Option-implied probability distributions for future inflation

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Beliefs about future inflation play a major role in determining the rate of inflation, and so it is important for the Monetary Policy Committee to take them into account when making their policy decisions. A number of measures of central expectations for inflation are available, such as surveys of inflation expectations or measures derived from financial markets. But until recently far fewer measures of beliefs about the full distribution of possible future inflation rates have been available. This article describes a new method for producing option-implied probability density functions for future inflation, which can be used as a measure of that distribution, and examines the recent rise in uncertainty about future inflation that they reveal.

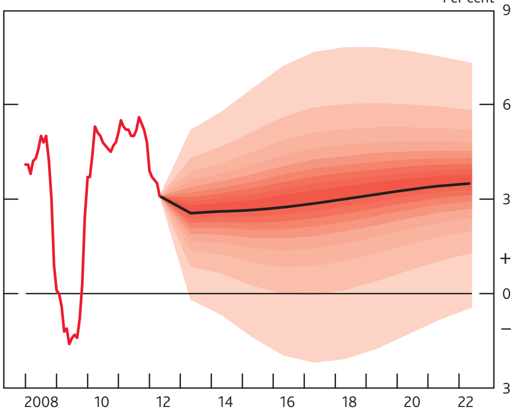
Introduction

Beliefs about future inflation play a major role in determining the rate of inflation. If people believe, for instance, that prices are likely to rise sharply in the future, they may demand higher wages today: this could push up prices, raising the current rate of inflation. So it is important for the Monetary Policy Committee (MPC) to monitor people's beliefs about future inflation, and to take them into account when making their policy decisions.

A number of measures of central expectations for inflation are available, such as surveys of households' and firms' inflation expectations or measures of implied inflation derived from financial markets. Measures like these are regularly used by the MPC to assess the risk to inflation from inflation expectations moving away from target.(2) But until recently far fewer measures of beliefs about the full distribution of possible future inflation rates have been available, particularly beyond a two or three-year horizon. Such measures would allow the MPC to examine in much more detail how close inflation expectations were to target, while also providing a source of information on investors' beliefs about the risks around the outlook for inflation.

In the past few years, however, a market has developed in inflation options, from which a measure of people's beliefs about the distribution of future inflation can be obtained. An inflation option is a financial instrument that allows investors to speculate on, or insure against, future inflation outturns. As with options on any asset, the prices of these options can be used to calculate implied probability density functions (pdfs), in this case for inflation. These pdfs summarise investors' beliefs about the distribution of future inflation rates. And by combining pdfs for inflation at different points in the future it is also possible to examine how investors believe inflation rates could evolve over a number of years — **Chart 1**, for example, illustrates investors' beliefs about how UK retail prices index (RPI) inflation is likely to evolve over the next decade.

Chart 1 Option-implied probability distributions for UK RPI inflation, 31 July 2012



Sources: Bloomberg, ONS and Bank calculations.

This article describes the technique developed at the Bank of England to produce these pdfs, and analyses what they reveal about investors' views on future inflation. The first section describes the underlying instruments and the markets in which they are traded. The second section discusses the interpretation of the implied pdfs and their relationship to the underlying distributions of investors' beliefs about inflation. The third section uses the pdfs to discuss the evolution of

(1) The author would like to thank Michael Chin for his help in producing this article.
(2) See Harimohan (2012) for more details.

uncertainty about UK inflation, and compares them with other measures of uncertainty.

The inflation option market

An option is a financial contract in which one party (the seller) pays the other (the buyer) only if a certain pre-agreed outcome occurs — often, but not always, if the price of some other asset ends up above (or below) some threshold, called a strike price. In exchange for that state-contingent future payout, the buyer pays the seller a price upfront. Inflation options are based on the rate of annual inflation in a given month. Typically, the seller pays out if inflation is higher than a pre-agreed strike rate.

Investors use inflation options to hedge against high or low inflation outturns or to speculate on the future path of inflation. For instance, an investor who is worried that he will suffer losses if inflation turns out to be particularly high can insure himself by buying an inflation option which will pay out in exactly that situation. The other investor who sells him the option may do so because he believes inflation is likely to be lower, and so is willing to bet against a high future rate.

The price agreed by the two investors in this example then reflects a collective judgement on their part on the probability that the option will pay out. Each investor will only be willing to enter into the option contract if he believes that the price he pays or receives upfront is worth taking on the risk of making or receiving an uncertain future payment. Fitting the implied pdfs shown in this article essentially amounts to reversing this process — that is, finding a probability distribution under which buyers of inflation options would be willing to pay the set of observed option prices in exchange for receiving the payouts that would be made by the sellers of those options.

For the simplest classes of options, it is easy to fit pdfs using standard techniques.(1) But most inflation options have a more complex structure. They are typically traded as caps and floors: bundles of simple inflation options called caplets and floorlets, all with the same strike price but each with a different expiry date. The structure of these options is explained in more detail in the box on page 226. Since caplets and floorlets are not directly traded, their prices are not observable; they must instead be recovered from the observed cap and floor prices. If those were available at a full range of annual maturities, this would be straightforward; but cap and floor prices are usually only available for a small number of maturities. The Bank's new technique (which is described in detail in the appendix) overcomes this problem by interpolating cap and floor prices at the missing maturities. The interpolated prices are used to decompose the caps and floors into caplets and floorlets, which are then used to produce the implied pdfs.(2)

Characteristics of inflation option markets

Most inflation options are traded on UK RPI, US consumer prices index (CPI) or euro-area CPI inflation, at maturities ranging from 1 to 30 years.(3)(4) The payouts from the options are determined by the annual inflation rate observed two (for the United Kingdom) or three months (for the United States and euro area) before contract expiry. For instance, UK inflation options expiring in April 2015 refer to the annual RPI inflation rate published for February 2015; and so, therefore, will the implied pdf calculated from those options.

