Real exchange rates and the relative prices of non-traded and traded goods: an empirical analysis

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Abstract

This paper provides an empirical analysis of the decomposition of UK real exchange rates into the relative price of traded goods and the ratio of the relative price of non-traded to traded goods, and tests the prediction that deviations from the law of one price in tradable goods dominate real exchange rate variability only in the short run. UK bilateral real exchange rates are examined relative to a sample of six main OECD partners. The existence of a long-run relationship between real exchange rates and these corresponding relative price ratios is analysed using cointegrated vector autoregressive models. These show only limited evidence of a cointegrating relationship. The paper quantifies the severity of the deviations from the law of one price, and shows that these deviations are persistent relative to the length of the sample period. This motivates the use of a multi-country panel cointegration-testing framework, which produces evidence of a long-run relationship between the real exchange rate and the non-tradable component.

Key words: Real exchange rates, Balassa-Samuelson, half-life measures, (panel) cointegration.

JEL classification: C32, F30, F31, F47.

Summary

Empirical real exchange rate studies mainly reflect one of two views of real exchange rate behaviour. Purchasing power parity (PPP) assumes that any measure of the real exchange rate is mean-reverting in nature and therefore constant in the long run. An alternative hypothesis makes a distinction between the empirical behaviour of the tradable and non-tradable components of the real exchange rate. This approach assumes that cross-country differences in the prices of tradable goods expressed in the same currency should eventually be eliminated, that is the Law of One Price (LOOP) across tradable goods between countries holds. In this case, the long-run movements in real exchange rates are related to movements in the ratio of the relative price of non-tradable and tradable goods between countries.

Based on evidence in the literature it seems sensible to assume that the real exchange rate contains a unit root. We carry out unit root tests on the data, which show this assumption is appropriate. Although this phenomenon is not consistent with PPP, it can be reconciled with the second approach; that national price indices have non-tradable components, which in turn affects real exchange rate behaviour. In this context, short to medium-run deviations between the real exchange rate and the ratio of the relative price of non-tradable and tradable components are possible. These occur as a consequence of temporary deviations from the LOOP. Hence LOOP deviations can only dominate the variability of the real exchange rate in the short to medium run.

In this paper we test this hypothesis for movements in UK real exchange rates relative to a sample of six main OECD partners. The identification of a long-run relationship between the real exchange rate and the ratio of the relative price of the non-tradable and tradable components requires us to choose a method for constructing these components. Determining precise indices that accurately capture the price of traded and non-traded goods is virtually impossible. Given these inevitable constraints we use two different methods to construct indices to capture movements in the prices of traded and non-traded goods in each country in our sample. One method decomposes the consumer prices index into its tradable and non-tradable components; the other uses the producer prices index as a proxy for tradable goods prices.

The analysis presented examines the existence of a long-run relationship between bilateral UK real exchange rates and the corresponding relative prices of non-traded to traded goods. Consistent with the findings elsewhere in the literature, using cointegrated vector

autoregressive (VAR) models for these series, otherwise known as vector error correction (VEC) models, we find little support for the LOOP; there is only limited evidence for a cointegrating relationship in the dollar and euro bilateral rates. Using an autoregressive model for the relative price of tradable goods, we quantify the severity of the deviations from the law of one price. This provides evidence that such deviations are persistent relative to the time span of our data set. This finding motivates the use of a multi-country panel cointegration-testing framework. It provides evidence for a cointegrating relationship between the real exchange rate and the relative price of non-tradable goods for the United Kingdom, using both the CPI and the PPI-based decompositions.

Out-of-sample evaluation shows that the estimated time series based cointegrating VAR models are inferior to a naive random-walk model. But we find evidence that a novel panel VEC approach can, for most bilaterals, provide a significantly more accurate prediction of movements in the real exchange rate than a random-walk model. Our results show that by using a panel-data framework we are able to identify a long-run relationship between bilateral UK real exchange rates and the corresponding relative prices of non-traded to traded goods.

1 Introduction

The behaviour of real exchange rates has long been a focus of academic research. Empirical real exchange rate studies mainly reflect one of two views of real exchange rate behaviour. Purchasing power parity (PPP) assumes that *any* measure of the real exchange rate is mean-reverting in nature and therefore constant in the long run. An alternative version, as first proposed by Balassa (1964) and Samuelson (1964), makes a distinction between the empirical behaviour of the tradable and non-tradable components of the real exchange rate. In this approach it is assumed that the 'Law of One Price' (LOOP) holds, ie cross-country differentials in the price of tradable goods expressed in the same currency should eventually be eliminated. Long-run movements in real exchange rates are therefore related to movements in the relative price ratio of non-tradable and tradable components.

The PPP view of real exchange rates is not founded on strong empirical evidence. Standard time series tests based on the augmented Dickey and Fuller (1979) (ADF) unit root test cannot in general reject that real exchange rates are non-stationary, see eg Mark (1990). As ADF tests are known to have low power, many researchers in the field have reverted to the use of panel unit root tests. However, when properly executed, these panel unit root tests do not provide overwhelming evidence for PPP either, see eg O'Connell (1998). Based on these observations from the literature it seems appropriate to assume that the real exchange rate contains a unit root. Although this phenomenon is not in compliance with PPP, it can be reconciled within the view that national price indices have non-tradable components and this in turn affects real exchange rate behaviour. Translated into non-stationary time series jargon, this implies that the real exchange rate and the relative price ratio of non-tradable and tradable components are cointegrated. In this context, short to medium-run deviations between the real exchange rate and the relative price ratio of non-tradable and tradable components are possible, and they occur as a consequence of temporary deviations of the LOOP. Hence, LOOP deviations can only dominate the variability of the real exchange rate in the short to medium run.

Several studies have tried to test the prediction that LOOP deviations dominate real exchange rate variability only in the short to medium run. Engel (1999), on a sample of US bilateral real exchange rates with other major OECD economies for the period 1962-95, shows that changes in the international relative price of traded goods account for the overwhelming majority of the overall variance of real exchange rate changes. The same conclusions can be drawn for other bilateral real exchange rate pairs, see eg Engel (2003). Using quarterly data from 1980 to 2000 for a 52-country sample; Betts and Kehoe (2001) have done the same and they find that their measure of the relative price ratio of non-tradable and tradable components is slightly better able to account for long-run real exchange rate variability for countries which have relatively close trade ties, albeit that the explained proportion never exceeds one third. Kakkar and Ogaki (1999), on the other hand, focus on the cointegration relationship between the real exchange rate and the relative non-tradables/tradables price ratio, and they are able to find evidence for this cointegration relationship in a 1929-88 sample of different bilateral pairs among a group comprising Canada, Italy, Japan, the United Kingdom and the United States. However, utilising both time series and panel techniques Drine and Rault (2002) have to reject the empirical appropriateness of cointegration between real exchange rates and relative non-tradables/tradables price ratios on a 1970-93 sample of annual effective real exchange rates for eleven OECD countries. Hence, both variance decompositions of tradable price ratios and cointegration tests on real exchange rates and relative non-tradables/tradables price ratios provide, at the most, mixed evidence in favour of LOOP.

In this paper we consider movements in UK bilateral real exchange rates relative to a sample of six main OECD partners from the perspective that price movements in the non-tradable components of the respective price indices have an impact on long-run real exchange rate behaviour. Determining precise indices that accurately capture the price of traded and non-traded goods is virtually impossible. Notwithstanding these inevitable constraints, we construct indices to capture movements in the prices of traded and non-traded goods in each country within our sample. This allows us to analyse the existence of a long-run relationship between UK bilateral real exchange rates and the relative price ratio of non-tradables and tradables.

The remainder of this paper is organised as follows. In Section 2 we briefly describe how the real exchange rate can be decomposed into an international relative price of tradable goods and a relative price ratio of traded and non-traded goods. We describe the data used in our analysis. A cointegration model for each of our bilateral pairs is utilised in Section 3 to test the empirical validity of the long-run real exchange rate relationships described in Section 2. We also try to quantify the severity of LOOP deviations through the estimation of the half-lives of LOOP deviations. Small multi-country panel structures are utilised in Section 4 in order to improve upon the time series methods used in Section 3. Both time series and panel versions of our long-run real exchange rate models are evaluated in an out-of-sample context in Section 5. Finally, Section 6 contains concluding remarks.

2 Real exchange rates: definitions and stylised facts

In general terms, the real exchange rate between the United Kingdom and another country can be described as the relative aggregate price levels of the countries expressed in the same currency, ie

$$q_t = s_t + p_t^{\text{UK}} - p_t^* \qquad \textbf{(1)}$$

where q_t is the logarithm of the real exchange rate Q_t, s_t is the logarithm of the nominal exchange rate S_t for the foreign currency expressed in pounds sterling,[(1)] p_t^{UK} is the logarithm of the UK aggregate price level P_t^{UK} and p_t^* is the logarithm of the foreign aggregate price level P_t^*. In order to link the movements in the real exchange rate to movements in the relative price ratio of the tradable and non-tradable components, we write the aggregate price level as the geometric weighted average of the price of the tradable goods component $P(T)_t^i$ and the non-tradable goods component $P(N)_t^i$, that is

$$P_t^i = C_t^i [P(T)_t^i]^{\alpha^i} [P(N)_t^i]^{1-\alpha^i}, \quad i = \text{UK or } * \qquad \textbf{(2)}$$

where C_t is a stationary measurement error. In logarithms equation **(2)** becomes:

$$p_t^i = c_t^i + \alpha^i p(T)_t^i + (1 - \alpha^i) p(N)_t^i \qquad \textbf{(3)}$$

The logarithm of the real exchange rate can be defined as the sum of the cross-country difference in measurement error, the cross-country traded good price ratio x_t, and the relative price ratio of non-tradable and tradable components y_t:

$$q_t = \theta_t + x_t + y_t \qquad \textbf{(4)}$$

where

$$\theta_t = c_t^{\text{UK}} - c_t^*$$
$$x_t = s_t + p(T)_t^{\text{UK}} - p(T)_t^*$$
$$y_t = (1 - \alpha^{\text{UK}})[p(N)_t^{\text{UK}} - p(T)_t^{\text{UK}}] - (1 - \alpha^*)[p(N)_t^* - p(T)_t^*]$$

Cross-country differences in tradable goods and the degree of nominal price stickiness induce deviations from the LOOP. According to the LOOP, cross-country price differentials expressed in one currency provide economic agents with arbitrage opportunities which would induce them to ship tradable goods to other countries and therefore these price differences would disappear.[(2)] We therefore assume that these LOOP deviations die out in the long run, ie

$$x_t = \varphi_t \qquad \textbf{(5)}$$

(1) As a consequence a rise in the nominal exchange rate indicates an appreciation of sterling and thus a rise in q_t indicates a real appreciation for the United Kingdom.
(2) In the LOOP world it is explicitly assumed that economic agents do not face transport costs.

where φ_t is a zero-mean $I(0)$ deviation. Combining **(4)** with **(5)** the log of the real exchange rate can now be written as

$$q_t = \bar{\theta} + y_t + \mu_t \tag{6}$$

where $\bar{\theta} = \mathrm{E}(c_t^{\mathrm{UK}} - c_t^*)$ reflects the average level of relative measurement error, and

$$\mu_t = [(c_t^{\mathrm{UK}} - c_t^*) - \bar{\theta}] + \varphi_t \sim I(0)$$

with mean zero. Thus, according to **(6)**, the real exchange rate can contain a stochastic trend when y_t contains a stochastic trend.

There are three potential explanations for why the non-traded/traded relative price ratio can contain a stochastic trend, which would induce the real exchange rate to be non-mean reverting. First, Balassa (1964) and Samuelson (1964) have argued that in fast-growing economies productivity growth in the traded goods sector is higher than in the non-traded goods sector and thus the relative price of non-traded/traded goods for such an economy, ie y_t, would rise quickly. Consequently, if for example the United Kingdom grows faster than a foreign economy, the corresponding bilateral UK real exchange rate will exhibit a sustained appreciation. Another explanation is based on relative factor endowments, as put forward by Bhagwati (1984). As services are relatively labour intensive in production and goods relatively capital intensive, capital abundant countries have a comparative advantage in producing goods, causing a sustained rise in the y_t for those countries. Finally, Bergstrand (1991) focuses on the relative demand structure among countries and its influence on the behaviour of y_t. This approach assumes non-homothetic tastes of agents and also that non-traded services are luxuries in consumption and traded commodities are necessities. Therefore, if a country becomes relatively wealthier than other countries it will exhibit a relatively higher demand for non-tradables, which in turn causes a sustained increase in the y_t for that country. Hence, based on one or more of these three explanations, we would expect to find cointegration between the log real exchange rate q_t and the log relative non-tradables/tradables price ratio y_t based on the proportionality between the two variables as in **(6)**.

