at the greatest solution — for banks prior to ring-fencing and for nRFBs:

$$p_{i,[t,T]}^{*} = 1 - \mathbb{V}(E_i^*|\mathcal{C}_i), \tag{15a}$$

$$p_{i,[t,T]}^{\mathrm{nRF},*} = 1 - \mathbb{V}(E_i^{\mathrm{nRF},*}|\mathcal{C}_i^{\mathrm{nRF}}) , \tag{15b}$$

whereas for RFBs:

$$p_{i,[t,T]}^{\mathrm{RF}} = 1 - \mathbb{V}(E_i^{\mathrm{RF}}|\mathcal{C}_i^{\mathrm{RF}}) . \tag{15c}$$

[11] Simple ex-ante valuation functions are non-increasing in the external leverage $1/\tilde{B} = A^e/E$, but are defined as functions of $\tilde{B}$ to avoid the discontinuity at $E = 0$.

4.3 Results on RFBs

In Proposition 2 we derive necessary and sufficient conditions so that leverage of RFBs and naive external leverage of nRFBs decrease or increase when compared with the bank prior to ring-fencing. The situation is less clear-cut for external leverages (i.e. for external leverages computed with fixed-point equities rather than with book-value equities).

We start by deriving sufficient conditions so that the leverage of RFBs decrease when compared with the external leverage of the bank prior to ring-fencing. For valuation functions that are non-decreasing with the external leverage (as in (15)), this naturally implies that the RFB is safer than the bank prior to ring-fencing.

Corollary 3. *Let bank i implement ring-fencing feasibly, i.e. let RFB i and nRFB i be naively solvent. If the allocation of assets and liabilities to the RFB is such that:*

$$\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i) \right) \geq \psi_i^L L_i^e \,,$$

then the leverage of RFB i is smaller than the external leverage of bank i prior to ring-fencing:

$$\lambda_i^{RF} \leq B_i^* \,.$$

Moreover, if probabilities of default are computed with simple ex-ante valuation functions (as in (15)), then the probability of RFB i is smaller than or equal to the probability of default of bank i prior to ring-fencing:

$$p_{i,[t,T]}^{RF} \leq p_{i,[t,T]}^* \,.$$

By comparing Corollary 3 with Proposition 1, we can now assess the impact of interbank lending and borrowing on (non-naive) external leverage. Corollary 3 implies that, when the net interbank lending of bank i is equal to zero ($\bar{A}_i = \bar{L}_i$), then the condition to make the leverage of RFB i smaller than the external leverage of bank i prior to ring-fencing is the same as when bank i does not lend to and borrow from other banks ($\psi_i^A > \psi_i^L$, under the mild assumption that $L_i^e > 0$). When bank i is a net lender to other banks ($\bar{A}_i > \bar{L}_i$), in order to lower the leverage of its RFB ($\lambda_i^{\mathrm{RF}} \leq B_i^*$), bank i needs to allocate proportionally more assets (or fewer liabilities) to the RFB with respect to the case in which its net interbank lending is zero. When bank i is a net borrower to other banks ($\bar{A}_i < \bar{L}_i$), in order to lower the leverage of its RFB ($\lambda_i^{\mathrm{RF}} \leq B_i^*$), bank i needs to allocate proportionally fewer assets (or more liabilities) to the RFB with respect to the case in which its net interbank lending is zero.

The reason why the condition $\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i) \right) \geq \psi_i^L L_i^e$ is only sufficient, but not necessary, for $\lambda_i^{\mathrm{RF}} \leq B_i^*$ is that B_i^* depends on the greatest solution E_i^*, which accounts for the valuation of interbank assets. In fact, B_i^* can be larger than λ_i^{RF} if the valuation of the interbank assets of bank i, and therefore E_i^*, is sufficiently small. This cannot happen if B_i^* is smaller than λ_i^{RF} regardless of the valuation of the interbank assets of bank i, that is if B_i^* is smaller than λ_i^{RF} even when the interbank assets of bank i are worth nothing. This intuition is formalised in the following proposition.

Proposition 3. *If the allocation of assets and liabilities to the RFB is such that:*

$$\psi_i^A \left(L_i^e + \bar{L}_i \right) \leq \psi_i^L L_i^e \,,$$

then the leverage of RFB i is greater than the external leverage of bank i prior to ring-fencing:

$$\lambda_i^{RF} \geq B_i^* \,.$$

Moreover, if probabilities of default are computed with simple ex-ante valuation functions (as in (15)), then the probability of RFB i is larger than or equal to the probability of default of bank i prior to ring-fencing:

$$p_{i,[t,T]}^{RF} \geq p_{i,[t,T]}^* \,.$$

We can now put together the results in Proposition 2, Corollary 3, and Proposition 3 and classify banks that implement ring-fencing in three groups. If bank i implements ring-fencing so that $\psi_i^L L_i^e \leq \psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i)\right)$, we have that $\lambda_i^{\mathrm{RF}} \leq B_i^0 \leq B_i^*$, i.e. the RFB has smaller leverage than the external leverage of the bank prior to ring-fencing. As a consequence, the probability of default of the RFB is smaller than the probability of default of the bank prior to ring-fencing. If bank i implements ring-fencing so that $\psi_i^A \left(L_i^e + \bar{L}_i\right) \leq \psi_i^L L_i^e$, we have that $\lambda_i^{\mathrm{RF}} \geq B_i^* \geq B_i^0$, i.e. the RFB has larger leverage than the external leverage of the bank prior to ring-fencing. As a consequence, the probability of default of the RFB is larger than the probability of default of the bank prior to ring-fencing. However, if banks i implements ring-fencing so that $\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i)\right) < \psi_i^L L_i^e < \psi_i^A \left(L_i^e + \bar{L}_i\right)$, we can still say that $\lambda_i^{\mathrm{RF}} \geq B_i^0$, but we cannot make any statement on the relationship between the leverage of RFB and the external leverage of the bank prior to ring-fencing.