The inflation option market is entirely 'over the counter': trading takes place directly between individual investors in the market, rather than being co-ordinated through a central exchange. As a result it is hard to measure characteristics such as trading volumes and market liquidity quantitatively. Instead this section uses qualitative information from the Bank's market-making contacts. **Table A** summarises the main features of the three markets considered here.

Table A Features of inflation option markets

	United Kingdom	United States	Euro area
Inflation rate	RPI	CPI	CPI (HICPxT)(a)
Liquidity	Fairly liquid	Least liquid	Most liquid
Major buyers and sellers	Pension funds (longer maturities)	Some structured product hedging	Pension funds (longer maturities); structured product hedging (shorter maturities)

(a) Harmonised index of consumer prices excluding tobacco.

All three of the markets considered in this article are relatively young and small. Trading activity is sporadic, and was particularly so in 2007 and 2008 when markets first developed. The UK inflation option market is more liquid than the US market, but less liquid than the market in the euro area.

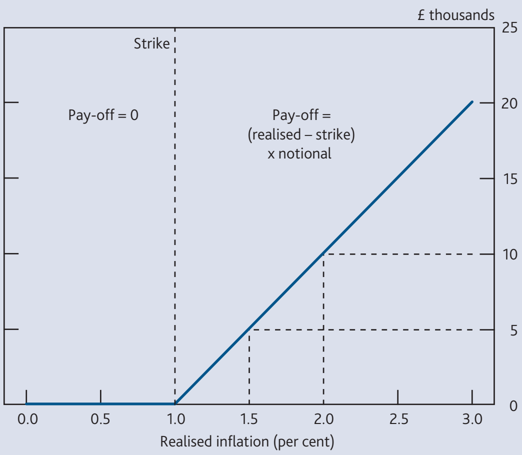
Lack of liquidity does not mean that the implied pdfs contain no information. But they may be noisy or slow to react to news, so care is needed when interpreting day-to-day movements in the implied pdfs. In addition, some of the observed prices can be affected by trading flows from major actors in the markets. In particular, many UK pension fund liabilities are linked to RPI inflation in a way which induces them to buy and sell certain caps and floors.(5) At long maturities, particularly at 20 to 30 years, these structural

(1) Clews, Panigirtzoglou and Proudman (2000) describe these techniques.
(2) One alternative would be to produce implied distributions for average inflation across the entire maturity of each option. Kitsul and Wright (2012) do something similar for US CPI inflation data, although using a slightly different underlying class of instruments.
(3) Almost all UK inflation-linked financial instruments are based on RPI rather than CPI. This is a legacy of the fact that, until recently, many UK pension schemes were linked to RPI.
(4) There is also a market in options on French CPI inflation, which this article does not discuss.
(5) This is because some pension schemes, under the terms of the Pensions Act 1995, have liabilities which increase in line with RPI inflation up to a maximum rate of 5%, and are never permitted to decrease — this is known as limited price indexation (LPI). That gives them an incentive to buy 0% RPI floors and sell 5% RPI caps.

Caps, caplets, floorlets and floors

When an inflation *caplet* is traded, the buyer and the seller agree an *expiry date*, a *strike rate* and a *notional* — a hypothetical sum of money which the seller agrees to insure for the buyer. The buyer pays the seller an upfront premium — the *option price*. On expiry the seller then pays the buyer the difference between the realised inflation rate and the strike rate multiplied by the notional, if realised inflation is above the strike rate. Otherwise no money changes hands. An example in which the notional is £1 million and the strike rate is 1% is shown in **Chart A**. If realised inflation on the expiry date was 1.5%, the seller would pay the buyer (1.5%–1%) x £1 million = £5,000. If realised inflation was 2%, the seller would pay £10,000. But if realised inflation was 1% or below, the buyer would receive nothing.

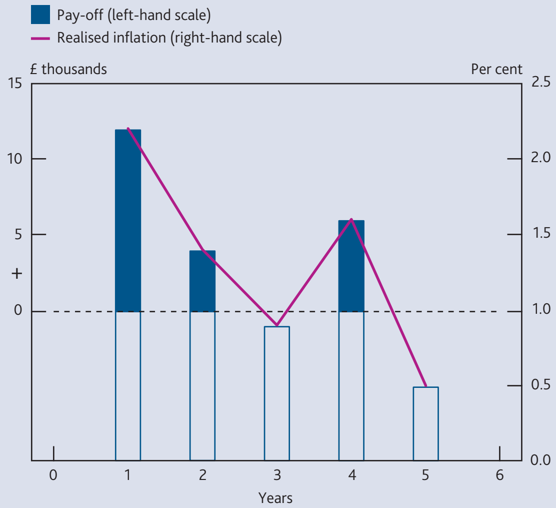
Chart A The pay-off from an inflation caplet



Thus if the buyer of the caplet really did have a liability equal to the rate of inflation multiplied by the notional value, the maximum he would actually have to pay would be capped at 1%, with the seller of the caplet bearing the cost of any additional payout. So caplets provide protection to the buyer against upside risks to inflation.