In modelling real exchange rate movements according to **(4)** and **(6)**, we have to choose how to measure the price of tradable and non-tradable goods within the respective economies. Following Engel (1999) and Betts and Kehoe (2001) we use two approaches. In the first approach, we assume that a producer price index (PPI) has a higher weight on traded goods than a consumer price index (CPI), we can proxy x_t and y_t by:

$$\begin{aligned} x_t &= s_t + p(\mathrm{PPI})_t^{\mathrm{UK}} - p(\mathrm{PPI})_t^* \\ y_t &= [p(\mathrm{CPI})_t^{\mathrm{UK}} - p(\mathrm{PPI})_t^{\mathrm{UK}}] - [p(\mathrm{CPI})_t^* - p(\mathrm{PPI})_t^*] \end{aligned} \tag{7}$$

We denote **(7)** as the PPI-based specification. One problem with this PPI-based decomposition is that it measures the relative price of non-traded goods as simply the difference between the real exchange rate and the tradables component; ie $y_t = q_t - x_t$. Measurement error in these two series may then imply a negative correlation between x_t and y_t. Our alternative approach decomposes the CPI into components that are related to tradable and non-tradable goods prices ($P(\text{CPI-T})$ and $P(\text{CPI-N})$ respectively). Consequently,

$$\begin{aligned} x_t &= s_t + p(\text{CPI-T})_t^{\text{UK}} - p(\text{CPI-T})_t^* \\ y_t &= (1-\alpha^{\text{UK}})[p(\text{CPI-N})_t^{\text{UK}} - p(\text{CPI-T})_t^{\text{UK}}] - (1-\alpha^*)[p(\text{CPI-N})_t^* - p(\text{CPI-T})_t^*] \end{aligned} \qquad \textbf{(8)}$$

A more detailed description of the data can be found in Appendix A.

We will now turn to the stylised properties of our data, before we subject it to a more rigorous analysis. UK bilateral real exchange rate movements appear to be without any mean-reverting tendencies over the period 1976 to 2002. In Table A we conduct ADF unit root tests on all the UK real exchange rate bilaterals, as well as their components y_t and x_t, over the 1976-2002 sample. The table shows that we are unable to reject the null of non-stationarity for the monthly real exchange rate for any bilateral pair.

This apparent lack of mean reversion implies, as may have been expected, the empirical failure of PPP for UK real exchange rates. However, LOOP may still hold, through a long-run relationship between the real exchange rate and the relative price ratio of non-tradables and tradables between countries. From Table A one notices that, in general, y_t contains a stochastic trend, irrespective of which price index is used to construct this measure. But the results in this table also indicate that the relative tradables price x_t has a unit root, indicating that LOOP also fails to hold across tradable goods.

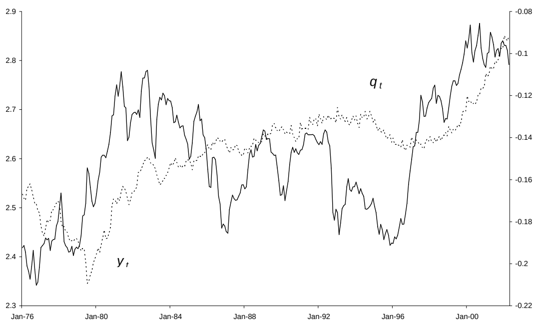
Charts 1 to 7 show the relationship between q_t, x_t and y_t over the sample period for the France/UK, Euro/UK, Germany/UK, Italy/UK, Canada/UK, Japan/UK and US/UK bilaterals using both our CPI and PPI-based decomposition of the real exchange rate in x_t and y_t. The q_t and y_t series show a certain amount of co-movement, although the strength of the relationship between q_t and y_t varies across the sample period, the bilaterals considered, and the method of decomposition used. If we look at the US/UK bilateral for both the CPI and the PPI-based measures, movements in the real exchange rate and the relative price ratio of tradable and non-tradable goods follow a similar trend during the early 1980s, but the disconnection between the movements of q_t and y_t increases towards the end of the sample. For the other sterling bilaterals the trends in the q_t and y_t series appear closer, but we still observe substantial persistent deviations between q_t and y_t. When we compare in Charts 1 to 7 the q_t and y_t series, on the one hand, with the x_t series, on the other, it becomes apparent that the q_t and x_t series share a comparable degree of

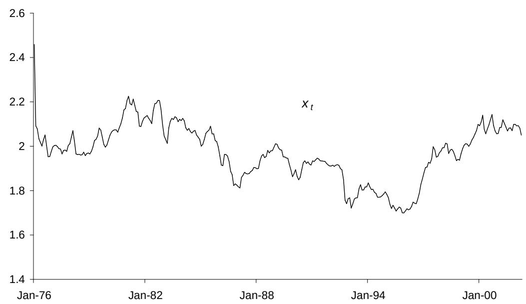
Table A: Unit root tests on the components of the major real sterling exchange rate relationships, 1976:01-2002:04[(a)]

	q_t		$y_{\mathrm{CPI},t}$		$y_{\mathrm{PPI},t}$		$x_{\mathrm{CPI},t}$		$x_{\mathrm{PPI},t}$	
Canada/UK	-2.25	(3)	-0.81	(12)	-1.98	(2)	-2.14	(3)	-2.48	(3)
EMU/UK	-2.15	(3)	-1.67	(12)	–		-2.16	(2)	–	
Germany/UK	-2.26	(7)	-2.07	(12)	-2.07	(12)	-1.73	(1)	-1.87	(1)
France/UK	-2.27	(3)	-0.51	(12)	–		-2.22	(2)	–	
Italy/UK	-1.95	(3)	-0.53	(12)	-3.23^{**}	(12)	-2.07	(3)	-1.37	(3)
Japan/UK	-2.10	(10)	-2.02	(9)	-1.45	(11)	-1.84	(10)	-2.69^{*}	(10)
US/UK	-2.53	(4)	-2.84^{*}	(8)	-0.89	(12)	-2.43	(3)	-2.08	(2)

[(a)] The columns contain values of the augmented Dickey and Fuller (1979) t-statistic of the null hypothesis that the series contains a unit root using the ADF test regression with an intercept term. The values in parentheses are the number of lagged first differences in the corresponding ADF test regression, where the appropriate number is selected by starting with 12 lagged first differences and decrease this number until the parameter of the last lagged first difference is statistically different from 0. Note that due to incomplete PPI data we do not report ADF test results for the PPI-based decomposition in the case of the EMU and France and only for the 1981-2002 sample in the case of Italy (see Appendix A). A * (**) [***] indicates a rejection of the null of a unit root at the 10% (5%) [1%] significance level, based on (finite sample) critical values from MacKinnon (1991).

Chart 1: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: France/UK





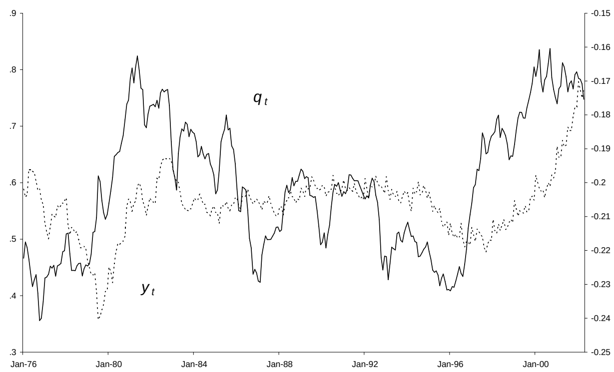
The charts compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. Due to incomplete French PPI data the PPI-based decomposition is not shown (see Appendix A).

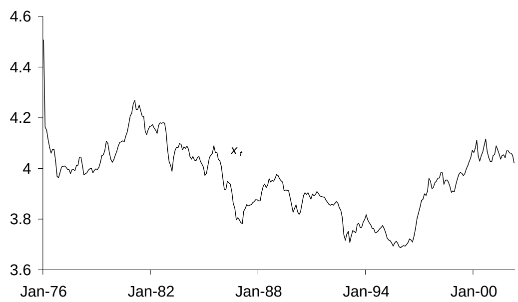
persistence, suggesting that deviations between q_t and y_t are most likely driven by deviations from LOOP. Note that although the 1976-2002 sample spans different exchange rate regimes, the dynamics of the relevant bilateral rates seems not to have been affected a lot by that, eg the fall in the real value of sterling relative to the European currencies during the 1992 ERM crisis is not exceptional from a historical point of view.

3 LOOP deviations in a time series context

In this section we analyse the existence of a long-run link between several bilateral UK real exchange rates and the corresponding relative price ratio of non-tradables and tradables, y_t in **(4)**. The analysis utilises both cointegrated vector autoregressive (VAR) models for q_t and y_t, as well as univariate autoregressive (AR) models for x_t. Section 3.1 describes both the underlying cointegrated VAR model and the AR-based estimates of the half-lives of shocks to the relative tradables price ratio x_t. Section 3.2 presents the results from both types of analysis.

Chart 2: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: Euro/UK





The charts compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. Due to incomplete French and Italian PPI data the PPI-based decomposition is not shown (see Appendix A).

3.1 Method

In order to test for cointegration between q_t and y_t in **(4)** the vector error correction (VEC) framework of Johansen (1991) is used, ie

$$\Delta Z_t = \sum_{s=2}^{12} \chi_s \bar{D}_s + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \quad -\boldsymbol{\beta_0}' \right) \tilde{Z}_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Z_{t-j} + \varepsilon_t \qquad \textbf{(9)}$$

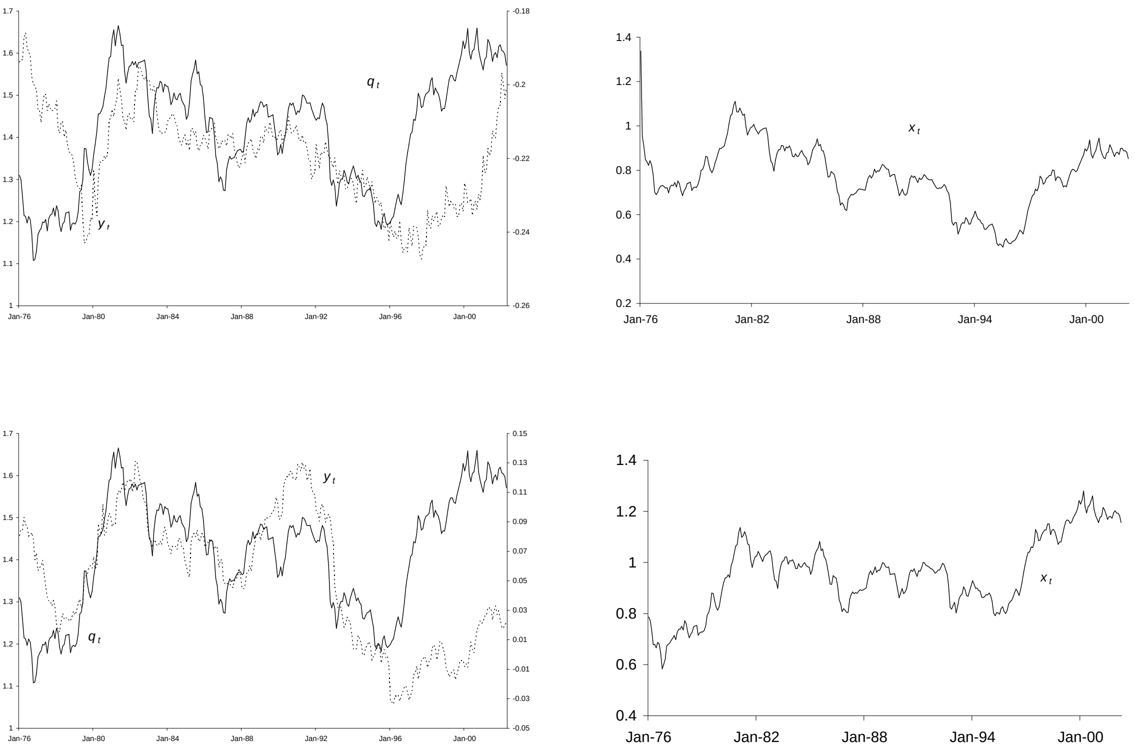
In **(9)**, the 2×1 vector Z_t is given by:

$$Z_t = (q_t \quad y_t)'$$

$\Delta Z_t = Z_t - Z_{t-1}$, $\tilde{Z}_{t-1} = (Z'_{t-1}\ 1)'$, $\bar{D}_s$ is a zero-mean seasonal dummy and ε_{it} is a 2×1 vector of white noise disturbances. The $1 \times r$ vector $\boldsymbol{\beta_0}$ is a vector of intercept terms, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $2 \times r$ matrices of adjustment parameters and cointegrating vectors, respectively, and r is the cointegrating rank value of VEC model **(9)**. Note that this specification of the deterministic part of **(9)** implies that the intercepts appear only in the long-run relationships.