Importantly, Corollary 3 and Proposition 3 allow us to make statements about whether the probability of default of the RFB has decreased or increased with respect to the bank prior to ring-fencing using only on quantities (ψ_i^A, ψ_i^L, $\bar{A}_i$, $\bar{L}_i$, L_i^e) that refer to those two banks. Even if the bank prior to ring-fencing is embedded in a financial network, and therefore its equity depends on other banks, Corollary 3 and Proposition 3 do not require the knowledge of any of these quantities, not even of the detailed breakdown of interbank assets (A_{ij}, for all banks j to which i is exposed) and liabilities (L_{ij}, for all banks j that are exposed to i) of bank i.

By comparing Corollaries 1 and 3 we can assess the impact of interbank lending and borrowing on the probability of default of the RFB. As long as the RFB has a smaller leverage than the external leverage of the bank prior to ring-fencing, it has also a smaller probability of default. From Corollary 3 we know that when bank i is a net lender (borrower) to other banks, in order to make the leverage of the RFB smaller than the external leverage of the bank prior to ring-fencing, bank i needs to allocate proportionally more (fewer) assets or fewer (more) liabilities to the RFB with respect to the case in which its net interbank lending is zero.

4.4 Results on nRFBs and banking groups

In contrast, we will see that in order to make statements about the probability of default of a nRFB, at least some knowledge of the topology of the financial network is required. We start by introducing the concept of asset risk orbit. Let $i \in \mathcal{N}$ and let $\mathbf{A}$ be a matrix of interbank assets. The asset risk orbit of i is[12]:

$$\mathcal{O}^A(i) = \left\{ j \in \mathcal{N} : \text{there exists a directed path from } i \text{ to } j \text{ in } \mathcal{G}^A \right\} .$$

For example, if i has invested in j, then $A_{ij} > 0$, i.e. i has an interbank asset towards j. This means that in $\mathcal{G}^A$ there is a directed path (consisting only of the edge $i \to j$) from i to j, and therefore that j is in the asset risk orbit of i. In fact, i holds an interbank asset corresponding to the investment made in j, and therefore it is exposed to changes in the creditworthiness of j. Now let us imagine that j has invested in k, i.e. that $A_{jk} > 0$. Also in this case there is a directed path from i to k ($i \to j \to k$), and therefore also k is in the asset risk orbit of i. Indeed, j is exposed directly to changes in the creditworthiness of k, but, since i is exposed directly to changes in the creditworthiness of j, also i is exposed to changes in the creditworthiness of k, but *indirectly*.

Similarly to the case of one bank, $E_i^{\mathrm{RF}} + E_i^{\mathrm{nRF},*}$ can be interpreted as the equity of the banking group that consolidates the RFB i and the nRFB i. When banks are embedded in

[12] The liability risk orbit has an analogous definition, but it is based on the matrix of interbank liabilities $\mathbf{L}$: $\mathcal{O}^L(i) = \left\{ j \in \mathcal{N} : \text{there exists a directed path from } i \text{ to } j \text{ in } \mathcal{G}^L \right\}$. It has been introduced in Eisenberg and Noe (2001) simply as *risk orbit*.

a financial network (3) does not necessarily hold, i.e. in general it is not true that the equity of the banking group is equal to E_i^* the equity of the bank prior to ring-fencing. However, as long as a bank is not exposed (either directly or indirectly) to any bank that has implemented ring-fencing, then the equity of the consolidated group is still equal to the equity of the bank prior to ring-fencing.

Proposition 4. *Let equity valuations of banks prior to ring-fencing and of nRFBs be the greatest solutions in a network valuation framework with simple ex-ante valuation functions.*

If no bank in the asset risk orbit of bank i implements ring-fencing:

$$\mathcal{O}^A(i) \cap \mathcal{R} = \varnothing \,,$$

then the sum of the equities of RFB i and nRFB i are equal to the equity of the bank prior to ring-fencing:

$$E_i^{RF} + E_i^{nRF,*} = E_i^* \,.$$

Intuitively, this happens because, after the implementation of ring-fencing, nothing has changed for any of the banks that could have a downstream impact on the nRFB bank i. What happens in the more general case in which banks that implement ring-fencing have a downstream impact on other banks? In the case in which *all banks in one asset risk orbit* implement ring-fencing consistently, i.e. when the leverage of all their RBFs is smaller than the naive external leverage (or larger than the external leverage) of their banks prior to ring-fencing, it is possible to prove the following result.