An inflation *cap* is a bundle of caplets all of which have the same strike rate and notional, but whose expiry dates fall on consecutive years. For example, a three-year cap bought on 1 October 2010 would consist of one caplet which expired on 1 October 2011, one which expired on 1 October 2012 and one which expired on 1 October 2013. **Chart B** shows an example set of pay-offs from a hypothetical five-year inflation cap, again with a 1% strike and a £1 million notional. The cap pays out only in years when realised inflation is above 1%.

Chart B The pay-offs from an inflation cap



The corresponding instruments which provide protection against downside risks to inflation are called *floorlets* and *floors*. The seller of an inflation floorlet pays the buyer the difference between the strike rate and the realised inflation rate multiplied by the notional if realised inflation is below the strike rate, and zero otherwise.

flows dominate the UK market. Similar structural flows are also present in US and euro-area markets, but they are less significant.

Despite some structural issues, there is likely to be enough information in published option prices to make the implied pdfs a useful measure of investors' beliefs about future inflation. Information from the Bank's market-making contacts suggests that quoted prices are a fair reflection of what investors are prepared to pay for inflation options.

Interpreting the implied distributions

The next section discusses what the option-implied inflation distributions say about the evolution of uncertainty around UK inflation in recent years. Before examining the data in detail, however, it is important to understand how the implied pdfs can and cannot be interpreted: this section sets out the key points.

Interpreting the data

Implied pdfs derived from option prices describe investors' collective beliefs about the future level of some asset price or macroeconomic variable. Particular features of those beliefs can then be described using standard summary statistics. The standard deviation of the implied pdf, for example, can be thought of as summarising investors' uncertainty about that price or variable, while the skewness of the pdf represents their collective views about the balance of risks around their central expectation.

In most cases, the mean of the distribution is equal to the price of another asset that it is linked to. For options whose payout is linked to the price of an underlying asset, this is the futures price of that asset. But for inflation options, whose payout is linked to the inflation rate rather than to an underlying asset, the mean of the implied distribution is the forward inflation swap rate. This is the price of a contract in which the seller pays a buyer a sum of money equal to the realised future inflation rate — effectively a futures contract on the inflation rate.(1)

As UK inflation options refer to RPI inflation, the implied pdfs obtained from them cannot be used as a direct measure of investors' beliefs about CPI inflation, the measure targeted by the Bank. It might appear as though this problem could be fixed by shifting the whole distribution by a constant 'wedge' based on the average historical difference between RPI and CPI inflation. But that would implicitly assume that there was no uncertainty about the future size of this wedge, which would be unrealistic. For example, the RPI inflation rate includes mortgage interest payments, which tend to vary with interest rates. By assuming a constant wedge, uncertainty about future interest rates would be ignored, as would the relationship between interest rates and the level of CPI inflation.(2) Nevertheless, RPI and CPI inflation do contain many common components, and so implied pdfs describing RPI inflation are still useful to policymakers, especially at long horizons where few other measures of the distribution of beliefs about inflation are available.

What the implied distributions measure

The probability distributions for inflation implied by option prices reflect the underlying probability distributions perceived by investors, but they are not exactly the same as those underlying distributions. That is because when people buy or sell options or other financial assets, they consider not only the probability that the option will pay out, but also how much they would value the payout in each state of the world.(3)

To understand why the distributions are different, consider the example of a fire insurance contract — this is the same as an inflation option in that it pays out different amounts in different states of the world. When someone buys fire insurance on their house, they are willing to pay much more than they expect to get back on average. That is because if the house did burn down, they would face high costs to rebuild or replace it, they would have lost a lot of their wealth, and they would suffer distress from the loss of their house and possessions, as well as significant inconvenience in the weeks that followed. In that state of the world, they would value the extra income from the insurance contract very highly; so they are willing to pay well over the odds to guarantee a payout. To an observer who only knew about the size of the possible payout, and did not understand how highly the payout would be valued in the event of a fire, it would appear that the owner thought that the property was much more likely to burn down than was really the case. In other words, the implied probability of a fire derived from the price of the insurance contract would be higher than the actual probability, because the event of a fire would be so painful to the buyer of the insurance.

In a similar way, people trading options adjust the prices that they are prepared to pay depending on how much they think they would value the payout from each option. Options that pay out in relatively painful states of the world, for instance in states when investors' consumption is likely to be relatively low, will be valued more than options that pay out in relatively painless states of the world, as investors want to hedge against the painful states. So the implied probabilities of painful events derived from those options will be relatively high compared to the underlying probabilities; likewise, the implied probabilities of painless events will be relatively low.(4)

The difference between the implied and underlying distributions has implications for how the implied distributions should be interpreted. For instance, the mean of the distribution — the futures price, or for inflation options, the inflation swap rate — will not be the same as the expected price of the underlying asset. The difference between inflation rates implied by asset prices and underlying inflation expectations is discussed in more detail in the article by Guimarães on pages 213–23 in this *Bulletin*.(5) More generally, since the implied probabilities do not match the underlying probabilities, the absolute levels of the implied probabilities contain little information by themselves. And changes in the implied probabilities will only reflect changes in the underlying probabilities if investors' desire to hedge against the most painful states has not changed.

Nevertheless, the implied distributions can still provide useful information about the underlying distributions. It is hard to make general statements about exactly how the two distributions are related, and there is no perfect method for separating them.(6) But by using the economic arguments outlined above, it is often possible to deduce something about the relationship between the two distributions.