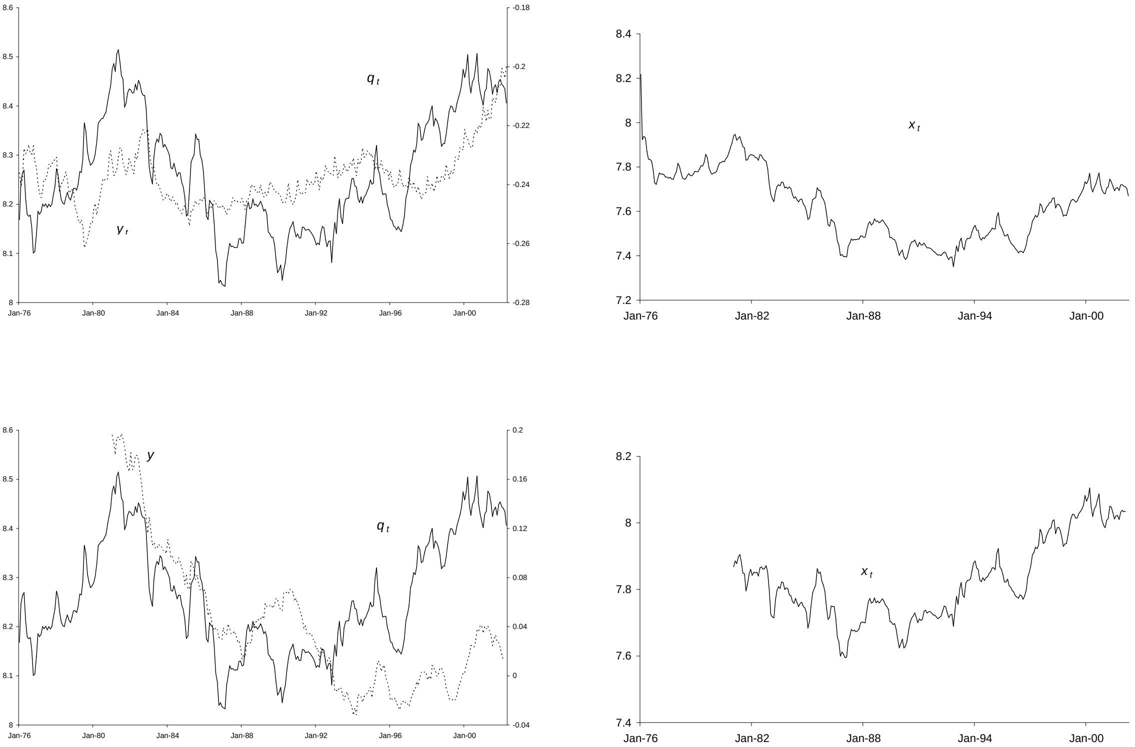
The Johansen (1991) likelihood ratio statistic for the null of r cointegrating vectors versus the alternative of a stationary VAR model can be used to determine the proper value of the cointegrating rank r in **(9)**. Once the proper cointegrating rank has been determined, likelihood ratio tests can be used to test restrictions on the r cointegrating vectors. As these

Chart 3: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: Germany/UK



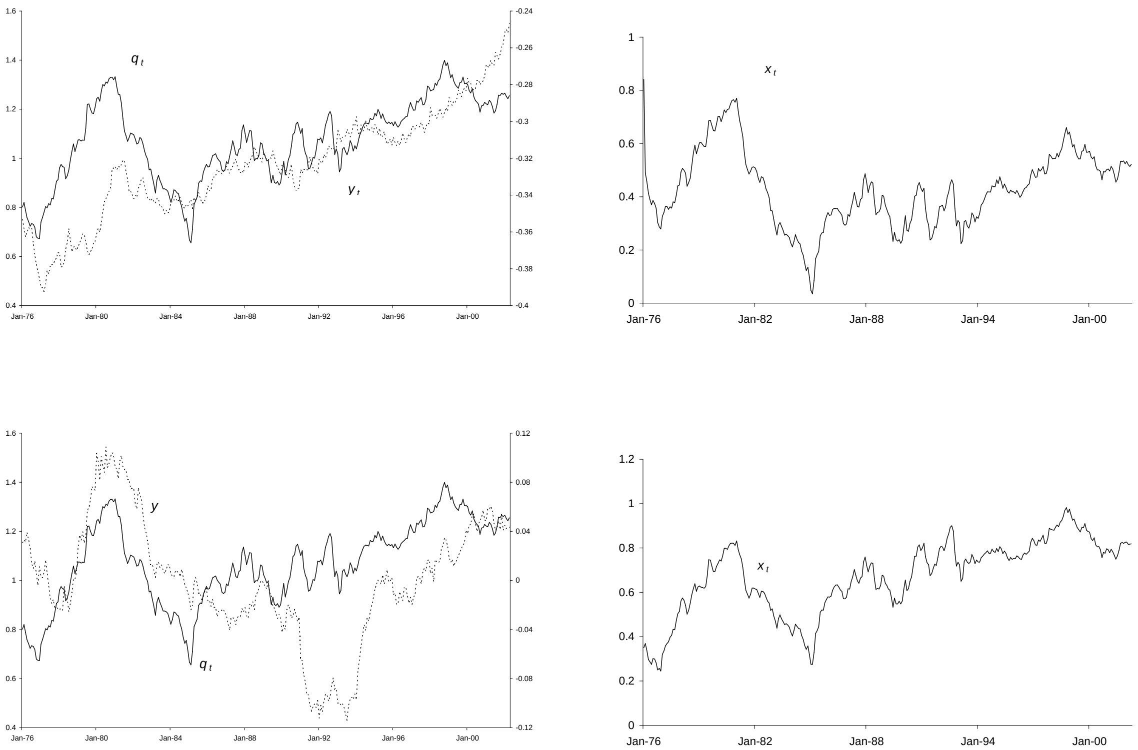
The charts in the first row compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. The second row contains the same comparisons using the PPI-based decomposition of the real exchange rate.

Chart 4: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: Italy/UK



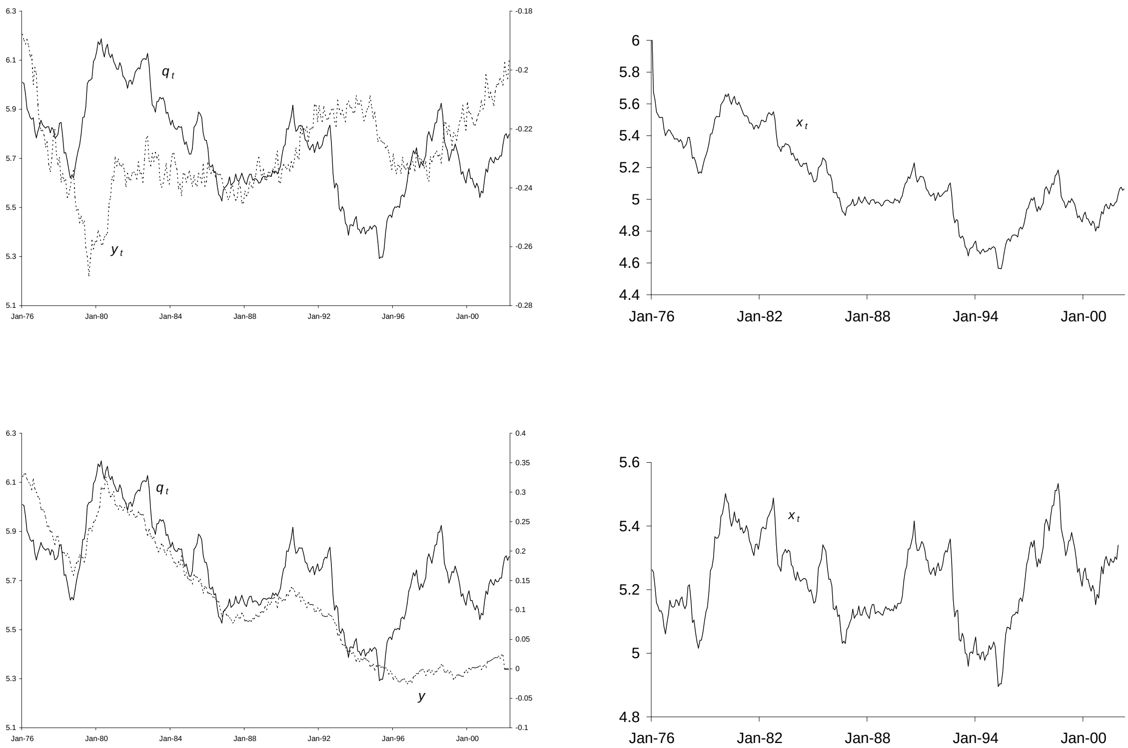
The charts in the first row compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. The second row contains the same comparisons using the PPI-based decomposition of the real exchange rate, which starts in 1981 due to incomplete Italian PPI data (see Appendix A).

Chart 5: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: Canada/UK



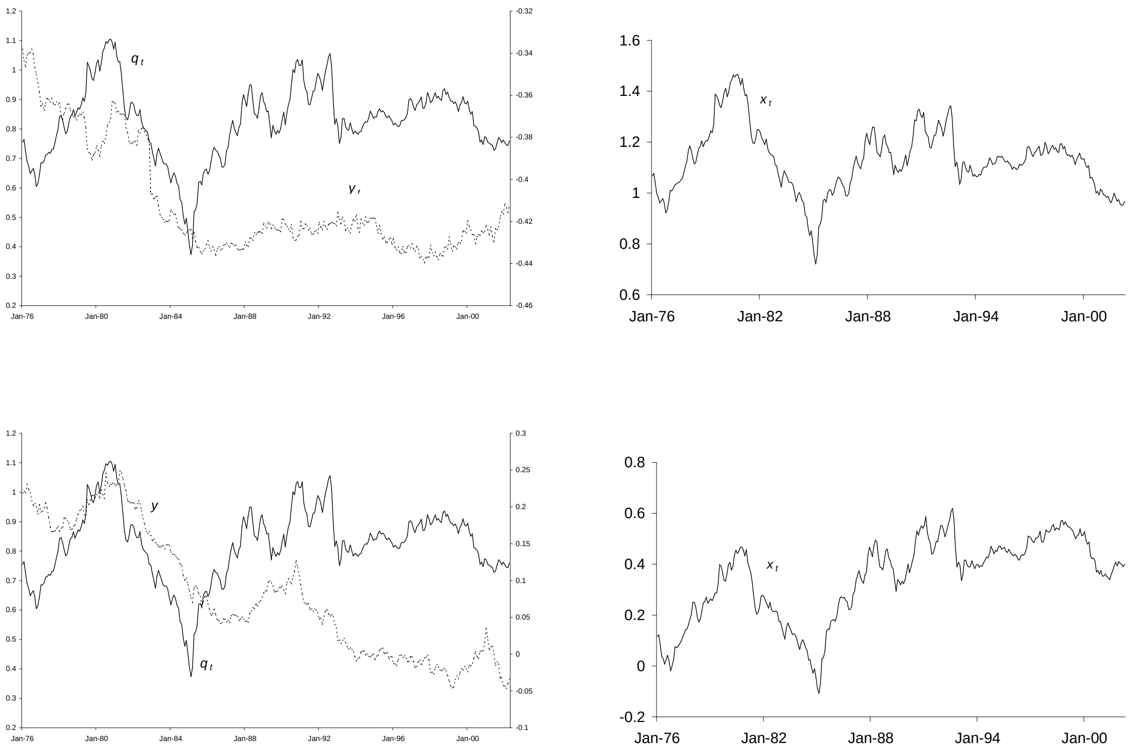
The charts in the first row compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. The second row contains the same comparisons using the PPI-based decomposition of the real exchange rate.