Theorem 1. *Let equity valuations of banks prior to ring-fencing and of nRFBs be the greatest solutions in a network valuation framework with simple ex-ante valuation functions.*

If all banks in the asset risk orbit of bank i either do not ring-fence or implement ring-fencing feasibly so that the leverage of their RFB is smaller than the naive external leverage of their bank prior to ring-fencing:

$$\forall j \in \mathcal{O}^A(i) \cap \mathcal{R} : \quad \lambda_j^{RF} \leq B_j^0 \,,$$

then the sum of the equities of RFB i and nRFB i is smaller than or equal to the equity of the bank prior to ring-fencing:

$$E_i^{RF} + E_i^{nRF,*} \leq E_i^* \,.$$

Vice versa, if all banks in the asset risk orbit of bank i either do not ring-fence or ring-fence feasibly so that the leverage of their RFB is larger than the external leverage of their bank prior to ring-fencing:

$$\forall j \in \mathcal{O}^A(i) \cap \mathcal{R} : \quad \lambda_j^{RF} \geq B_j^* \,,$$

then the sum of the equities of RFB i and nRFB i is larger than or equal to the equity of the bank prior to ring-fencing:

$$E_i^{RF} + E_i^{nRF,*} \geq E_i^* \,.$$

An important implication of Theorem 1 is that there are cases in which $E_i^{\mathrm{RF}} + E_i^{\mathrm{nRF},*} \neq E_i^*$ i.e. in which the equity of the group that consolidates RFB and nRFB is different from the equity of the bank prior to ring-fencing. In other words, the implementation of ring-fencing can either decrease or increase the equity of a banking group. This happens because, when banks are embedded in a financial network, equities are the product of a collective (self-consistent) valuation, and allocating assets and liabilities to RFBs and nRFBs can alter the valuation process. More precisely, equity valuations of nRFBs and of banks prior to ring-fencing depend on how much their interbank assets are worth, and therefore on how risky their counterparties are.

Intuitively, if enough assets are allocated to a RFB to lower its leverage and make it safer, fewer assets will be available to the corresponding nRFB. Everything else equal, this will make the nRFB riskier. A riskier nRFB will have a downstream impact on the other nRFBs exposed to it, lowering the valuation of the corresponding interbank assets. This will ultimately lead to smaller equities for those nRFBs exposed to the riskier nRFB and for their groups. According to Theorem 1, this happens when banks in the asset risk orbit of bank i implement ring-fencing by decreasing the leverage of their RFBs with respect to their *naive* external leverage prior to ring-fencing. Since the naive external leverage is always smaller than the external leverage, we have that those RFBs are safer than their banks prior to ring-fencing. However, the converse is not true. In fact, one bank in the asset risk orbit of bank i can make its RFB safer than its bank prior to ring-fencing (by making leverage of its RFB smaller than the external leverage of its bank prior to ring-fencing), but the leverage of its RFB can be still larger than its *naive* external leverage prior to ring-fencing, making Theorem 1 not applicable.

Theorem 1 also tells us that, when banks in the asset risk orbit of bank i implement ring-fencing by increasing the leverage of their RFBs with respect to their external leverage prior to ring-fencing, thereby making them less safe, the equity of the group that consolidates RFB and nRFB is larger than or equal to the equity of the bank prior to ring-fencing.

In our model RFBs only hold external liabilities, which means that we do not explicitly model agents who are holding these liabilities. If those agents were to mark-to-market the value of the corresponding assets accounting for the creditworthiness of RFBs, when RFBs are safer than their banks prior to ring-fencing the value of those agents' equities could increase. Whether this increase would offset the reduction of the equity of those exposed to the riskier nRFBs remains an open question that we do not address here.

We also observe that the hypotheses of Theorem 1 are only about banks in the asset risk orbit of bank i, not about bank i itself. In fact, it is irrelevant whether bank i implements ring-fencing at all. How bank i implements ring-fencing is relevant only if bank i is in its own asset risk orbit, i.e. if $i \in \mathcal{O}^A(i)$. At the same time, we stress that Theorem 1 applies only if *all* banks in the asset risk orbit of bank i either do not implement ring-fencing, or implement ring-fencing consistently. For example, if all banks in the asset risk orbit of bank i implement ring-fencing by making the leverage of their RFBs smaller than the naive external leverage of their banks prior to ring-fencing, except for one bank that makes the leverage of their RFB larger, then Theorem 1 does not apply.

An immediate consequence of Theorem 1 is that, if *all* banks implement ring-fencing consistently, the inequalities of Theorem 1 hold also for the aggregate equity. For example, if all RFBs have a smaller leverage than the naive external leverage of their banks prior to ring-fencing, then the aggregate equity after ring-fencing has been implemented is smaller than or equal to the aggregate equity prior to ring-fencing. Vice versa, if all RFBs have a larger leverage than the external leverage of their banks prior to ring-fencing, then the aggregate equity after ring-fencing has been implemented is larger than or equal to the aggregate equity prior to ring-fencing.

Corollary 4. *Let equity valuations of banks prior to ring-fencing and of nRFBs be the greatest solutions in a network valuation framework with simple ex-ante valuation functions.*

If all banks either do not ring-fence or implement ring-fencing feasibly so that the leverage of their RFB is smaller than the naive external leverage of their bank prior to ring-fencing:

$$\forall j \in \mathcal{N} \cap \mathcal{R} : \quad \lambda_j^{RF} \leq B_j^0 \,,$$

then the sum of the equities of RFB i and nRFB i is smaller than or equal to the equity of *the bank prior to ring-fencing:*

$$\sum_i E_i^{RF} + \sum_i E_i^{nRF,*} \leq \sum_i E_i^* \,.$$

Vice versa, if all banks either do not ring-fence or ring-fence feasibly so that the leverage of their RFB is larger than the external leverage of their bank prior to ring-fencing:

$$\forall j \in \mathcal{N} \cap \mathcal{R} : \quad \lambda_j^{RF} \geq B_j^* \,,$$

then the sum of the equities of RFB i and nRFB i is larger than or equal to the equity of the bank prior to ring-fencing:

$$\sum_i E_i^{RF} + \sum_i E_i^{nRF,*} \geq \sum_i E_i^* \,.$$

We have already pointed out that Theorem 1 requires only banks in the asset risk orbit of bank i to ring-fence in a certain way, and not bank i itself. However, if *also* bank i implements ring-fencing consistently with the banks in its asset risk orbit, then also the external leverage of its nRFB will change accordingly with respect to its bank prior to ring-fencing. More specifically, when the leverage of RFB i and of all RFBs in the asset risk-orbit of i[13] decrease with respect to the naive external leverage of bank i prior to ring-fencing, then the external leverage of nRFB i increases with respect to the external leverage of bank i prior to ring-fencing. Therefore, the probability of default of RFB i will be smaller than (or equal to) the probability of default of bank i prior to ring-fencing, which in turn will be smaller than (or equal to) the probability of default of the nRFB i. Similarly, when the leverage of RFB i and of all RFBs in the asset risk-orbit of i increase with respect to the external leverage of bank i prior to ring-fencing, then the external leverage of nRFB i decreases with respect to the external leverage of bank i prior to ring-fencing. Therefore, the probability of default of RFB i will be larger than (or equal to) the probability of default of bank i prior to ring-fencing, which in turn will be larger than (or equal to) the probability of default of the nRFB i.

Proposition 5. *Let external leverages of banks prior to ring-fencing and of nRFBs be computed with equity valuations corresponding to the greatest solutions in a network valuation framework with simple ex-ante valuation functions.*

If:

- *all banks in the asset risk orbit of bank i either do not ring-fence or implement ring-fencing feasibly so that the leverage of their RFB is smaller than the naive external leverage of their bank prior to ring-fencing:*

$$\forall j \in \mathcal{O}^A(i) \cap \mathcal{R} : \quad \lambda_j^{RF} \leq B_j^0 \,,$$

- *bank i implements ring-fencing feasibly so that the leverage of its RFB is smaller than the naive external leverage of its bank prior to ring-fencing:*

$$\lambda_i^{RF} \leq B_i^0 \,,$$

then the external leverage of bank i prior to ring-fencing is smaller than or equal to the external leverage of nRFB i:

$$B_i^* \leq B_i^{nRF,*} \,.$$

[13] RFBs do not have interbank liabilities and therefore cannot be in asset risk-orbits. By "RFBs in the risk-orbit of i" we simply mean RFBs corresponding to nRFBs in the risk-orbit of nRFB i.

Moreover, if probabilities of default are computed with simple ex-ante valuation functions (as in (15)), then the probability of default of RFB i is smaller than or equal to that of bank i prior to ring-fencing, which in turn is smaller than or equal to that of nRFB i:

$$p_{i,[t,T]}^{RF} \leq p_{i,[t,T]}^{*} \leq p_{i,[t,T]}^{nRF,*} \,.$$

Vice versa if:

- *all banks in the asset risk orbit of bank i either do not ring-fence or implement ring-fencing feasibly so that the leverage of their RFB is larger than the external leverage of their bank prior to ring-fencing:*

$$\forall j \in \mathcal{O}^A(i) \cap \mathcal{R} : \quad \lambda_j^{RF} \geq B_j^* \,,$$

- *bank i implements ring-fencing feasibly so that the leverage of its RFB is smaller than the naive external leverage of its bank prior to ring-fencing:*

$$\lambda_i^{RF} \geq B_i^* \,,$$

then the external leverage of bank i prior to ring-fencing is larger than or equal to the external leverage of nRFB i:

$$B_i^* \geq B_i^{nRF,*} \,.$$

Moreover, if probabilities of default are computed with simple ex-ante valuation functions (as in (15)), then the probability of default of RFB i is larger than or equal to that of bank i prior to ring-fencing, which in turn is larger than or equal to that of nRFB i:

$$p_{i,[t,T]}^{RF} \geq p_{i,[t,T]}^{*} \geq p_{i,[t,T]}^{nRF,*} \,.$$

5 Conclusion

We build a simple framework to assess the impact of ring-fencing on banks interconnected in a financial network of mutual investments. To summarise, we find that, making RFBs safer with respect to their banking group prior to ring-fencing can make nRFBs riskier and reduce the overall equity valuation of banking groups exposed to those riskier nRFB. In particular, this happens when all groups to which one group is directly or indirectly exposed implement ring-fencing so that the leverage of their RFBs is sufficiently below the external leverage of their groups prior to ring-fencing.

In our model risks for banks' balance sheets come from two sources. First, from external assets, that is from investments in the real economy. The larger the external leverage, the larger the risk. Second, from interbank assets, that is from investments in other banks. The larger the exposures to other banks or their probability of default, the larger this risk. While nRFBs and banks prior to ring-fencing are subject to both risks, implementing ring-fencing insulates RFBs from risks arising from interbank assets, leaving them exposed only to risks from external assets. In order to make those risks sufficiently small, one can allocate a sufficiently large amount of external assets to RFBs to make their leverage sufficiently small. By doing so, fewer or riskier assets will be allocated to nRFBs, exposing them more to risks from external assets. However, because nRFB also holds interbank assets, this increased riskiness has a downstream impact on other nRFBs. If a nRFB is exposed to other riskier nRFBs, its interbank assets will be worth less. This will make the equity of that nRFB and of the group to which it belongs smaller.

nRFBs can react in several ways. First, they can reduce their interbank assets, and therefore their exposure to other nRFBs. Second, they can reduce their exposure to risks from external assets. This can be achieved either by deleveraging — selling some external assets to repay liabilities — or by rebalancing their portfolio, by divesting from riskier assets and investing in safer assets. Third, they could improve their resilience by raising additional capital that would act as an additional buffer to withstand shocks to assets and therefore to equity. Finally, reallocating some external assets from RFBs to nRFBs would reduce the external leverage of nRFBs, but it would do so at the expense of RFBs, whose leverage and therefore their probability of default would increase.