(1) It is a 'swap' in the sense that the buyer swaps a payment which is known in advance — the price — for a payment of uncertain size.
(2) That is, it ignores the last two terms in the identity var(RPI) = var(CPI) + 2 cov(CPI, wedge) + var(wedge).
(3) In the language of finance theory, the implied distribution is the risk-neutral pdf. This differs from the underlying pdf due to the presence of a stochastic discount factor.
(4) From this perspective, the 'trading flow' effects described in the previous section can be seen as particularly large price adjustments. Pension funds with LPI-linked liabilities, for example, would suffer losses if their RPI-linked asset returns fell below 0%. So they find those states of the world particularly painful, and are prepared to pay a high price to insure against them.
(5) For an explanation in elementary terms of why the futures price does not equal the expected spot price, see Nixon and Smith (2012).
(6) Some progress has been made, however, both in the Bank and elsewhere (Bliss and Panigirtzoglou (2004) and de Vincent-Humphreys and Noss (2012)).

- In many cases, events in the tails of the distribution seem likely to be relatively painful. Inflation is one such case: very high inflation, for instance, erodes investors' nominal savings, while very low inflation often coincides with periods of recession. The implied tail probabilities and the standard deviation of the implied pdfs will then be relatively high compared to those of the underlying distribution.(1)

- That in turn means that implied uncertainty, as measured by the standard deviation, may well increase more than one-for-one with investors' uncertainty. Small increases in the tails of the underlying distribution will be amplified by investors' dislike of tail events, leading to larger increases in the tails of the implied pdf.

- It may also be the case that events in one tail of the distribution are more painful than those in the other tail. If investors thought that episodes of deflation were likely to be even more painful than episodes of high inflation, then the left tail of the implied pdf would be larger than the left tail of the underlying pdf; or *vice versa*. In the first case, the implied pdf would then be more negatively skewed than the underlying distribution, and the mean of the implied pdf lower than that of the underlying distribution.

There are also some technical factors that affect the interpretation of the implied distributions. Option prices are only available for strike prices between 0% and 6%, so pdfs cannot realistically be produced when inflation is more likely than not to fall outside that range — in particular, this makes it difficult to produce short-maturity pdfs for trading dates in late 2008 and early 2009, when realised RPI inflation was negative. More generally, the fact that the pdfs are extrapolated outside that range is another reason to be cautious about interpreting the exact level of any summary statistic. In addition, the precise shape of the individual pdfs for the first couple of years depends quite heavily on the technique used to fit them.(2)

Overall, there are a number of reasons why the implied pdfs cannot be thought of as direct measures of the underlying distributions for inflation perceived by investors. But the pdfs are still a valuable source of information on investors' beliefs about the distribution of future inflation. The next section presents the implied pdfs and examines the information that they do contain.

The evolution of uncertainty about UK inflation

Charts 2–7 show implied pdfs for UK RPI inflation produced using the Bank's new technique for selected dates from early 2008 to July 2012. The central, darkest coloured band on each chart contains the 10% of the implied distribution which includes the mode: that is, the set of inflation rates that the pdfs imply are the most likely outturns. Each pair of identically coloured bands around that central band contains a further 10% of the distribution, with the coloured bands covering 90% of the distribution in total. The final 10% of the distribution lies outside the coloured bands. The black line on each chart shows the mean of the implied distribution, which, as explained in the previous section, corresponds to the inflation swap rates.

There are several striking features of the distributions over this period. The mean of the distributions — the RPI swap rate — beyond the first three years has been relatively stable over time. But the uncertainty around inflation, measured by the standard deviation of the distributions, has increased substantially since 2008, particularly between three and seven years ahead. Of course, as discussed in the previous section, underlying uncertainty among investors is likely to be lower than the implied pdfs would suggest. But unless investors' desire to hedge against the most painful outcomes has changed substantially, this seems likely to reflect a genuine increase in underlying uncertainty about RPI inflation.

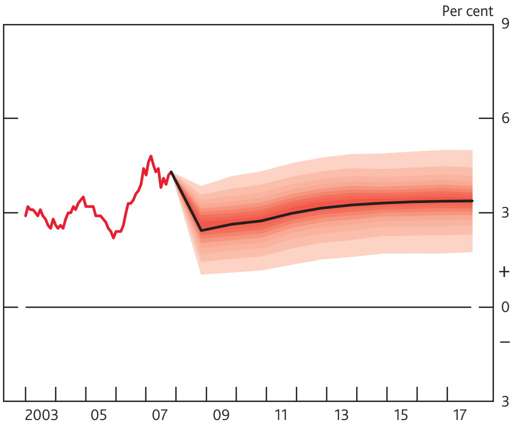
The balance of risks has been more variable, particularly at short maturities. Between Autumn 2008 and Summer 2010 there was a strong downside skew in the distributions at short maturities, reflecting investors' worries that there would be an episode of RPI deflation. This is consistent with the negative inflation risk premia discussed in the article by Guimarães on pages 213–23 in this *Bulletin*. But more recently the pdfs have been broadly symmetrical.

Understanding inflation uncertainty during the financial crisis

There are at least two possible factors underlying the rise in uncertainty. One possibility is that investors believe that inflation will continue, as in the past four years, to be subject to the effects of larger or more persistent shocks than in the pre-crisis period for at least the next few years. That could be because investors anticipate larger or more persistent exogenous shocks. Or it could be that market participants believe that any given shock will have a larger or more persistent impact on inflation than it would have done before the crisis. A second possibility is that the rise in uncertainty reflects a perception that policymakers have become more willing to tolerate — or, without excessive volatility in output, less able to prevent — deviations of inflation from target.