Chart 6: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: Japan/UK



The charts in the first row compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. The second row contains the same comparisons using the PPI-based decomposition of the real exchange rate.

Chart 7: Logarithms of real exchange rate vs. relative non-tradables/tradables goods price ratio and relative tradable goods price: US/UK



The charts in the first row compare the log real exchange rate (q_t) with the log relative non-tradables/tradables goods price ratio (y_t) and the log relative tradables goods price (x_t), using the CPI-based decomposition of the real exchange rate. The second row contains the same comparisons using the PPI-based decomposition of the real exchange rate.

tests are conducted conditional on the cointegrating rank they have standard limiting distributions. Validity of one of the real exchange rate models from Section 2 within VEC **(9)** implies a reduced rank value $r = 1$ and a cointegrating vector, normalised on q_t, equal to $(\bar{\boldsymbol{\beta}}' \;\; -\bar{\boldsymbol{\beta}}_{\mathbf{0}}') = (1 \;\; -1 \;\; -c)$, which complies with **(6)**. Hence, this implies testing the restriction

$$\beta_q + \beta_y = 0 \qquad \textbf{(10)}$$

on the unnormalised cointegrating vector (ie $\boldsymbol{\beta}$ in **(9)**).

VEC model **(9)** is a bivariate system of the log real exchange rate q_t and the log relative non-tradable/tradable price ratio y_t, indicating that the cointegration model for q_t does not allow for an I(1) log relative tradables price x_t to have an effect on q_t. If an I(1) x_t had been allowed for, then this would have meant that international goods arbitrage opportunities would never be utilised, ie LOOP deviations **(5)** would be permanent. None of the economic models available in the literature allow for this, as it essentially assumes the absence of any international trade linkages. Phenomena like shipping costs, pricing-to-market and so on can cause persistent I(0) movements in x_t and **(5)**, however, and when we compare the equilibrium errors from **(9)** with **(6)** one can observe that the degree of equilibrium error correction in **(9)** is implicitly determined by the persistence of both LOOP deviations φ_t in **(5)** and relative measurement errors θ_t (see **(4)**). If one or both of these sources of equilibrium errors are very persistent relative to the available span of the data, Otero and Smith (2000) show that the VEC-based cointegration framework is often unable to detect the presence of cointegration. From an economic point of view, identifying the degree of persistence of LOOP deviations is of interest as it gives an indication of the severity of the failure of the 'Law of One Price'. Hence, as a complement to the cointegration test results, the half-life of a shock to the relative tradables price x_t is also computed.

In order to compute the half-life of a shock to a bilateral x_t, an AR(p) model is estimated for x_t:

$$x_t = \delta_0 + \sum_{j=1}^{p} \delta_j x_{t-j} + \nu_t \qquad \textbf{(11)}$$

where $\nu_t \sim$ i.i.d.$(0, \sigma_\nu^2)$. Based on **(11)**, one can compute the impulse response function of a unity shock to x_t, ie how x_t behaves in the future when there is a unity shock now *ceteris paribus*. The half-life of x_t can now be defined as the number of years necessary for a unity impulse to x_t to have dissipated by 50%. In contrast to the more traditional measure ln$(0.5)/$ln$(\sum_{j=1}^{p} \delta_j)$, the impulse response-based measure of the half-life is valid irrespective of whether x_t contains a unit root or not as the individual δ_js in **(11)** always have a normal limiting distribution, see Inoue and Kilian (1999). As the impulse response function of **(11)** is a non-linear function of the δ_js, it is convenient to use a bootstrap procedure to compute confidence intervals for the estimates of the half-life in order to deal with the small sample bias due to this non-linearity.

When x_t is very persistent, however, OLS estimates of the parameters in **(11)** are biased downwards in finite samples. This small bias in the parameter estimates, in combination with the non-linearity of the impulse response function coefficients, will result in a skewed and biased estimate of the impulse response coefficients. As a consequence, the half-life estimate will also have a skewed and biased distribution. Computing confidence intervals around this biased half-life estimate through traditional bootstrap procedures will not circumvent this, as traditional bootstrap procedures are based on the biased OLS estimates of **(11)**. As an alternative, the impulse response-based half-life estimate of x_t is constructed, following Kilian (1998), through a bootstrap-based mean-bias corrected estimate of **(11)**. This bootstrap-based mean-bias corrected estimate of **(11)** in turn is used in a double bootstrap procedure, in order to generate confidence intervals around the mean-bias corrected half-life estimate of x_t. Appendix B provides a more detailed description of how the bootstrap-based mean-bias corrected half-life estimates and the corresponding confidence intervals were computed.

3.2 Cointegration results

Monthly data over the 1976-2002 period for the bilateral relationships of Canada, France, Germany, Italy, Japan and the United States *vis-à-vis* the United Kingdom were used to test the long-run appropriateness of the real exchange rate relationship **(4)** through cointegration analysis. As explained in Section 2, there are several ways in which the real exchange rate can be decomposed into tradable and non-tradable parts. The PPI-based decomposition **(7)** and the CPI-based decomposition **(8)** were both used to get different measures of the log relative non-tradables/tradables price ratio y_t.[(3)]

We constructed a VEC model like **(9)** for each bilateral relationship based on our different measures of the log relative non-tradables/tradables price ratio y_t. The lag order p in **(9)** was selected based on a general-to-specific approach in which a likelihood ratio test statistic was used to test the appropriateness of an upper-bound lag order in an unrestricted VAR of q_t and y_t in levels. If the upper-bound lag order is insignificant, the lag order was decreased and again a likelihood ratio test was conducted to see whether this lag order is appropriate and so on. In this study, the upper-bound lag order is set equal to $p = 12$ and tested downwards. For robustness, the presence of residual autocorrelation at the selected

(3) Appendix A contains a more detailed description of how the data were constructed.

lag order was checked for, and if any residual autocorrelation is detected, the lag order p in this unrestricted VAR in levels was increased until this phenomenon disappears. The overall results can be found in Table B.

The upper panel of Table B reports the results for the PPI-based decomposition **(7)** of the real exchange rate. In general, the null of no cointegration could not be rejected. Only in the case of the US/UK relationships was evidence for one cointegrating vector found. For this bilateral relationship the restriction of long-run proportionality between q_t and y_t (ie restriction **(10)** on the cointegrating vector in **(9)**) was accepted.

As can be observed from the lower panel of Table B, the evidence for a cointegrating relationship between the log real exchange rate q_t and the relative price ratio of non-tradables/tradables y_t was more frequent for the CPI-based decomposition **(8)**. With the exception of the Canada/UK, Italy/UK and Japan/UK relationships the null of no cointegration for the remaining bilateral relationships could be rejected. However, for the EMU/UK relationship, as shown in the third column of the table, the possibility that the corresponding VEC system has more than one cointegrating vector could not be rejected, and thus it cannot be assumed that the representation in **(9)** is valid. For the remaining 3 cointegrating relationships identified, the restriction of long-run proportionality between q_t and y_t for the France/UK and Germany/UK bilaterals can be accepted. Hence, there seems to be somewhat stronger evidence of a valid cointegrating relationship between q_t and y_t for the bilateral rates *vis-à-vis* European countries, which might reflect the fact that transportation costs for goods are lower between the United Kingdom and other European countries, than for inter-continental trade.[4] These findings are consistent with those in Engel (2003).

3.3 Half-lives of LOOP deviations

The traded goods exchange rate, x_t, measures the deviations of the real exchange rate, q_t, from the relative price ratio of non-tradables/tradables, y_t. The cointegrating relationship as modelled in **(9)** and **(10)** implies stationarity in the deviations from the LOOP. The results in Section 3.2 provide mixed evidence that this model is valid for all the real exchange rates within the sample. In some cases it is difficult to identify a proportional relationship between the real exchange rate, q_t, and the relative price ratio of

[4] In an attempt to model real exchange rate dynamics across OECD countries through real interest rate differentials Chortareas and Driver (2001) are more successful when they focus on small open economies than when they focus on the G7 economies. This may suggest that a large degree of openness to international trade of the countries used in a data sample could improve our ability to model real exchange rate dynamics properly.

Table B: Cointegration tests for the long-run real exchange rate relationship (6), 1976:01-2002:04[(a)]

	p	LR(0|2)	LR(1|2)	LR($\beta_q + \beta_y = 0$)
		PPI-based decomposition		
Canada/UK	6	15.47	4.61	
Germany/UK	6	12.91	3.11	
Italy/UK	3	15.72	4.30	
Japan/UK	9	16.90	6.30	
US/UK	1	18.85^{*}	3.13	2.04
				(0.15)
		CPI-based decomposition		
Canada/UK	8	15.45	4.62	
EMU/UK	9	27.16^{***}	8.52^{*}	
France/UK	10	32.21^{***}	4.63	0.01
				(0.94)
Germany/UK	7	23.21^{**}	7.44	1.31
				(0.25)
Italy/UK	6	8.90	1.92	
Japan/UK	8	9.80	2.28	
US/UK	3	20.78^{**}	4.10	3.00
				(0.08)
90%		17.79	7.50	
95%		19.99	9.13	
99%		24.74	12.73	

[(a)] The column denoted with 'p' contains the order of first differences in **(9)**. LR($r|2$) denotes the values of the Johansen (1991) likelihood ratio test statistic for H_0: rank($\boldsymbol{\alpha\beta'}$) = r versus H_1: rank($\boldsymbol{\alpha\beta'}$) = 2 in **(9)**. The symbol * (**) [***] indicates rejection of H_0 at the 10% (5%) [1%] significance level. The row '90%' ('95%') ['99%'] contains the asymptotic 90% (95%) [99%] quantile for LR($r|2$) under the null, see Johansen (1996, Table 15.2). The column denoted with LR($\beta_q + \beta_y = 0$) contains, if $r = 1$ is accepted, the likelihood ratio test of the restriction $\hat{\beta}_q + \hat{\beta}_y = 0$ and the corresponding $\chi^2(1)$ p-values are reported in parentheses. Note that due to incomplete PPI data we do not report half-life estimates for the PPI-based decomposition in the case of the EMU and France and only for the 1981-2002 sample in the case of Italy (see Appendix A).

non-tradables/tradables, y_t. This lack of significant evidence could be due to a lack of statistical power in the time series analysis. If so, it would be useful to consider the properties of the data if the restriction of long-run proportionality is imposed, ie to assume that the deviations from the relationship, as measured by x_t, are stationary.

Half-life analysis calculates the expected time period for a LOOP deviation to decay by 50%, and therefore allows the severity of the LOOP deviations in the sample to be quantified. As explained in Section 3.1, an AR model like **(11)** with corrected estimates of the AR parameters was used to produce mean unbiased half-life estimates for x_t in the presence of serial correlation and small sample bias. Table C presents the estimated mean unbiased half-lives for each of the relative tradable goods prices using both the CPI and the PPI-based measures. The point estimates suggest the half-lives are large, ranging from 2 to 50 years. Although for the majority of the sample the estimated half-life is less than 10 years, for Italy it is consistently much larger than that across the different decompositions, which possibly reflects the large and persistent real depreciation of the Italian currency *vis-à-vis* other European currencies after the ERM crisis in 1992. Bootstrap-based confidence intervals which measure the precision of the implied estimated half-lives were also calculated, see also Appendix B. The width of the confidence intervals reported are consistent with the findings for the cointegration analysis in Section 3.2. The absence of a proportional relationship implies that the existence of a unit root in the tradable goods price cannot be ruled out for these data, which is also consistent with the ADF tests in Section 2. The 95% confidence intervals for the half-life estimates in Table C indicate that in general the upper bound of the estimated confidence intervals, with the sole exception of the Japan/UK bilateral using the PPI-based decomposition, is approaching infinity.

The standard time series-based cointegration approach used in Section 3.2 seems unable to identify a proper long-run relationship between the real exchange and the relative non-tradable/tradable price ratio as in **(6)**. Given that the point estimates of the half-lives generated for most relative price ratios *vis-à-vis* the United Kingdom in the sample are large relative to the time span of the data, this may not be surprising. These high and variable half-lives imply that the sample may not be long enough to capture any mean reversion in the deviation between the real exchange rate and the relative non-tradable/tradable price ratio.