Some current limitations of our model naturally outline possible directions for future research. First, our results descend purely from allocating assets and liabilities of the banking group into two different entities. Even though we are able to suggest actions that individual banking groups could implement to counter some adverse effects of ring-fencing, we do not approach this point quantitatively. For example, how much does a nRFB need to deleverage to become as risky as the banking group prior to ring-fencing? Second, we assume that RFBs are fully insulated from the financial network, on the asset side — they can only invest in the real economy — and on the liability side — they cannot be funded by other banks. In practice, RFBs could still invest and be funded by other RFBs. This means that also RFBs could hold interbank assets and liabilities, albeit safer than those held by nRFBs, and therefore they would be part of a parallel financial network. Third, we assume that the intrinsic riskiness (i.e. the volatility) of assets held by RFBs and nRFBs is the same. In reality, it is reasonable to expect that RFBs would hold safer assets than nRFBs. We leave the case in which RFBs and nRFBs hold different assets to future extensions of this work, as it would considerably complicate the analysis.

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A Valuation framework: Existence of and convergence to the greatest solution

We adapt Theorems 3.1 and 3.2 in Barucca et al. (2020) to banks prior to ring-fencing and to nRFBs.

Theorem A.1 (Existence of and convergence to the greatest solution)**.** *If all valuation functions are feasible, the set of equations (11a) and (11b) admit the greatest solutions* $\mathbf{E}^+$ *and* $\mathbf{E}^{nRF,+}$*. Moreover, the sequences* $\{\mathbf{E}^\kappa\}$ *and* $\{\mathbf{E}^{nRF,\kappa}\}$*, defined, for all* i*, as follows:*

$$\begin{aligned} E_i^0 &= A_i^e + \sum_{j=1}^N A_{ij} - L_i^e - \sum_{j=1}^N L_{ij} \\ E_i^{nRF,0} &= (1-\psi_i^A)A_i^e + \sum_{j=1}^N A_{ij} - (1-\psi_i^L)L_i^e - \sum_{j=1}^N L_{ij}\,, \end{aligned}$$

and for $\kappa \geq 1$*:*

$$\begin{aligned} E_i^{\kappa+1} &= A_i^e + \sum_{j=1}^N A_{ij}\mathbb{V}(E_j^\kappa|\mathcal{C}_j) - L_i^e - \sum_{j=1}^N L_{ij} \\ E_i^{nRF,\kappa+1} &= (1-\psi_i^A)A_i^e + \sum_{j=1}^N A_{ij}\mathbb{V}(E_j^{nRF,\kappa}|\mathcal{C}_j^{nRF}) - (1-\psi_i^L)L_i^e - \sum_{j=1}^N L_{ij}\,, \end{aligned}$$

are monotonically non-increasing and convergent to the greatest solutions:

$$\begin{aligned} \lim_{\kappa\to\infty} \mathbf{E}^\kappa &= \mathbf{E}^+ \\ \lim_{\kappa\to\infty} \mathbf{E}^{nRF,\kappa} &= \mathbf{E}^{nRF,+}\,. \end{aligned}$$

One consequence of Theorem A.1 is that we can interpret the sequences $\{\mathbf{E}^\kappa\}$ and $\{\mathbf{E}^{\text{nRF},\kappa}\}$ as incremental adjustments to equity valuations. In the beginning, equities are naive, in the sense that they incorporate a naive valuation of interbank assets, which are taken at face value. In the first iteration, $\{\mathbf{E}^1\}$ and $\{\mathbf{E}^{\text{nRF},1}\}$ incorporate the valuation of interbank assets only of their direct counterparties. In the second iteration, $\{\mathbf{E}^2\}$ and $\{\mathbf{E}^{\text{nRF},2}\}$ incorporate the valuation of interbank assets of their direct counterparties and of the direct counterparties of their counterparties, and so on, until convergence. Since the $\{\mathbf{E}^\kappa\}$ and $\{\mathbf{E}^{\text{nRF},\kappa}\}$ are non-increasing, all incremental adjustments to equity valuations are downwards, i.e. $\mathbf{E}^+ \leq \mathbf{E}^0$ and $\mathbf{E}^{\text{nRF},+} \leq \mathbf{E}^{\text{nRF},0}$.

B Proofs

Proof of Proposition 1. We also prove that $\psi^A \geq \psi^L$ is equivalent to the following two statements:

- The equity of the RFB is bounded from below by:

$$\psi^A E \leq E^{\text{RF}}$$

- The equity of the nRFB bank is bounded from above by:

$$E^{\text{nRF}} \leq (1-\psi^A)E\,.$$

Analogous inequalities hold when $\psi^A < \psi^L$.