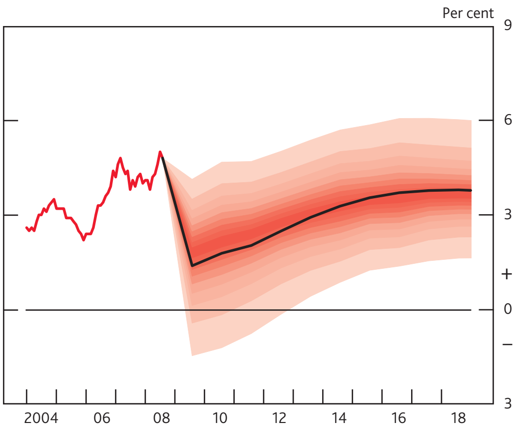
(1) Consistent with this, Kitsul and Wright (2012) find that their option-implied pdfs for US CPI inflation put more weight on very high or very low inflation than would be implied by statistical forecasts.
(2) Specifically, the (weighted) sum of the pdfs for the first three years is pinned down by the observed prices, but how that sum is distributed between those three years depends to a large extent on how the option prices are interpolated. In theory this is true for all the pdfs but in practice it is much less of an issue at longer maturities.

Chart 2 Option-implied probability distributions for UK RPI inflation, 2 January 2008



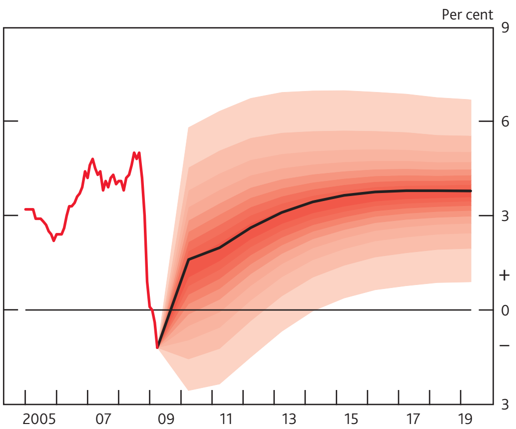
Sources: Bloomberg, ONS and Bank calculations.

Chart 3 Option-implied probability distributions for UK RPI inflation, 28 October 2008



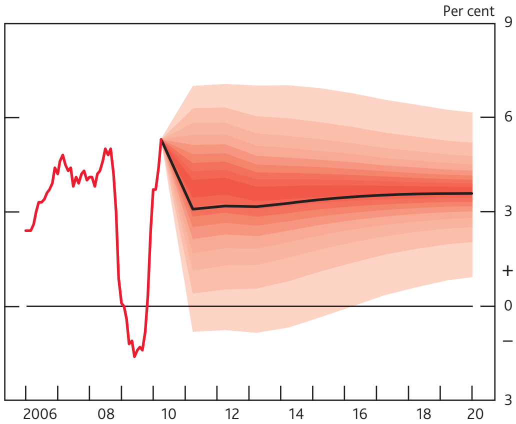
Sources: Bloomberg, ONS and Bank calculations.

Chart 4 Option-implied probability distributions for UK RPI inflation, 4 June 2009



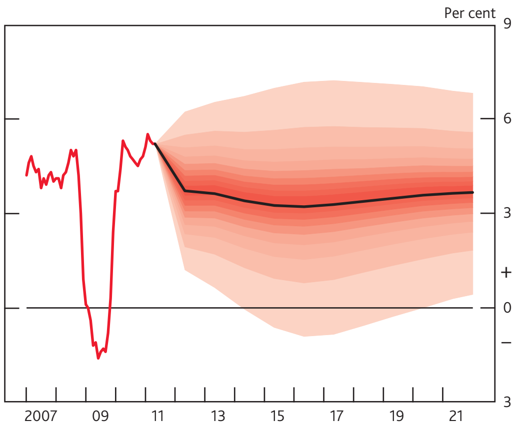
Sources: Bloomberg, ONS and Bank calculations.

Chart 5 Option-implied probability distributions for UK RPI inflation, 2 June 2010



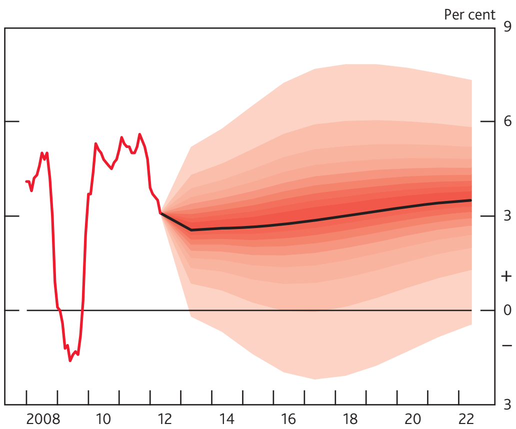
Sources: Bloomberg, ONS and Bank calculations.

Chart 6 Option-implied probability distributions for UK RPI inflation, 1 July 2011



Sources: Bloomberg, ONS and Bank calculations.