There are a number of reasons why the rate of mean reversion in the deviation between the real exchange rate and the relative non-tradable/tradable price ratio can be perceived to be low. First, in our framework the deviation between the real exchange rate and the relative non-tradable/tradable price ratio is assumed to be composed of the relative tradable price

Table C: Estimated half-lives for relative tradable goods prices, 1976:01-2002:04[(a)]

	CPI-based decomposition			*PPI-based decomposition*		
	Lags	HL	95% CI	Lags	HL	95% CI
Canada/UK	4	5.25	[1.58, ∞)	4	5.92	[1.75, ∞)
France/UK	3	5.00	[1.92, ∞)	–	–	–
Germany/UK	9	6.42	[1.83, ∞)	5	11.00	[1.50, ∞)
Italy/UK	7	23.92	[1.50, ∞)	4	47.30	[1.30, ∞)
Japan/UK	11	35.17	[3.08, ∞)	10	2.17	[1.33, 7.58]
US/UK	3	8.75	[1.58, ∞)	3	8.80	[1.60, ∞)
EMU/UK	8	14.30	[1.90, ∞)	–	–	–

[(a)] Columns denoted with 'HL' are the mean unbiased impulse response-based half-life estimates of the tradable price ratio x_t based on **(11)** with corresponding lag order reported in columns 'Lags', whereas columns denoted with '95% CI' are the corresponding 95% confidence intervals. All the point and interval estimates are based on the procedures of Appendix B. Note that due to incomplete PPI data we do not report half-life estimates for the PPI-based decomposition in the case of the EMU and France and only for the 1981-2002 sample in the case of Italy (see Appendix A).

ratio and the relative demeaned price index measurement error, as in **(6)**. Compositions of price indices can change systematically, which can result in very persistent behaviour of the relative measurement error term θ_t. On the other hand, the results of the half-life analysis on the relative tradable goods prices are consistent with the hypothesis that deviations from LOOP could be due to shipping costs. Frictions in the costs of transferring physical goods between countries imply that the difference in the common currency price of tradable goods at home and abroad has to reach a certain level before it becomes profitable for producers to ship their produce abroad. Dumas (1992) and Obstfeld and Rogoff (2001) build theoretical models in which this type of non-linear adjustment in relative prices of tradable goods due to shipping costs can increase the adjustment time for shocks to the relative tradable price and/or real exchange rate. Obstfeld and Taylor (1997) and Taylor, Peel and Sarno (2001) find some empirical support for this using non-linear time series models. Finally, the measures of the relative tradable goods price could be affected by dynamic aggregation bias. The dynamics of the individual relative prices which make up the measures of x_t can potentially be heterogeneous in nature. Imbs, Mumtaz, Ravn and Rey (2002) claim that ignoring the heterogeneous dynamics of the different components in constructing x_t could result in overstating the degree of persistence in this x_t.

Given these possible factors it is likely that both the deviations between q_t and y_t as well as the deviations from LOOP are quite persistent in the sample period. This, in combination with the relatively short length of the available sample, results in a lack of statistical power to model the relationship between the real exchange rate and the relative non-tradable/tradable price ratio properly. One way to circumvent this problem is to use a multi-country panel model.

4 LOOP deviations in a panel context

From the previous section it becomes clear that empirical evidence on the theoretically appropriate long-run link between real exchange rates and the corresponding relative non-tradable/tradable price ratio for the sample of bilateral sterling real exchange rates is limited. This finding is especially apparent when the PPI-based non-tradables/tradables decomposition of the real exchange rate is used. LOOP deviations are an important source of disturbances in the long-run link between real exchange rates and the relative non-tradables/tradables price ratio. Indeed, the half-life estimates for the bilateral tradable price ratios in Section 3.3 indicate that it can take up to several decades for shocks to the LOOP relationship to die out. Moreover, the variability of the half-lives is very large.

High and variable half-lives of LOOP deviations would make it very hard to measure any significant mean-reversion in the tradable price ratios and consequently in the deviations between the real exchange rate and the relative non-tradable/tradable price ratio in a relatively short data span. Under such circumstances Shiller and Perron (1985) and Otero and Smith (2000) have shown through Monte Carlo studies that the power of unit root tests and VAR-based cointegration tests to reject the null of either non-stationarity and no cointegration in the face of a persistent alternative hypothesis indeed depends on the span of the data sample. As the 1976-2002 sample period is relatively short, methods such as those of Johansen (1991), would be expected to have difficulty in verifying the cointegration restriction as summarised in **(6)**, especially since the short-run dynamics could be affected by the different exchange regimes prevailing during the sample period. Alternatively, panel-based techniques can be applied in which inference is based on an artificially extended number of observations, and that is done in this section.

In Section 4.1 we describe the multi-country panel cointegration-testing framework, which is basically a panel generalisation of the Johansen (1991) approach. Section 4.2 reports the estimation results from this panel framework.

4.1 Method

The VEC model **(9)** in Section 3.2 is used to test the cointegration restriction in **(6)** for each bilateral sterling real exchange rate relationship separately. However, log real exchange rates *vis-à-vis* the same base country co-move with each other, as they are by definition contemporaneously correlated. This phenomenon in itself is an argument to analyse all the bilateral sterling real exchange rates in our sample simultaneously as applying the VEC **(9)** for each bilateral relationship separately will then be based on inefficiently estimated parameters. This, plus the problem of the short sample period, could decrease the power of the pure time series Johansen (1991) cointegrated VAR approach.

As an alternative the implied long-run real exchange rate relationship **(6)** can be analysed within a multi-country panel setting. Drine and Rault (2002), for example, use an 11-country OECD panel of annual data to analyse the long-run relationship between real exchange rates and the relative non-tradable/tradable price ratio using the Pedroni (1996) panel cointegration framework, and reject the appropriateness of the that relationship. The Pedroni (1996) framework, however, is basically a panel version of the static OLS regression residuals-based Engle and Granger (1987) cointegration test. Monte Carlo experiments in Groen (2002) indicate that, for multi-country panels with a small cross-section dimension, the panel Engle and Granger (1987) class of panel cointegration tests has low power to reject the null hypothesis of no cointegration relative to an alternative of persistent but temporary deviations from cointegration. A panel version of the VAR-based Johansen (1991) cointegration approach, on the other hand, is shown to have much more power in such circumstances. Given the limited amount of bilateral sterling relationships in our sample, we therefore use the latter approach.

There are several ways in which a panel version of the VAR-based Johansen (1991) cointegration test can be constructed. One way would be to follow Larsson, Lyhagen and Löthgren (2001) and simply construct normalised cross-country averages of the individual trace statistics from Section 3.2. Under the assumption of no contemporaneous cross-unit correlation Larsson *et al* (2001) show that these normalised cross-country averages have a standard-normal distribution. However, given that one rationale for using the panel approach to test our long-run real exchange rate relationship **(6)** is the presence of significant contemporaneous correlation across UK bilateral real exchange rates, this assumption of no cross-unit correlation is invalid. Indeed, Monte Carlo experiments in Groen and Kleibergen (2003, Section 5) indicate that applying the Larsson *et al* (2001) test when the no cross-unit correlation assumption is violated severely biases the corresponding test results. A different approach is to construct a panel version of the Johansen (1991) test, which takes into account the presence of contemporaneous cross-unit dependence, and this alternative approach is now discussed in more detail.

VEC models like **(9)**, constructed for each of our N bilateral sterling real exchange rates, can be stacked into one system:

$$\begin{array}{ccccccc} \Delta Z_{1t} = & \sum_{s=2}^{12}\chi_{1,s}\bar{D}_s + & \boldsymbol{\alpha}_1\left(\boldsymbol{\beta}_1' \quad -\boldsymbol{\beta}_{0,1}'\right)\tilde{Z}_{1,t-1} & + & \sum_{j=1}^{p_1-1}\Gamma_{j,1}\Delta Z_{1,t-j} & + & \varepsilon_{1t} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ \Delta Z_{Nt} = & \sum_{s=2}^{12}\chi_{N,s}\bar{D}_s + & \boldsymbol{\alpha}_N\left(\boldsymbol{\beta}_N' \quad -\boldsymbol{\beta}_{0,N}'\right)\tilde{Z}_{N,t-1} & + & \sum_{j=1}^{p_N-1}\Gamma_{j,N}\Delta Z_{N,t-j} & + & \varepsilon_{Nt} \end{array} \quad \textbf{(12)}$$

where we assume [5]

$$\begin{pmatrix}\varepsilon_{1t}\\ \vdots \\ \varepsilon_{Nt}\end{pmatrix} \sim \mathrm{N}(\mathbf{0},\Omega_\varepsilon), \quad \Omega_\varepsilon = \frac{1}{T-1-p_{\max}}\sum_{t=1}^{T-1-p_{\max}}\left[\begin{pmatrix}\varepsilon_{1t}\\ \vdots \\ \varepsilon_{Nt}\end{pmatrix}\begin{pmatrix}\varepsilon_{1t}\\ \vdots \\ \varepsilon_{Nt}\end{pmatrix}'\right]$$

and $p_{max} = \max(p_1,\ldots,p_n)$. Note that the disturbance covariance matrix Ω_ε of **(12)** is unrestricted and the panel framework, in contrast, for example, to the Pedroni (1996) and the Larsson *et al* (2001) approaches, therefore allows for cross-country correlations. Maximum likelihood estimation of the parameters in system **(12)** under the assumption of a *common* cointegrating rank $r_1 = \cdots = r_N = r$ can be achieved with the iterative

(5) In order to facilitate a convenient estimation of the disturbance covariance matrix, we utilise a balanced multi-country panel to estimate all the parameters in **(12)**.

Generalised Method of Moment (GMM) estimators of Groen and Kleibergen (2003). Different possible specifications of the cointegrating vectors in our panel VEC model **(12)** are tested:

Definition 1 In the N-country panel VEC model which corresponds with **(12)** the following hypotheses are tested:

- $\mathbf{B}(r)|\mathbf{A}$ tests the null hypothesis that for each cross-sectional unit i the cointegrating rank value equals r versus the alternative hypothesis that for each cross-sectional unit i has a full rank value while not assuming $\boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_N$ in **(12)**.

- $\mathbf{C}(r)|\mathbf{A}$ tests the null hypothesis that for each cross-sectional unit i the cointegrating rank value equals r versus the alternative hypothesis that for each cross-sectional unit i we have a full rank value assuming $\boldsymbol{\beta}_1 = \cdots = \boldsymbol{\beta}_N = \boldsymbol{\beta}$ in **(12)**.

In order to construct the corresponding likelihood ratio test statistics, **(12)** has to be estimated both under the null and alternative hypotheses with the Groen and Kleibergen (2003) estimation approach. These test statistics have non-standard asymptotic distributions and the corresponding critical values are computed with the procedures from Groen (2002, Appendix).

A crucial assumption underlying the panel VEC system **(12)** is that there is no cross-unit cointegration, ie the I(1) series from country *i* cannot be cointegrated with those of country *j* for $i \neq j$. In order to establish that the results based on the aforementioned panel cointegration test statistics are not contaminated by the presence of cross-unit cointegration, a two-step procedure, based on Gonzalo and Granger (1995), is followed to test whether the cross-country structure in **(12)** is misspecified or not:[(6)]

1. If the null hypothesis under $\mathbf{B}(1)|\mathbf{A}$ (see Definition 1) can be accepted, it implies that for each country-specific bivariate subsystem the non-stationarity of both $q_{i,t}$ and $y_{i,t}$ is due to one common unit root process, which we denote as the common I(1) factor. For each country-specific block in **(12)** the common I(1) factor is extracted through
$$f_{i,t} = \boldsymbol{\alpha}'_{\perp,i} \begin{pmatrix} q_{i,t} \\ y_{i,t} \end{pmatrix} \quad \text{for} \quad i = 1, \ldots, N, \qquad \textbf{(13)}$$
where $\boldsymbol{\alpha}_{\perp,i}$ is 2×1 such that $\boldsymbol{\alpha}'_{\perp,i}\boldsymbol{\alpha}_i \equiv \mathbf{0}$ and $\boldsymbol{\alpha}'_{\perp,i}\boldsymbol{\alpha}_{\perp,i} \equiv 1$. Following Gonzalo and Ng (2001, pages 1,542-43) we estimate $\boldsymbol{\alpha}_{\perp,i}$ in **(13)** by setting it equal to the eigenvector

(6) As suggested by Banerjee, Marcellino and Osbat (2000) in the context of panel VEC models.

which corresponds with the smallest eigenvalues of

$$\boldsymbol{\alpha}_i \boldsymbol{\alpha}_i' \quad \text{for} \quad i = 1, \ldots, N,$$

where the $\boldsymbol{\alpha}_i$'s are the country-specific error-correction parameters which result from the Groen and Kleibergen (2003) iterative GMM estimation of **(12)**. The specification in **(13)** indicates that each of the N common I(1) factors associated with **(12)** is for each cross-section unit i proxied by a linear combination of $q_{i,t}$ and $y_{i,t}$, which is orthogonal to the cointegrating I(0) combination of these two series.