We have that:

$$\begin{aligned}\psi^A E \le E^{\mathrm{RF}} &\iff \psi^A A - \psi^A L \le \psi^A A - \psi^L L \\ &\iff -\psi^A L \le -\psi^L L \\ &\iff \psi^A \ge \psi^L\,,\end{aligned}$$

and analogously for $E^{\mathrm{nRF}} \le (1-\psi^A)E$. Moreover, since (ψ^A, ψ^L) is feasible:

$$\begin{aligned}\psi^A E \le E^{\mathrm{RF}} &\iff \frac{\psi^A}{E^{\mathrm{RF}}} \le \frac{1}{E} \\ &\iff \frac{\psi^A A}{E^{\mathrm{RF}}} \le \frac{A}{E} \\ &\iff \lambda^{\mathrm{RF}} \le \lambda\,,\end{aligned}$$

and analogously for $E^{\mathrm{nRF}} \le (1-\psi^A)E \iff \lambda \le \lambda^{\mathrm{nRF}}$. □

Proof of Corollary 1. The proof follows immediately from Proposition 1 and from the fact that the probability of default is a non-increasing function of the leverage. □

Proof of Proposition 2. We also prove that $\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i)\right) \ge \psi_i^L L_i^e$ is equivalent to the following two statements:

- The equity of the RFB is bounded from below by:

$$\psi_i^A E_i^0 \le E_i^{\mathrm{RF}}.$$

- The naive equity of the nRFB bank is bounded from above by:

$$E_i^{\mathrm{nRF},0} \le (1-\psi_i^A)E_i^0\,.$$

Analogous inequalities hold when $\psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i)\right) < \psi_i^L L_i^e$.

The proof is analogous to the proof of Proposition 1. We have that:

$$\begin{aligned}\psi_i^A E_i^0 \le E_i^{\mathrm{RF}} &\iff \psi_i^A \left(A_i^e - L_i^e + \bar{A}_i - \bar{L}_i\right) \le \psi_i^A A_i^e - \psi_i^L L_i^e \\ &\iff \psi_i^A \left(-L_i^e + \bar{A}_i - \bar{L}_i\right) \le -\psi_i^L L_i^e \\ &\iff \psi_i^A \left(L_i^e - (\bar{A}_i - \bar{L}_i)\right) \ge \psi_i^L L_i^e\,.\end{aligned}$$

Moreover, by using (9):

$$\begin{aligned}\psi_i^A E_i^0 \le E_i^{\mathrm{RF}} &\iff -\psi_i^A E_i^0 \ge -E_i^{\mathrm{RF}} \\ &\iff E_i^0 - \psi_i^A E_i^0 \ge E_i^0 - E_i^{\mathrm{RF}} \\ &\iff (1-\psi_i^A)E_i^0 \ge E_i^{\mathrm{nRF},0}\,.\end{aligned}$$

Since (ψ_i^A, ψ_i^L) is feasible:

$$\begin{aligned}\psi_i^A E_i^0 \le E_i^{\mathrm{RF}} &\iff \frac{\psi_i^A}{E_i^{\mathrm{RF}}} \le \frac{1}{E_i^0} \\ &\iff \frac{\psi_i^A A_i^e}{E_i^{\mathrm{RF}}} \le \frac{A_i^e}{E_i^0} \\ &\iff \lambda_i^{\mathrm{RF}} \le B_i^0\,,\end{aligned}$$

and analogously for $E_i^{\mathrm{nRF},0} \le (1-\psi_i^A)E_i^0 \iff B_i^0 \le B_i^{\mathrm{nRF},0}$. □

Proof of Corollary 2. This is a straightforward consequence of Proposition 2. □

Proof of Corollary 3. From Proposition 2 and from the fact that the greatest solution is smaller than or equal to the naive equity ($\mathbf{E}^* \leq \mathbf{E}^0$) it follows that:

$$\psi_i^A E_i^* \leq E_i^{\mathrm{RF}} .$$

Therefore, we immediately have also that: $\lambda_i^{\mathrm{RF}} \leq B_i^*$. The result on probabilities of default comes from the fact that simple ex-ante valuation functions are non-decreasing with external leverage (see Definition 2) and therefore probability of default computed as in (15) are non-increasing with external leverage. □

Proof of Proposition 3. Under the hypotheses we also prove that:

$$E_i^{\mathrm{RF}} \leq \psi_i^A E_i^* .$$

We have that:

$$\begin{aligned} \psi_i^A \left(L_i^e + \bar{L}_i\right) \leq \psi_i^L L_i^e \iff & -\psi_i^A \left(L_i^e + \bar{L}_i\right) \geq -\psi_i^L L_i^e \\ \iff & \psi_i^A \left(A_i^e - L_i^e - \bar{L}_i\right) \geq \psi_i^A A_i^e - \psi_i^L L_i^e \\ \iff & \psi_i^A \left(A_i^e - L_i^e - \bar{L}_i\right) \geq E_i^{\mathrm{RF}} . \end{aligned}$$

At the same time:

$$\begin{aligned} E_i^* = A_i^e + \sum_{j=1}^{N} A_{ij} \mathbb{V}\left(E_j^* | \mathcal{C}_j\right) - L_i^e - \bar{L}_i \implies & E_i^* \geq A_i^e - L_i^e - \bar{L}_i \\ \iff & \psi_i^A E_i^* \geq \psi_i^A \left(A_i^e - L_i^e - \bar{L}_i\right) , \end{aligned}$$

therefore, if $\psi_i^A \left(L_i^e + \bar{L}_i\right) \leq \psi_i^L L_i^e$:

$$\psi_i^A E_i^* \geq \psi_i^A \left(A_i^e - L_i^e - \bar{L}_i\right) \geq E_i^{\mathrm{RF}} .$$

If $E_i^* > 0$ and since (ψ_i^A, ψ_i^L) is feasible we have:

$$\begin{aligned} \psi_i^A E_i^* \geq E_i^{\mathrm{RF}} \iff & \frac{\psi_i^A}{E_i^{\mathrm{RF}}} \geq \frac{1}{E_i^*} \\ \iff & \frac{\psi_i^A A_i^e}{E_i^{\mathrm{RF}}} \geq \frac{A_i^e}{E_i^*} \\ \iff & \lambda_i^{\mathrm{RF}} \geq B_i^* . \end{aligned}$$

The result on probabilities of default comes from the fact that simple ex-ante valuation functions are non-decreasing with external leverage (see Definition 2) and therefore probability of default computed as in (15) are non-increasing with external leverage. □

Proof of Proposition 4. In order to prove that:

$$E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} = E_i^* , \tag{B.1}$$

it is sufficient to prove that:

$$\sum_{j \in \mathcal{N}(i)} A_{ij} \left[\mathbb{V}(E_j^{\mathrm{nRF},*} | \mathcal{C}_j^{\mathrm{nRF}}) - \mathbb{V}(E_j^* | \mathcal{C}_j)\right] = 0 , \tag{B.2}$$

where we denote with $\mathcal{N}(i)$ the neighbours of i.