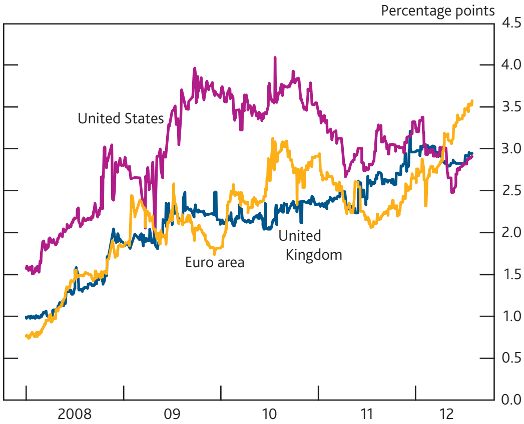
Chart 7 Option-implied probability distributions for UK RPI inflation, 31 July 2012



Sources: Bloomberg, ONS and Bank calculations.

These two possibilities are not mutually exclusive, and they are hard to disentangle. But there is some evidence to support the first possibility rather than the second, although it is by no means conclusive. For example, comparing the standard deviation of the UK inflation pdfs with those in the United States and euro area suggests that uncertainty has increased in a broadly similar way across all three markets (**Chart 8**). So, if investors do believe that policymakers have become more willing to tolerate, or less able to prevent, deviations of inflation from target, they must believe this to be the case for all three markets. Moreover, such a belief might be expected to have a very persistent impact on inflation uncertainty. The fact that the pdfs become narrower at longer maturities perhaps suggests that this has not happened. And again, this pattern is evident across all three markets.

Chart 8 Standard deviation of five year ahead option-implied inflation pdfs

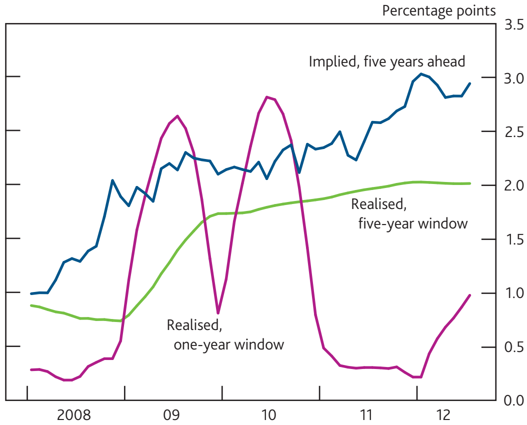


Sources: Bloomberg, Eurostat, ONS, US Bureau of Labor Statistics and Bank calculations.

But there are other possible explanations for the rise in inflation uncertainty. One other possibility is that the rise in uncertainty reflects increased disagreement among individual investors about future inflation. The implied pdf aggregates the beliefs of all investors; so if all investors became less certain about future inflation rates, then the implied pdf would become wider. But even if no individual investor had become more uncertain about future inflation, an increase in the dispersion of individual views could also cause the pdf to become wider.

Another possibility is that the rise in uncertainty simply reflects the unexpectedly large movements in inflation seen since 2008. There probably is a mechanical link between the size of past movements in inflation and the uncertainty embodied in inflation option prices, as dealers and investors often use the volatility of realised inflation rates to calibrate their pricing models. But that does not imply that the option prices are purely backward looking, or that they contain no useful information about the future. In any case, the relationship between implied uncertainty and realised volatility is not a precise one: as **Chart 9** shows, the rise in option-implied uncertainty since mid-2011 does not seem to reflect a rise in the realised volatility. And the use of realised volatility by investors still represents a judgement that the recent volatility in inflation is likely to persist, for the next few years at least.

Chart 9 Realised and implied standard deviation of UK RPI inflation



Sources: Bloomberg, ONS and Bank calculations.

Comparing the implied distributions with other measures of beliefs about inflation

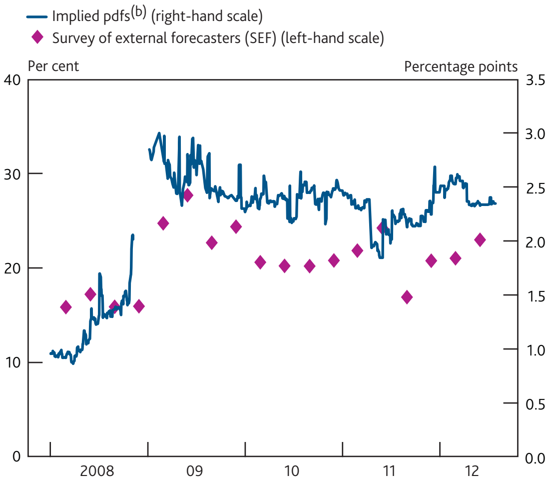
One way of checking that the messages from the implied distributions are plausible is to compare them against the messages from other measures of beliefs about inflation. At longer horizons this is not possible: there are relatively few measures of inflation expectations available beyond five-year maturities, and no measures of inflation uncertainty. And while for some surveys the underlying dispersion of individual responses is observable, this measure will reflect disagreement between individuals rather than aggregate uncertainty. But there are some comparable measures available at shorter horizons.

One such measure is available from the Bank's survey of external forecasters (SEF). As part of this survey, a sample of external forecasters are asked every three months for an assessment of the risks around their central projections for CPI inflation. **Charts 10** and **11** compare option-implied measures of uncertainty and of the balance of risks for UK RPI inflation three years ahead with measures of uncertainty and of the balance of risks around CPI inflation from the probability distributions given by that survey.