2. A VEC system like **(9)** is constructed for the N common I(1) factors $f_{1,t}, \ldots, f_{N,t}$, with the common factors computed in the previous step, and conduct the Johansen (1991) likelihood ratio test for testing the cointegrating rank of this system. In order for the structure of **(12)** to be valid, this VAR-based cointegration test on the N common I(1) factors should indicate that these common factors are **not cointegrated**.

4.2 Results

In this subsection, the validity of the long-run real exchange rate relationship **(6)** is tested within a multi-country panel setting as summarised by **(12)**. However, we do not use all six bilateral real sterling exchange rate pairs, as the UK real exchange rates relative to France, Germany and Italy are likely to be cointegrated among each other. We therefore use a 4-country panel data set, and thus $N = 4$ in **(12)**. For of the CPI-based non-tradables/tradables decomposition of the real exchange rate, the behaviour of the real sterling exchange rate relationships *vis-à-vis* France, Germany and Italy is summarised by a synthetic real EMU/UK rate.[7] We therefore use in our panel system **(12)** data on the Canada/UK, EMU/UK, Japan/UK and US/UK pairs. Due to gaps in the PPI data for France and Italy, the PPI-based non-tradables/tradables real exchange rate decomposition data on the Canada/UK, Germany/UK, Japan/UK and US/UK pairs are used in the multi-country panel data set.

The first row of Table D reports the panel cointegration test results for the CPI-based non-tradables/tradables decomposition of the real exchange rate relationships of the United Kingdom *vis-à-vis* Canada, the EMU-zone, Japan and the United States. The lag orders of the panel VEC system for this particular panel are set equal to those of the individual VEC models from Section 3.2 for each of the bilateral relationships.[8] The test results indicate

[7] This EMU/UK rate is a weighted average of the French, German and Italian rates based on time-varying relative common-currency GDP weights. See also Appendix A.

[8] The usage of the individual lag orders is motivated by the 'bottom-up' modelling strategy for restricted VAR models from Lütkepohl (1993, pages 182-83), where the panel VEC model **(12)** can be considered as a restricted VAR.

Table D: Panel cointegration tests for the long-run real exchange rate relationship (6), 1976:01-2002:04[(a)]

	LR($\mathbf{B}(0)\|\mathbf{A}$)	LR($\mathbf{B}(1)\|\mathbf{A}$)	LR($\mathbf{C}(1)\|\mathbf{A}$)	LR($\beta_q + \beta_y = 0$)
CPI-based decomp.	67.60^{**}	16.22	24.52	0.66 (0.42)
PPI-based decomp.	60.95^{*}	16.73	26.79	0.19 (0.67)
Critical values for the Groen and Kleibergen (2003) panel cointegration rank tests				
90%	59.58	23.35	27.05	
95%	63.25	25.88	29.80	
99%	70.51	31.10	35.42	

(a) The results in the table are based on either a panel of data on the Canada/UK, EMU/UK, Japan/UK and US/UK relationships in case of the CPI-based decomposition or a panel of data on the Canada/UK, Germany/UK, Japan/UK and US/UK relationships in case of the PPI-based decomposition. Columns LR($\mathbf{B}(r)|\mathbf{A}$) and LR($\mathbf{C}(r)|\mathbf{A}$) report the Groen and Kleibergen (2003) likelihood ratio test statistics for H_0: common cointegration rank = r versus H_1: common cointegration rank = 2 in **(12)** under heterogeneous and homogeneous cointegrating vectors respectively, see Definition 1. The column denoted with LR($\beta_q+\beta_y = 0$) contains, if H_0: $\mathbf{C}(1)|\mathbf{A}$ is accepted, the likelihood ratio test of the restriction $\beta_q + \beta_y = 0$ and the corresponding $\chi^2(1)$ p-values are reported in parentheses. The row '90%' ('95%') ['99%'] contains the asymptotic 90% (95%) [99%] quantile for either LR($\mathbf{B}(r)|\mathbf{A}$) and LR($\mathbf{C}\ (r)|\mathbf{A}$) under the null, which are computed with the procedures of Groen (2002, Appendix), and the symbol * (**) [***] indicates rejection of H_0 at the 10% (5%) [1%] significance level.

Table E: Cointegration tests on the N common I(1) factors in (12), 1976:01-2002:04[(a)]

	CPI-based	PPI-based	90%	95%	99%
LR(0|4)	45.12	46.59	49.92	53.42	60.42
LR(1|4)	20.41	21.46	31.88	34.80	40.84
LR(2|4)	8.13	10.53	17.79	19.99	24.74
LR(3|4)	2.59	1.65	7.50	9.13	12.73
$p-1$	7	9			

[(a)] The row denoted with $p-1$ contains the order of first differences determined with AIC in a VEC system like **(9)** but now for the common I(1) factors $f_{1,t}, \ldots, f_{4,t}$ associated with **(12)** under $r = 1$, where $f_{1,t}, \ldots, f_{4,t}$ are computed through **(13)**. LR($r|4$) denotes the values of the Johansen (1991) likelihood ratio test statistic for H_0: rank($\boldsymbol{\alpha\beta}'$) = r versus H_1: rank($\boldsymbol{\alpha\beta}'$) = 4 in the VEC system for $f_{1,t}, \ldots, f_{4,t}$. The symbol * (**) [***] indicates rejection of H_0 at the 10% (5%) [1%] significance level. The row '90%' ('95%') ['99%'] contains the asymptotic 90% (95%) [99%] quantile for LR($r|4$) under the null, see Johansen (1996, Table 15.2).

that the null of no cointegration can be rejected across the four bilateral real sterling relationships, whereas the null of 1 cointegrating vector (and thus 1 equilibrium relationship) between q_t and y_t can be accepted across the panel. The results for the CPI-based panel in the first row of Table D, and in particular the third and fourth columns, further show that the cointegrating vectors are identical across the four bilateral relationships and that there is proportionality between q_t and y_t in this common cointegrating vector, as is implied by **(6)**.

From the second row of Table D it is clear that the results for the PPI-based non-tradables/tradables decomposition of the real exchange rate for the panel data set on the Canada/UK, Germany/UK, Japan/UK and US/UK real rates are similar. Again, the null of no cointegration can be rejected whereas the null that the four real rates share a common long-run relationship between q_t and y_t can be accepted, which complies with **(6)**.

The results reported in Table D are conditional on the validity of the assumption in panel

VEC model **(12)** that there is no cointegration across the individual VEC systems. In order to assess the robustness of our panel cointegration analysis, a Gonzalo and Granger (1995)-type test procedure is conducted to test for the absence of cross-unit cointegration across the stochastic trends in the two multi-country panel data sets, see Section 4.1, and the test results are reported in Table E. Comparing the results for the CPI-based and PPI-based panels in the first and second columns of Table E, it becomes apparent that for neither of the two panels can the null hypothesis that the country-specific common I(1) factors are not cointegrated with each other be rejected. These results provide an indication that the assumption of no cross-unit cointegration in **(12)** is most likely an appropriate assumption for both panels. This confirms the economic intuition that the behaviour of the UK relative to the European economies should be summarised in a synthetic EMU/UK relationship to avoid cross-unit cointegration due to the strong economic interdependence between France, Germany and Italy.

5 Out-of-sample evaluation

Since the seminal papers of Meese and Rogoff (1983, 1988) on out-of-sample evaluation of structural models of nominal and real exchange rate behaviour respectively, it has become standard practice in empirical exchange rate research to conduct this kind of analysis. In this paper, the out-of-sample evaluation serves two purposes. First, it is a check on the robustness of the in-sample results on the long-run relationship between the real exchange rate q_t and the relative non-tradables/tradables price ratio y_t from Sections 3.2 and 4.2. Second, it can give an indication, in particular for the panel-based estimates, after approximately how many periods a reversion of the tradables price ratio x_t to LOOP-consistent levels will 'kick in'.

The out-of-sample evaluation method is described in Section 5.1. The results are reported in Section 5.2.

5.1 Method

Meese and Rogoff (1988) compared post-sample predictions of both PPP-based and real uncovered interest rate parity-based models of real exchange rate behaviour to a random walk or 'no change' model at forecasting horizons up to one year. Mark and Choi (1997) conduct a similar exercise in which they compare the out-of-sample exchange rate change predictions of current error-correction terms, based on several structural real exchange rate models, with those of the random-walk model at horizons up to four years. This approach has become standard in empirical exchange rate analysis, and so is followed here; we compare the out-of-sample exchange rate forecasts (in levels) of either **(9)** or **(12)**. Our evaluation criterion for the log exchange rate level is the root of the mean of squared forecast errors [RMSE]

$$\text{RMSE} = \sqrt{\frac{1}{T - t_0 - h} \sum_{t=t_o}^{T-h} e_{s,t+h}^2}, \qquad \textbf{(14)}$$

where t_0 is the first observation in the forecast period, h is the forecasting horizon and $e_{s,t+h}$ is the forecast error of the model-generated prediction of the log real exchange rate level relative to the *observed* log real exchange rate level.

The forecasts are generated in a recursive manner. Our first h-period ahead forecast is generated at observation t_0 ($t_0 < T$). Thus, we first estimate for a sample which runs up to t_0 either **(9)** under $r = 1$ for each of our bilateral rates separately, or **(12)** based on one common equilibrium relationship jointly for all our bilateral rates. Both **(9)** and **(12)** impose proportionality between q_t and y_t. Based on these estimates we generate forecasts for the log exchange rate levels at all forecasting horizons h. For $h > 1$, the exchange rate forecasts in the individual cointegrated VAR system **(9)** and the panel VEC model **(12)** are generated in a dynamic manner, ie forecasts for q_t and y_t in the previous month are used to generate the exchange rate forecast for the current month. These two steps are repeated for the observations $t_0 + 1, t_0 + 2, \ldots, T - h$. In order to evaluate the behaviour of our relative non-tradable/tradable price ratio model-based forecasts, we construct the ratio of RMSE **(14)** based on our recursively generated predictions from either **(9)** or **(12)** relative to that of the random-walk model. For the non-tradable/tradable price ratio model-based cointegrated VEC models to be valid, these ratios should be smaller than 1.

Note that (panel) VEC models, as well as the random-walk model, impose an identical order of integration for the log real exchange rate, ie I(1). Christoffersen and Diebold (1998) show that for $h \to \infty$, the forecast error variance tends to infinity. Hence, a ratio of RMSE measures, which itself is a measure of the forecast error variance, for two forecast models that impose I(1) on the level forecasts could potentially be very close to 1. In order to circumvent this problem, we follow Groen (2004) and simulate p-values for each of our estimated RMSE ratios for H_0: RMSE ratio = 1 versus H_1: RMSE ratio $<$ 1, according to a parametric bootstrap procedure. In this parametric bootstrap procedure, an artificial I(1) series of q_t and y_t is generated for each bilateral relationship, based on a random walk for the log real exchange rate and an autoregressive model for y_t (with identical lag order as used in Section 3.2). These series are not cointegrated with each other in order to comply with the null hypothesis. We then apply both **(9)** and **(12)** through the recursive forecasting procedure on these artificial series to generate artificial equivalents of our RMSE ratios *vis-à-vis* the random walk model. The artificial RMSE ratios from 5,000 parametric bootstrap simulations are then combined with the empirical estimates of the RMSE to compute the p-values. A more detailed description of this parametric bootstrap procedure can be found in Groen (2004, Appendix B).