Let us prove that:

$$E_p^{\text{nRF},\kappa} = E_p^{\kappa} \qquad \forall p \in \mathcal{O}^A(i)\,, \forall \kappa\,. \tag{B.3}$$

We proceed by induction. Eq. (B.3) holds for $\kappa = 0$ because:

$$\mathbf{E}^{\text{nRF},0} + \mathbf{E}^{\text{RF}} = \mathbf{E}^{0}$$

and $E_p^{\text{RF}} = 0$, for all $\forall p \in \mathcal{O}^A(i)$. Let us now assume that (B.3) holds for $\kappa > 0$, proving that holds (B.3) for $\kappa + 1$ is equivalent to proving that:

$$\sum_{s \in \mathcal{N}(p)} A_{ps} \left[\mathbb{V}(E_s^{\text{nRF},\kappa} | \mathcal{C}_s^{\text{nRF}}) - \mathbb{V}(E_s^{\kappa} | \mathcal{C}_s) \right] = 0\,. \tag{B.4}$$

Our induction hypothesis holds for all nodes in the asset risk orbit of i, and therefore also for all the nodes in the asset risk orbit of one of the nodes in the asset risk orbit of i: $E_s^{\text{nRF},\kappa} = E_s^{\kappa}$, for all $s \in \mathcal{N}(p)$, for all $p \in \mathcal{O}^A(i)$. Moreover, since all banks in the asset risk orbit of i do not ring-fence, then $\mathcal{C}_s = \mathcal{C}_s^{\text{nRF}}$, for all $s \in \mathcal{N}(p)$, for all $p \in \mathcal{O}^A(i)$. As a consequence, (B.4) holds and therefore also (B.3). By plugging (B.3) into (B.2) and since $\mathcal{C}_j = \mathcal{C}_j^{\text{nRF}}$ also for all $j \in \mathcal{N}(i)$, we have that (B.2), and therefore (B.1) holds. □

The following result relates valuation functions for nRFBs to valuation functions for banks prior to ring-fencing, and it is needed to prove Theorem 1.

Lemma B.1. *Let $\mathbb{V}$ be a simple ex-ante valuation function and let $\mathcal{C} = \{A^e, \sigma\}$ and $\mathcal{C}^{nRF} = \{(1-\psi^A)A^e, \sigma\}$, with $A^e > 0$, $\sigma > 0$, and $\psi^A \in [0, 1)$. Then:*

$$\mathbb{V}\left(E | \mathcal{C}^{nRF}\right) = \mathbb{V}\left(\frac{E}{1-\psi^A} \middle| \mathcal{C}\right)\,.$$

Proof. We have:

$$\begin{aligned}\mathbb{V}\left(E | \mathcal{C}^{\text{nRF}}\right) &= f\left(\frac{E}{(1-\psi^A)A^e}, \sigma\right) \\ &= \mathbb{V}\left(\frac{E}{1-\psi^A} \middle| \mathcal{C}\right)\,.\end{aligned}$$

□

Proof of Theorem 1. We start with the first statement. In order to prove that:

$$E_i^{\text{nRF},*} + E_i^{\text{RF}} \leq E_i^{*}\,, \tag{B.5}$$

it is sufficient to prove that a similar inequality holds for every iteration κ:

$$E_i^{\text{nRF},\kappa} + E_i^{\text{RF}} \leq E_i^{\kappa}\,, \tag{B.6}$$

which in turn is equivalent to:

$$\sum_{j \in \mathcal{N}(i)} A_{ij} \left[\mathbb{V}(E_j^{\text{nRF},\kappa} | \mathcal{C}_j^{\text{nRF}}) - \mathbb{V}(E_j^{\kappa} | \mathcal{C}_j) \right] \leq 0\,, \tag{B.7}$$

where we denote with $\mathcal{N}(i)$ the neighbours of i.

Let us prove that:

$$E_p^{\text{nRF},\kappa} + E_p^{\text{RF}} \leq E_p^{\kappa} \qquad \forall p \in \mathcal{O}^A(i)\,, \forall \kappa\,. \tag{B.8}$$

We proceed by induction. Eq. (B.8) obviously holds for $\kappa = 0$, simply because:

$$\mathbf{E}^{\text{nRF},0} + \mathbf{E}^{\text{RF}} = \mathbf{E}^0 \,.$$

Let us now assume that (B.8) holds for $\kappa > 0$, proving that holds (B.8) for $\kappa+1$ is equivalent to proving that:

$$\sum_{s\in\mathcal{N}(p)} A_{ps}\left[\mathbb{V}(E_s^{\text{nRF},\kappa}|\mathcal{C}_s^{\text{nRF}}) - \mathbb{V}(E_s^{\kappa}|\mathcal{C}_s)\right] \leq 0\,. \tag{B.9}$$