These measures are not directly comparable — the SEF measures pertain to CPI inflation, not RPI; and the SEF measures are in different units to the option-implied data.(1) But the SEF measures track the option-implied measures relatively well. As in the option-implied pdfs, uncertainty

(1) The SEF asks respondents to assign probabilities to inflation outturns falling within discrete buckets, making it hard to construct comparable measures.

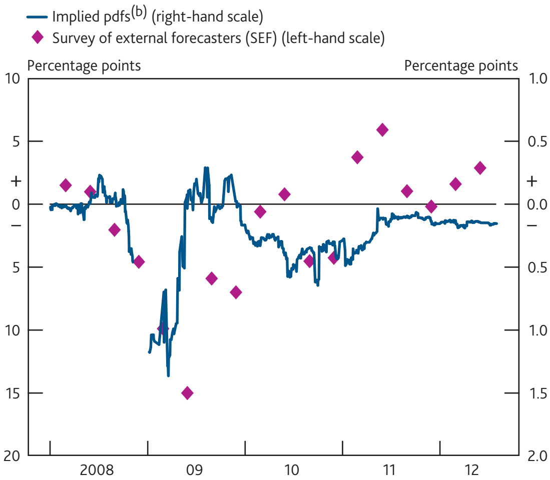
Chart 10 Measures of uncertainty about inflation three years ahead(a)



Sources: Bloomberg, ONS and Bank calculations.

(a) For SEF: average weight given by respondents to CPI inflation being more than 1 percentage point away from target. For implied pdfs: standard deviation.
(b) Missing data in late 2008 and early 2009 are due to technical problems relating to the very low level of RPI inflation — see previous section for details.

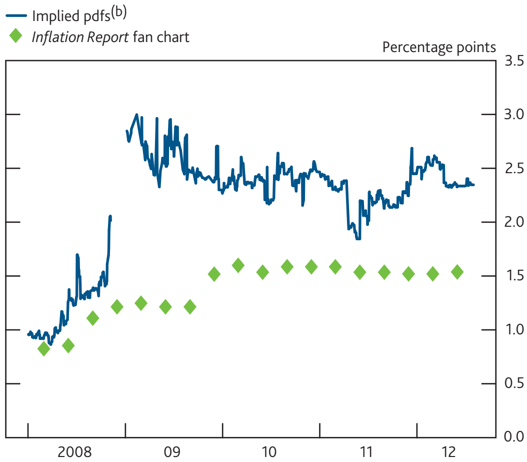
Chart 11 Measures of the balance of risks to inflation three years ahead(a)



Sources: Bloomberg, ONS and Bank calculations.

(a) For SEF: measured as net balance of average weights given by respondents to CPI inflation being more than 1 percentage point above or below target. For implied pdfs: measured as mean minus mode.
(b) Missing data in late 2008 and early 2009 are due to technical problems relating to the very low level of RPI inflation — see previous section for details.

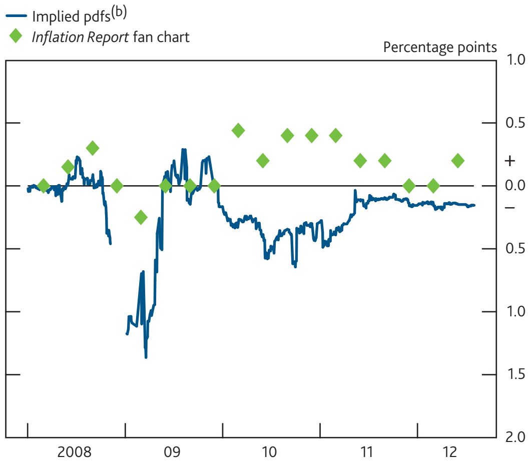
Chart 12 Measures of uncertainty about inflation three years ahead(a)



Sources: Bloomberg, ONS and Bank calculations.

(a) Measured as standard deviation for both distributions.
(b) Missing data in late 2008 and early 2009 are due to technical problems relating to the very low level of RPI inflation — see previous section for details.

Chart 13 Measures of the balance of risks to inflation three years ahead(a)



Sources: Bloomberg, ONS and Bank calculations.

(a) Measured as mean minus mode for both distributions.
(b) Missing data in late 2008 and early 2009 are due to technical problems relating to the very low level of RPI inflation — see previous section for details.

among external forecasters about inflation three years ahead rose through 2008 and 2009 and has remained relatively flat since then. And both measures of the balance of risks fell sharply in late 2008 and early 2009, rose again later in 2009 and have remained relatively stable since then.

Another measure is based on the fan charts for inflation published in the Bank's *Inflation Report*. **Charts 12** and **13** compare the option-implied three year ahead measures with the standard deviation and skewness parameter of those fan charts.(1) Again, these measures are not directly comparable. The fan charts describe CPI inflation, not RPI inflation. And the fan charts are produced using conditioning assumptions for the paths of some financial market variables, such as policy rates, whereas the implied distributions will include investor uncertainty about these variables.

The uncertainty embodied in the *Inflation Report* fan charts has also risen over the past few years, although it has increased by somewhat less than that from the option-implied distributions. That could reflect the differences between the two measures.

(1) The standard deviation of the *Inflation Report* fan chart is not strictly the same as the uncertainty parameter published by the Bank, except in the special case when the skewness parameter is zero. The difference between the two is usually small. See Wallis (1999) for details.

But, as described in the previous section, the rise in investors' actual uncertainty is also likely to have been amplified in the implied data by their desire to hedge against the worst outturns — this would not affect the *Inflation Report* fan charts.