5.2 Results

One of the most elaborate out-of-sample studies of Balassa-Samuelson type real exchange rate models can be found in Mark and Choi (1997), and their results indicate that these models have predictive power for real exchange rates at horizons of three to four years. However, Mark and Choi (1997) *impose* the cointegration restrictions of their structural real exchange rate models without having tested these restrictions. In the context of monetary fundamentals-based nominal exchange rate models, Groen (1999) and Berkowitz and Giorgianni (2001) have shown that imposing empirically invalid cointegration restrictions will render findings of long-horizon exchange rate predictability to be spurious. Hence, in our analysis, the specifications which yielded appropriate cointegration restrictions in Sections 3.2 and 4.2 may be expected to have the most robust long-run predictive power for our real sterling bilateral exchange rates.

Out-of-sample evaluation results can be found in Table F. The first column reports the RMSE ratios for the log real exchange rate level of an individual cointegrated VAR system using the CPI-based decomposition, the same decomposition is used in the second column which reports the ratio for panel VEC model relative to random walk forecasts. Similarly, the third and fourth columns again contain the ratios of RMSE of respectively the individual VEC models and the panel VEC model, but now based on the PPI-based decomposition. Irrespective of the choice of real exchange rate decomposition, the results are similar for the Euro(Germany)/UK and Japan/UK real exchange rates. At shorter horizons neither the individual VEC models nor the panel VEC models are able to significantly outperform random-walk forecasts. However, the results for the Euro(Germany)/UK and Japan/UK rates at the three and four year horizons indicate that in contrast to the pure time series approach the panel VEC forecasts provide a significant improvement over random-walk forecasts.

The results in Table F for the Canada/UK and US/UK real exchange rates are more diverse in nature. Again, as in the case of the Euro(Germany)/UK and Japan/UK real exchange rates, none of the (panel) VEC models are able to structurally outperform random walk-based forecasts at the shorter horizons. However, in contrast to the Euro(Germany)/UK and Japan/UK real exchange rates, the type of real exchange rate decomposition is of importance for the forecasting performance of the Canada/UK and

Table F: Forecasting evaluation of long-run real exchange rate model (6), 1991:04-2002:04[(a)]

	h	*CPI-based decomp.* Ind.VEC	Pan.VEC	*PPI-based decomp.* Ind.VEC	Pan.VEC
Ca/UK	1	1.053	0.984	1.045	0.988
		(0.644)	(0.060)	(0.827)	(0.040)
	12	1.025	0.913	1.192	1.051
		(0.131)	(0.030)	(0.977)	(0.730)
	24	0.997	0.829	1.113	1.154
		(0.102)	(0.020)	(0.703)	(0.830)
	36	0.953	0.727	1.080	1.239
		(0.090)	(0.020)	(0.458)	(0.820)
	48	0.771	0.642	0.872	1.276
		(0.042)	(0.020)	(0.128)	(0.810)
Euro/UK (PPI-based: Germany/UK)	1	1.011	0.964	0.994	0.968
		(0.680)	(0.200)	(0.072)	(0.170)
	12	0.997	1.046	1.020	1.038
		(0.139)	(0.510)	(0.287)	(0.390)
	24	0.980	1.014	1.036	1.007
		(0.137)	(0.360)	(0.286)	(0.230)
	36	0.985	0.908	1.059	0.950
		(0.170)	(0.200)	(0.308)	(0.140)
	48	0.995	0.761	1.098	0.870
		(0.196)	(0.010)	(0.344)	(0.040)
Japan/UK	1	0.983	0.970	0.984	0.946
		(0.080)	(0.170)	(0.050)	(0.120)
	12	1.015	0.994	1.033	0.958
		(0.325)	(0.180)	(0.215)	(0.140)
	24	1.016	0.963	1.054	0.930
		(0.313)	(0.130)	(0.213)	(0.080)
	36	1.036	0.894	1.102	0.870
		(0.351)	(0.080)	(0.250)	(0.060)
	48	1.066	0.787	1.139	0.764
		(0.388)	(0.001)	(0.268)	(0.001)
US/UK	1	0.988	0.944	1.006	0.957
		(0.071)	(0.021)	(0.380)	(0.060)
	12	1.036	1.034	1.061	0.958
		(0.259)	(0.530)	(0.462)	(0.140)
	24	1.150	1.094	1.161	0.804
		(0.517)	(0.640)	(0.562)	(0.050)
	36	1.259	1.108	1.276	0.782
		(0.558)	(0.580)	(0.598)	(0.010)
	48	1.455	1.106	1.480	0.759
		(0.643)	(0.530)	(0.676)	(0.001)

[(a)] The entries in the table are the RMSE ratio of long-run model **(6)**-based versus random walk predictions in case of predicted exchange rate levels, see **(14)**, whereas the values in parentheses are the corresponding p-values for H_0 : RMSE ratio $=1$ versus H_1 : RMSE ratio <1 computed through parametric bootstrap procedures similar to those described in Groen (2004, Appendix B). The forecasting horizons (in months) can be found under the heading "h". Columns with "Ind.VEC" report the outcomes for the individual country VEC models and "Pan.VEC" those for the panel VEC model.

US/UK panel VEC models at longer horizons, ie beyond three years. In the case of the real US/UK rate, the sensitivity to the type of decomposition could be due to the large US/UK nominal exchange rate appreciation/depreciation cycle of the 1980s which, according to Chart 7, seems to have been accompanied by a more prolonged depressing effect on the level of the CPI-based y_t than for the PPI-based counterpart. The relatively large share of commodities in Canadian output could have influenced the out-of-sample analysis of the real Canada/UK rate. Some commodities prices behave in a manner similar to asset prices, and thus are much more volatile than other prices. These commodities could potentially influence the Canadian PPI index, and this could mean that the Canadian CPI/PPI-based proxy for the movements in the Canadian non-tradables/tradables ratio would be more volatile than the CPI tradable versus non-tradable components proxy, as is manifested by the behaviour of the different y_t measures in Chart 5. The degree of predictability for the real Canada/UK and US/UK rates could thereby have been pushed beyond the four-year horizon for certain real exchange rate decompositions.

6 Concluding remarks

The identification of a long-run relationship between the real exchange rate and the relative price ratio of the non-tradable and tradable components requires us to choose a method for constructing these components. In this paper the indices for the tradable and non-tradable goods prices are constructed in two alternative ways; one which decomposes the consumer price index into its tradable and non-tradable components, and one which uses the producer price index as a proxy for tradable goods.

The case for movements in real exchange rates being determined by movements in the relative prices of non-traded and traded goods is not supported by strong empirical evidence. Using a wide sample of bilateral exchange rates Engel (2003) shows that it is changes in the prices of traded goods between countries which accounts for nearly all the movements in real exchange rates. And he concludes that deviations from LOOP occur because of the existence of transportation costs and sticky nominal prices so that nominal exchange rate changes are not passed through to consumer prices in the local currency. These findings are consistent with other studies that use alternative methods for constructing the non-traded and traded goods indices, such as the PPI-based measure used by Betts and Kehoe (2001), and approaches that utilise both time series and panel techniques such as Drine and Rault (2002).

The analysis presented here examines the existence of a long-run relationship between UK bilateral real exchange rates and the corresponding relative prices of non-traded to traded goods. Using cointegrated VAR models for these series, the findings are not strong; there is only limited evidence for a cointegrating relationship in the US and euro bilateral rates. Using an AR model for the difference between these components, the tradable exchange rate, quantifies the severity of the deviations from the law of one price, and provides evidence that such deviations are persistent relative to the length of the sample. This motivates the use of a multi-country panel cointegration testing framework. Such a framework provides evidence for a cointegrating relationship between the real exchange rate and the relative price of non-tradable goods for the United Kingdom using both the CPI and the PPI-based decompositions. This confirms that structural changes in sterling bilateral real equilibrium exchange rates, which could be caused by a number of different possible factors, as discussed in Benigno and Thoenissen (2002), cannot be ruled out.

Perhaps unsurprisingly, out-of-sample evaluation shows that the estimated time series based cointegrating VAR models are inferior to a naive random-walk model. But utilising the panel VEC approach of Groen and Kleibergen (2003) can for most bilaterals provide a significantly more accurate prediction of movements in the real exchange rate than a random-walk model. Hence, our results describe the behaviour of UK bilateral real exchange rates with respect to the Law of One Price over the past 25 years, and show that using a panel modelling approach is superior to modelling the individual time series.

Appendix A: Data

We construct the UK real exchange as defined in **(1)** *vis-à-vis* the six main OECD trading partners. We also construct a Euro/UK real exchange rate that prior to 1999 is a weighted average of the German, French and Italian real exchange rates. The weights used are time varying, as in Beyer, Doornik and Hendry (2001), determined by relative GDP measured in a common currency, where industrial production indices are used to interpolate quarterly GDP series to construct a monthly GDP series. The nominal exchange rate data are taken from the International Monetary Fund's International Financial Statistics CD-ROM. The consumer price indices for each country are taken from Datastream's OECD database. The data are monthly from January 1976 to April 2002, giving us 316 observations for each exchange rate.

As noted in Section 2, in modelling real exchange rate movements according to equation **(4)** we must choose how to measure the price of tradable and non-tradable goods in each country. Two approaches are used in this paper. The first assumes that a producer price index (PPI) has a higher weight on traded goods than a consumer price index (CPI). The measures used are given by **(7)**. The consumer price index (CPI) and producer price index (PPI) are also taken from the International Monetary Fund's International Financial Statistics CD-ROM, and again the data are monthly from January 1976 to April 2002. The exceptions to this are Italy where we only have PPI data from January 1981, and France where we only have PPI data from 1993 onwards. Due to the lack of data for Italy and France for the PPI-based measure, we do not construct a tradable and non-tradable decomposition for the euro. Within the panel-data model in Section 4 we use the German/UK bilateral in place of the euro.

In the second approach we decompose the CPI into components that can be related to the tradable and non-tradable goods as in equation **(8)**. The tradables component is constructed from the commodities and food subindices. The non-tradables component is constructed from services and housing. The data for all countries except the United Kingdom are taken from Datastream's OECD database. Seasonally unadjusted monthly data for all items (*ai*), all goods less food (*aglf*), food (*f*), services less rent (*slr*), and rent (*r*) are used. UK consumer price data are not available. Instead we use seasonally unadjusted monthly data for the UK retail price index, which is equivalent and is obtained from the ONS. The services less rent price index is only available for the United Kingdom from 1976 onwards, as service data are only available from this point. Hence, our data set cannot begin earlier.

The weights used to construct the price indices for tradable and non-tradable good for each country are taken from the OECD's Main Economic Indicators. The weights used for each country are the weights used in constructing the CPI for that country. We construct the tradable and non-tradable price indices for each country by

$$\begin{aligned} p(\text{CPI-T})_t &= \left(\frac{\phi_1}{\phi_1+\phi_2}\right) aglf_t + \left(\frac{\phi_2}{\phi_1+\phi_2}\right) f_t \\ p(\text{CPI-N}) &= \left(\frac{\phi_3}{1-\phi_1-\phi_2}\right) slr_t + \left(\frac{1-\phi_1-\phi_2-\phi_3}{1-\phi_1-\phi_2}\right) r_t \end{aligned} \qquad \textbf{(A-1)}$$

where

$\phi_1 =$ weight of all goods less food in price index

$\phi_2 =$ weight of food in price index

$\phi_3 =$ weight for services less rent in price index

$\phi_4 =$ weight for rent in price index.

This method differs slightly from that used by Engel (2003) where the price indices for all countries are constructed using the weights used in the 2001 US consumer price index. We construct a Euro CPI-based decomposition by taking a weighted average of the France, Germany and Italy series, again using relative GDP measured in a common currency to construct time-varying weights as in Beyer *et al* (2001).

Appendix B: Mean unbiased half-life estimates

Bias corrections for the univariate AR model **(11)** are crucial for getting appropriate, unbiased impulse response-based estimates of the half-life of the tradable price ratio x_t. As in Kilian (1998), we use data resampling algorithms to estimate both the OLS mean bias of the parameters in **(11)** and the confidence intervals of the bias-corrected impulse response-based half-life estimates. The resampling algorithms are based on the bootstrapping principle, and within the context of a linear regression model bootstrapping implies that one generates artificial data from the estimated model by random drawing 'new' disturbance series from the original regression residuals and feeding these into the original estimated model.