For all banks s that are neighbours of a node p in the asset risk orbit of i that do not ring-fence we have that:

$$\begin{aligned}\mathbb{V}(E_s^{\text{nRF},\kappa}|\mathcal{C}_s^{\text{nRF}}) &\leq \mathbb{V}(E_s^{\kappa} - E_s^{\text{RF}}|\mathcal{C}_s^{\text{nRF}}) \\ &\leq \mathbb{V}\left(E_s^{\kappa}|\mathcal{C}_s\right)\,,\end{aligned}$$

where the second line comes from the fact that for banks that do not ring-fence $E_s^{\text{RF}} = 0$ and $\mathcal{C}_s^{\text{nRF}} = \mathcal{C}_s$. Instead, for all other banks s that are neighbours of a node p in the asset risk orbit of i we have that:

$$\begin{aligned}\mathbb{V}(E_s^{\text{nRF},\kappa}|\mathcal{C}_s^{\text{nRF}}) &\leq \mathbb{V}(E_s^{\kappa} - E_s^{\text{RF}}|\mathcal{C}_s^{\text{nRF}}) \\ &= \mathbb{V}\left(\frac{E_s^{\kappa} - E_s^{\text{RF}}}{1-\psi_s^A}|\mathcal{C}_s\right) \\ &\leq \mathbb{V}\left(E_s^{\kappa}|\mathcal{C}_s\right)\end{aligned}$$

where the first line comes from our induction hypothesis (B.8), which holds for all nodes in the asset risk orbit of i, and therefore for all neighbours of all nodes in the asset risk orbit of i (which are also part of the asset risk orbit of i), and the second step from Lemma B.1. The third step comes from the fact that for all nodes in the asset risk orbit of i that do ring-fence we have that $\lambda_s^{\text{RF}} \leq B_s^0$. In fact:

$$\begin{aligned}\frac{E_s^{\kappa} - E_s^{\text{RF}}}{1-\psi_s^A} \leq E_s^{\kappa} &\iff \\ E_s^{\kappa} - E_s^{\text{RF}} \leq E_s^{\kappa} - \psi_s^A E_s^{\kappa} &\iff \\ E_s^{\text{RF}} \geq \psi_s^A E_s^{\kappa}\,,&\end{aligned}$$

but Proposition 2 implies that $E_s^{\text{RF}} \geq \psi_s^A E_s^0 \geq \psi_s^A E_s^{\kappa}$. We have proved (B.9), and thus also (B.8). In particular, (B.8) holds for all neighbours of i. Therefore, one can easily prove (B.7) by reproducing the same steps used to prove (B.9).

The proof proceeds analogously also in the second case. In this case, instead of Proposition 2 one uses Proposition 3, according to which, for all banks s for which $\lambda_s^{\text{RF}} \geq B_s^*$, we have that $E_s^{\text{RF}} \leq \psi_s^A E_s^* \leq \psi_s^A E_s^{\kappa}$. □

Proof of Corollary 4. For the first statement, all banks satisfy the conditions of the first case of Theorem 1, and therefore $E_i^{\text{nRF},*} + E_i^{\text{RF}} \leq E_i^*$, for all i. The proof follows by summing on both sides of the equation over all i. Analogously for the second case. □

Proof of Proposition 5. Let us start with the first statement. Since $\lambda_i^{\text{RF}} \leq B_i^0$, using Proposition 2 and Corollary 3 we have that: $\psi_i^A E_i^* \leq E_i^{\text{RF}}$. At the same time the first statement of Theorem 1 is applicable: $E_i^{\text{nRF},*} + E_i^{\text{RF}} \leq E_i^*$. Combining the two we have:

$$E_i^{\text{nRF},*} + E_i^{\text{RF}} \leq E_i^* \leq \frac{E_i^{\text{RF}}}{\psi_i^A}\,.$$

We note that assuming that $E_i^{\mathrm{RF}} > 0$ implies that some external assets must have been allocated to the RFB, i.e. $\psi_i^A > 0$. Combining the two inequalities above also yields:

$$\begin{aligned}
\psi_i^A \left(E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \right) \leq E_i^{\mathrm{RF}} &\iff \\
-\psi_i^A \left(E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \right) \geq -E_i^{\mathrm{RF}} &\iff \\
E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} - \psi_i^A \left(E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \right) \geq E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} - E_i^{\mathrm{RF}} &\iff \\
(1 - \psi_i^A) \left(E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \right) \geq E_i^{\mathrm{nRF},*} &\iff \\
E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \geq \frac{E_i^{\mathrm{nRF},*}}{1 - \psi_i^A} .
\end{aligned}$$

Putting all inequalities together we have:

$$\frac{E_i^{\mathrm{nRF},*}}{1 - \psi_i^A} \leq E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \leq E_i^* \leq \frac{E_i^{\mathrm{RF}}}{\psi_i^A} ,$$

from which the inequality of external leverages easily follows.

The second statement is analogous when noting that, since $\lambda_i^{\mathrm{RF}} \geq B_i^*$, by using Proposition 3 we have that $\psi_i^A E_i^* \geq E_i^{\mathrm{RF}}$. In this case we have:

$$\frac{E_i^{\mathrm{nRF},*}}{1 - \psi_i^A} \geq E_i^{\mathrm{nRF},*} + E_i^{\mathrm{RF}} \geq E_i^* \geq \frac{E_i^{\mathrm{RF}}}{\psi_i^A} .$$

The results on probabilities of default comes from the fact that simple ex-ante valuation functions are non-decreasing with external leverage (see Definition 2) and therefore probability of default computed as in (15) are non-increasing with external leverage. □