In contrast, the balance of risk measures have evolved somewhat differently; the skewness of the MPC's forecast underlying the fan charts fell only a little in 2008–09, unlike the option-implied skewness, and in 2010–11 the two measures moved in roughly opposite directions.

Conclusion

The option-implied inflation distributions presented in this article are a valuable tool for examining people's beliefs about future inflation rates. In particular they are available at much longer time horizons than any other measures, and so provide a unique window onto investors' views about the range of possible inflation outturns for many years ahead. If the inflation option market continues to develop it may become possible to use the implied pdfs to extract much more detailed information about investors' beliefs about possible outturns for inflation, including over the MPC's forecast horizon.

The implied pdfs demonstrate that uncertainty around inflation has risen substantially since 2008 at all maturities. That does not necessarily mean that central inflation expectations have become less well anchored. Indeed, the mean of the distributions has been relatively stable over time. Instead, much of the increase in uncertainty seems likely to reflect investors' beliefs that the volatility in inflation seen since the financial crisis will persist for at least the next few years.

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Appendix
How the implied pdfs are produced

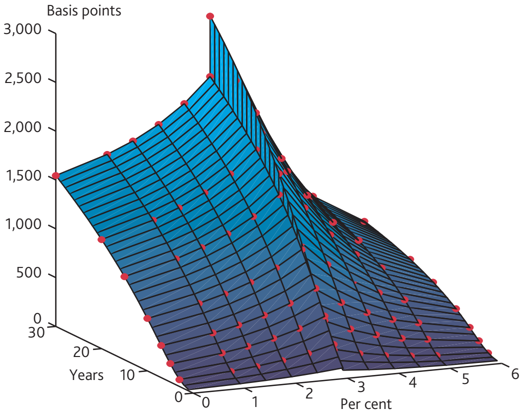
Fitting option-implied pdfs relies on a well-known result in finance, ultimately due to Breeden and Litzenberger (1978). The result relates the price $c(K)$ of a call option with strike price K and expiry date t to the probability density function f describing the value of the underlying variable at t, via the equation

$$f(K) = e^{rt} \frac{\partial^2 c(K)}{\partial K^2}$$

where r is the risk-free rate. That is, the probability density function is proportional to the second derivative of the *call-price function* with respect to the strike. For most assets, option prices are observed at a finite number of strike prices and expiry dates. Once a call-price function has been interpolated, implied pdfs can be obtained by numerically calculating the second derivative and rescaling it.(1)

For inflation, prices for annual call and put options (ie caplets and floorlets) are not directly available; instead, cap and floor prices are quoted at a small number of maturities. So calculating inflation pdfs requires the extra step of *caplet stripping*: interpolating cap and floor prices at the unobserved maturities (shown in **Chart A1**), and then recovering the caplet and floorlet prices which make up those caps.

Chart A1 Fitted and observed option prices



Sources: Bloomberg, ONS and Bank calculations.

There is no standard way to carry out this procedure. The Bank's method is as follows:

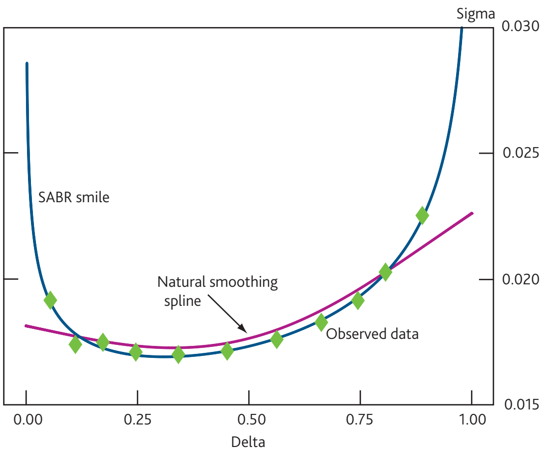
- First, convert the upfront option prices into the annual premia that would be paid if the payments were made on a per-year basis (like credit default swap contracts). This effectively standardises the option prices, stripping out the effects of contract length and the shape of the yield curve.

- Second, interpolate across time to obtain the annual premia for the missing maturities. Each interpolation is performed using a natural smoothing spline: this is a standard curve fitting technique which attempts to fit the data while keeping the curve as smooth as possible. To allow for the fact that beliefs about inflation are likely to vary more at short maturities, more deviation from smoothness is permitted at the short end of the curves.

- Finally, convert all the fitted premia back into upfront prices and calculate the associated caplet and floorlet prices. This is now straightforward: each caplet price is equal to the difference between two caps with consecutive maturities. These prices are then passed on to the standard toolkit for processing as usual.

There is one other notable difference between the technique used for these options and the standard toolkit. Before the standard toolkit interpolates across the observed prices, it transforms the price-strike pairs into *sigma-delta space*; the interpolated curve in this space is called the *volatility smile*. Like the transformation described above for the inflation option prices, this is purely for fitting convenience. However, it turns out that the natural smoothing spline usually used for interpolation does not fit the inflation option data well. A technique called a *SABR smile* gives a much better fit, as **Chart A2** shows.(2)

Chart A2 Fitting the volatility smile



Sources: Bloomberg, ONS and Bank calculations.

(1) This is the 'non-parametric method' described in more detail in Clews, Panigirtzoglou and Proudman (2000).
(2) SABR stands for Stochastic Alpha Beta Rho: the fitting method was originally derived from a stochastic volatility model for interest rates. The model is set out in Hagan *et al* (2002).