The bootstrap-based mean unbiased half-life estimates are obtained as follows. Estimate **(11)** with OLS, and use the corresponding OLS estimates $\hat{\delta}_0,\ \hat{\delta}_1, \ldots, \hat{\delta}_p$ and OLS residuals $\hat{\nu}_1, \ldots, \hat{\nu}_T$ to generate $T + 50$ artificial observations of x_t through

$$x_t^* = \hat{\delta}_0 + \sum_{j=1}^{p} \hat{\delta}_j x_{t-j}^* + \nu_t^* \qquad \textbf{(B-1)}$$

where we use the first p observations on x_t from the historical sample as the initial values and the ν_t^*'s are $T + 50$ random drawings from $\hat{\nu}_1, \ldots, \hat{\nu}_T$. We delete the first 50 observations of x_t^* in order to deal with initial value bias, and use the remaining T observations on x_t^* to estimate **(11)** yielding a new set of parameter estimates $\hat{\delta}_0^*,\ \hat{\delta}_1^*, \ldots, \hat{\delta}_p^*$. We repeat this 1,000 times and based on the 1,000 sets of parameter estimates $\hat{\delta}_0^*(s),\ \hat{\delta}_1^*(s), \ldots, \hat{\delta}_p^*(s)$ ($s = 1, \ldots, 1000$) we can approximate the mean OLS bias as

$$\Psi = \frac{1}{1000} \sum_{s=1}^{1000} \begin{pmatrix} \hat{\delta}_0^*(s) - \hat{\delta}_0 \\ \hat{\delta}_1^*(s) - \hat{\delta}_1 \\ \vdots \\ \hat{\delta}_p^*(s) - \hat{\delta}_p \end{pmatrix} \qquad \textbf{(B-2)}$$

Utilising **(B-2)** we can construct the mean unbiased parameter values for **(11)**,

$$\begin{pmatrix} \tilde{\delta}_0 \\ \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_p \end{pmatrix} = \begin{pmatrix} \hat{\delta}_0 \\ \hat{\delta}_1 \\ \vdots \\ \hat{\delta}_p \end{pmatrix} - \Psi, \qquad \textbf{(B-3)}$$

and based on these we construct the mean unbiased impulse response function for x_t under a unity impulse, which in turn provides us with an unbiased estimate of the half-life of x_t.

In order to get the confidence intervals for our unbiased half-life estimates, we have to generate bootstrap samples of x_t by random drawings from the mean unbiased residuals of **(11)**

$$\tilde{\nu}_t = x_t - \tilde{\delta}_0 - \sum_{j=1}^{p} \tilde{\delta}_j x_{t-j}, \quad t = 1, \ldots, T \qquad \textbf{(B-4)}$$

The confidence intervals are computed through the following steps:

1. Generate $T + 50$ artificial observations on x_t through

$$x_t^* = \tilde{\delta}_0 + \sum_{j=1}^{p} \tilde{\delta}_j x_{t-j}^* + \nu_t^{**}, \qquad \textbf{(B-5)}$$

   where $(\tilde{\delta}_0, \tilde{\delta}_1, \ldots, \tilde{\delta}_p)$ are defined in **(B-3)**, the ν_t^{**}'s are $T + 50$ random drawings from **(B-4)** and the initial values are the first p historical values of x_t. After deleting the first 50 observations we estimate **(11)** resulting in new set of parameter estimates $\tilde{\delta}_0^*,\ \tilde{\delta}_1^*, \ldots, \tilde{\delta}_p^*$.
2. Generate 1,000 artificial samples of $T + 50$ observations through

$$x_t^{**} = \tilde{\delta}_0^* + \sum_{j=1}^{p} \tilde{\delta}_j^* x_{t-j}^{**} + \tilde{\nu}_t^{**}, \qquad \textbf{(B-6)}$$

   where $\tilde{\delta}_0^*,\ \tilde{\delta}_1^*, \ldots, \tilde{\delta}_p^*$ are the result of step 1 and $\tilde{\nu}_t^{**}$ is a random drawing from the residuals of $x_t^* - \tilde{\delta}_0^* - \sum_{j=1}^{p} \tilde{\delta}_j^* x_{t-j}^*$ for $t = 1, \ldots, T$ (the x_t^*'s are the artificial x_t's from step 1). For each of the 1,000 artificial samples we delete the first 50 observations and estimate **(11)**, yielding 1,000 sets of new parameter estimates. We use these through **(B-2)** and **(B-3)** to construct mean unbiased equivalents of $\tilde{\delta}_0^*,\ \tilde{\delta}_1^*, \ldots, \tilde{\delta}_p^*$ from step 1, which we denote as $\check{\delta}_0^*,\ \check{\delta}_1^*, \ldots, \check{\delta}_p^*$.
3. Using $\check{\delta}_0^*,\ \check{\delta}_1^*, \ldots, \check{\delta}_p^*$ from step 2 we construct the impulse response function under a unity impulse. This impulse response function in turn is used to compute a half-life estimate, ie the number of periods after which 50% of the unity shock has been dissipated.

We repeat the aforementioned steps 5,000 times, which gives us a sample of 5,000 half-life estimates. These 5,000 estimates are used to compute the α and $1 - \alpha$ percentile interval endpoints of the mean unbiased half-lives. Hence, in order to get both a point estimate of the mean unbiased half-life and the corresponding confidence intervals we use in total $1000 + (5000 \times 1000)$ simulations.

References

Balassa, B (1964), 'The purchasing power parity doctrine: a reappraisal', *Journal of Political Economy*, Vol. 72, pages 584–96.

Banerjee, A, Marcellino, M and Osbat, C (2000), 'Some cautions on the use of panel methods for integrated series of macro-economic data', *European University Institute Working Paper in Economics no. 00/20*.

Benigno, G and Thoenissen, C (2002), 'Equilibrium exchange rates and supply-side performance', *Bank of England Working Paper no. 156*.

Bergstrand, J H (1991), 'Structural determinants of real exchange rates and national price levels: some empirical evidence', *American Economic Review*, Vol. 81, pages 325–34.

Berkowitz, J and Giorgianni, L (2001), 'Long-horizon exchange rate predictability?', *Review of Economics and Statistics*, Vol. 83, pages 81–91.

Betts, C M and Kehoe, T J (2001), 'Real exchange rate movements and the relative price of nontraded goods', University of Minnesota, *mimeo*.

Beyer, A, Doornik, J A and Hendry, D F (2001), 'Constructing historical Euro-zone data', *Economic Journal*, Vol. 111, pages F102–F121.

Bhagwati, J N (1984), 'Why are services cheaper in the poor countries?', *Economic Journal*, Vol. 94, pages 270–86.

Chortareas, G E and Driver, R L (2001), 'PPP and the real exchange rate - real interest rate differential puzzle revisited: evidence from non-stationary panel data', *Bank of England Working Paper no. 138*.

Christoffersen, P F and Diebold, F X (1998), 'Cointegration and long-horizon forecasting', *Journal of Business & Economic Statistics*, Vol. 16, pages 450–58.

Dickey, D F and Fuller, W A (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association*, Vol. 74, pages 427–31.

Drine, I and Rault, C (2002), 'How sure are we about the Balassa-Samuelson hypothesis? Time series versus panel data approach for OECD countries', Université de Paris I, *mimeo*.

Dumas, B (1992), 'Dynamic equilibrium and the real exchange rate in a spatially seperated world', *Review of Financial Studies*, Vol. 5, pages 153–80.

Engel, C (1999), 'Accounting for U.S. real exchange rate changes', *Journal of Political Economy*, Vol. 107, pages 507–38.

Engel, C (2003), 'Expenditure switching and exchange rate policy', in Gertler, M and Rogoff, K (eds), *NBER Macroeconomics Annual 2002*, Cambridge (Massachusetts): NBER and MIT Press.

Engle, R F and Granger, C W J (1987), 'Co-integration and error correction: representation, estimation and testing', *Econometrica*, Vol. 55, pages 251–76.

Gonzalo, J and Granger, C W J (1995), 'Estimation of common long-memory components in cointegrated systems', *Journal of Business & Economic Statistics*, Vol. 13, pages 27–35.

Gonzalo, J and Ng, S (2001), 'A systematic framework for analyzing the dynamic effects of permanent and transitory shocks', *Journal of Economic Dynamics and Control*, Vol. 25, pages 1,527–46.

Groen, J J J (1999), 'Long horizon predictability of exchange rates: is it for real?', *Empirical Economics*, Vol. 24, pages 451–69.

Groen, J J J (2002), 'Cointegration and the monetary exchange rate model revisited', *Oxford Bulletin of Economics and Statistics*, Vol. 64, pages 361–80.

Groen, J J J (2004), 'Exchange rate predictability and monetary fundamentals in a small multi-country panel', *Journal of Money, Credit, and Banking*, forthcoming.

Groen, J J J and Kleibergen, F (2003), 'Likelihood-based cointegration analysis in panels of vector error correction models', *Journal of Business & Economic Statistics*, Vol. 21, pages 295–318.

Imbs, J, Mumtaz, H, Ravn, M O and Rey, H (2002), 'PPP strikes back: aggregation and the real exchange rate', *NBER Working Paper no. 9372*.

Inoue, A and Kilian, L (1999), 'Bootstrapping autoregressive processes with possible unit roots', University of Michigan, *mimeo*.

Johansen, S (1991), 'Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models', *Econometrica*, Vol. 59, pages 1,551–80.

Johansen, S (1996), *Likelihood-based inference in cointegrated vector autoregressive models*, *2nd edition*, Oxford: Oxford University Press.

Kakkar, V and Ogaki, M (1999), 'Real exchange rates and nontradables: a relative price approach', *Journal of Empirical Finance*, Vol. 6, pages 193–215.

Kilian, L (1998), 'Small-sample confidence intervals for impulse response functions', *Review of Economics and Statistics*, Vol. 80, pages 218–30.

Larsson, R, Lyhagen, J and Löthgren, M (2001), 'Likelihood-based cointegration tests in heterogeneous panels', *Econometrics Journal*, Vol. 4, pages 109–42.

Lütkepohl, H (1993), *Introduction to multiple time series analysis, 2nd edition*, Berlin: Springer-Verlag.

MacKinnon, J G (1991), 'Critical values for cointegration tests', in Engle, R F and Granger, C W J (eds), *Long run economic relationships: readings in cointegration*, Oxford: Oxford University Press, pages 267–76.

Mark, N C (1990), 'Real exchange rates in the long-run: an empirical investigation', *Journal of International Economics*, Vol. 28, pages 115–36.

Mark, N C and Choi, D-Y (1997), 'Real exchange-rate prediction over long horizons', *Journal of International Economics*, Vol. 43, pages 29–60.

Meese, R A and Rogoff, K (1983), 'Empirical exchange rate models of the Seventies: do they fit out of sample?', *Journal of International Economics*, Vol. 14, pages 3–74.

Meese, R A and Rogoff, K (1988), 'Was it real? The exchange rate-interest rate differential relation over the modern floating period', *Journal of Finance*, Vol. 43, pages 933–48.

Obstfeld, M and Rogoff, K (2001), 'The six major puzzles in international macroeconomics: is there a common cause?', in Bernanke, B and Rogoff, K (eds), *NBER Macroeconomics Annual 2000*, Cambridge (Massachusetts): NBER and MIT Press.

Obstfeld, M and Taylor, A (1997), 'Non-linear aspects of goods market arbitrage and adjustment: Heckscher's commodity points revisited', *Journal of the Japanese and International Economies*, Vol. 11, pages 441–79.

O'Connell, P (1998), 'The overvaluation of purchasing power parity', *Journal of International Economics*, Vol. 44, pages 1–19.

Otero, J and Smith, J (2000), 'Testing for cointegration: Power versus frequency of observation - further Monte Carlo results', *Economics Letters*, Vol. 67, pages 5–9.

Pedroni, P (1996), 'Fully modified OLS for heterogeneous cointegrated panels and the case of purchasing power parity', *Indiana University Working Paper no. 96-020*.

Samuelson, P A (1964), 'Theoretical notes on trade problems', *Review of Economics and Statistics*, Vol. 46, pages 145–54.

Shiller, R and Perron, P (1985), 'Testing the random walk hypothesis: power versus frequency of observation', *Economics Letters*, Vol. 18, pages 381–86.

Taylor, M, Peel, A and Sarno, L (2001), 'Nonlinear mean-reversion in real exchange rates: toward a solution to the purchasing power parity puzzles', *International Economic Review*, Vol. 42.